Paper Number(s): **ISE2.4**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2001

ISE PART II: M.Eng. and B.Eng.

COMMUNICATIONS 2

Tuesday, 1 May 2:00 pm

There are FOUR questions on this paper.

Answer THREE questions.

Time allowed: 2:00 hours

CORRECTED
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Examiners: Turner,L.F.

4. Figure 1 shows the constellation diagrams associated with two modems. The figure also shows the appropriate decision boundaries used by the modem receiver. The two modems operate using the same average transmitted signal power in the presence of additive noise whose sampled amplitude value, x, has a probability density function, $P(x)$, given by

$$P(x) = \frac{1}{3.4}; \quad -1.7 \le x \le 1.7$$
$$= 0; \quad \text{elsewhere.}$$

Calculate the average probability of symbol error for each modem.
From the point of view of modem design, what is the significance of your results?

You may assume that the noise affects each dimension (the X and Y directions) in a statistically independent manner and that the symbols are transmitted with equal probability.

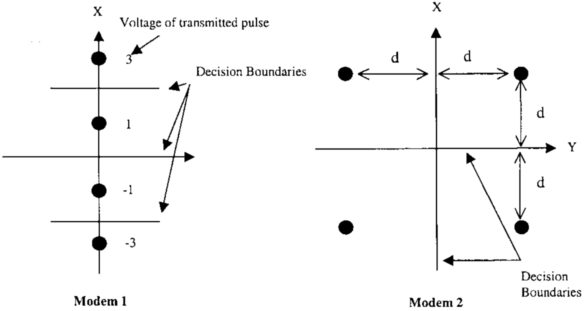


Figure 1

ISE 2.4 Solutions

marks

Q1. The single-parity check code will detect any pattern of 1 or 3 errors, So the probability of asking for a retransmission is

$$P_e = \binom{5}{1} p(1-p)^4 + \binom{5}{3} p^3(1-p)^2$$

$$P_e = 5p(1-p)^4 + 10\,p^3(1-p)^2$$

(4)

The probability that there will be i requests for retransmission and that the block will be accepted on the $(i+1)^{th}$ transmission is

$$P_e^i . (1-P_e)$$

So the average number of transmissions is

$$\bar{X} = 1(1-P_e) + 2P_e(1-P_e) + 3P_e^2(1-P_e) + \cdots + nP_e^{n-1}(1-P_e)$$

$$= (1-P_e)\left[1 + 2P_e + \cdots + nP_e^{n-1}\right]$$

Now consider $1 + 2P_e + \cdots + nP_e^{n-1}$ and let this sum be S then we have

$$S = 1 + 2P_e + \cdots + nP_e^{n-1}$$
$$P_e S = \quad P_e + \cdots + (n-1)P_e^{n-1} + nP_e^n$$

and hence $(1-P_e)S = \underbrace{1+P_e+P_e^2+\dots+P_e^{n-1}}_{\text{This is a geometric series}} - nP_e^n$ 2/8

Sum of the geometric series is $\dfrac{1-P_e^n}{1-P_e}$

So we have $S = \dfrac{1-P_e^n}{(1-P_e)^2} - \dfrac{nP_e^n}{(1-P_e)}$

and hence

$$\bar{X} = (1-P_e)S = \frac{1-P_e^n}{1-P_e} - \frac{nP_e^n}{1}$$ (13)

and since $P_e < 1$ we have in the limit as $n \to \infty$

$$\boxed{\bar{X} = \frac{1}{1-P_e}}$$

Now with $P = 0.6$ it follows that

$$P_e = 5 \times 0.6 \times 0.4^4 + 10 \times 0.6^3 \times 0.4^2$$
$$= .4224$$

and hence $\bar{X} = \dfrac{1}{1-.4224} = \underline{1.73 \text{ transmissions}}$

The rate of transmission is therefore

$$\frac{4}{1.73 \times 5} = 0.46$$ (3)

The code rate $= 4/5 = 0.8$

So overall rate $\simeq$ 1/2 code rate

Q2. The $M = 2^L$ be the number of levels in the quantizer, where L is the number of bit in the code word.

Let V_K be the mid-point of the K^{th} quantization level. — I expect a proof that this is the best position to which the quantizer value should be assigned.

marks

Let the quantization step-size (interval) be Δ.

Then for the K^{th} level the mse is

$$mse = \int_{V_K - \Delta/2}^{V_K + \Delta/2} (V - V_K)^2 P(V)\, dV.$$

$$= \int_{V_K - \Delta/2}^{V_K + \Delta/2} (V - V_K)^2 \cdot P_{V_K} \cdot dV_K$$

; P_{V_K} is pdf within level (same for all levels in this case same signal pdf instant)

$$= \frac{\Delta^3}{12} \cdot P_{V_K} = \frac{\Delta^2}{12} \cdot P_K$$

where P_K is probability of finding sample in K^{th} level.

$$\therefore \text{Overall mse} = \sum_{K=1}^{M} \frac{\Delta^2}{12} \cdot P_K = \frac{\Delta^2}{12} = N_q$$

Quantization noise

6

The quantized signal level is 4/8

$$S_q = \frac{2\Delta^2}{m} \cdot \left\{ 0^2 + 1^2 + \cdots + \left(\frac{m-1}{2}\right)^2 \right\}$$

and using the fact that $\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

we have that $S_q = \frac{2\Delta^2}{m} \left\{ \frac{m}{6} \cdot \left(\frac{m-1}{2}\right) \cdot \left(\frac{m+1}{2}\right) \right\}$

$$= \frac{\Delta^2}{12} \left\{ m^2 - 1 \right\}$$

and since $S = S_q + N_q$ we have that $S = \frac{\Delta^2}{12} \cdot m^2$ (8)

and hence $S/N_q = m^2$

Finally; $(S/N_q)\,dB's = 10 \log_{10} m^2$

and if we increase the length of the codeword by one digit we double the number of levels ie $m \to 2m$ (6)
hence

$$(S/N_q)\,dB = 10 \log_{10} (2m)^2 = 10 \log_{10} m^2 + 10 \log_{10} 4$$

$= \text{increase of } 6.02\,dB$

Q3.

Case 1 Conventional AM.

$$S_{direct} = a_1[1 + m\, s(t)]\cos\omega_c t$$

$$S_{reflected} = a_2[1 + m\, s(t-\tau)]\cos\omega_c(t-\tau)$$ marks

$$\therefore S_{point\ receiver} = a_1[1 + m\, s(t)]\cos\omega_c t$$
$$+ a_2\{1 + m\, s(t-\tau)]\cos\omega_c\tau\cos\omega_c t$$
$$+ a_2\{1 + m\, s(t-\tau)]\sin\omega_c\tau\sin\omega_c t$$

$$= \{\overbrace{a_1[1 + m\, s(t)] + a_2[1 + s(t-\tau)]\cos\omega_c\tau}^{A}\}\cos\omega_c t$$
$$+ \{\overbrace{a_2[1 + m\, s(t-\tau)]\sin\omega_c\tau}^{B}\}\cdot\sin\omega_c t$$

$$= \sqrt{\{a_1[1+m\, s(t)] + a_2[1+s(t-\tau)\cos\omega_c\tau]\}^2 + \{a_2[1+m\, s(t-\tau)\sin\omega_c\tau\}^2}$$

$$\cos(\omega_c t - \tan^{-1}\alpha(t)) \qquad \alpha(t) = \tan^{-1}\left(\frac{B}{A}\right)$$ (10)

$\therefore$ Output from envelope detector is

$$\sqrt{\{a_1[1+m\, s(t)] + a_2[1+s(t-\tau)\cos\omega_c\tau]\}^2 + \{a_2[1+m\, s(t-\tau)\sin\omega_c\tau\}^2}$$ (1)

Case II DSB-SC 6/8

$$S_{direct} = a_1 s(t) \cos \omega_c t$$

$$S_{reflected} = a_2 s(t-\tau) \cos \omega_c (t-\tau)$$

$$\therefore S_{received} = a_1 s(t) \cos \omega_c t + a_2 s(t-\tau) \cos \omega_c (t-\tau)$$

After demodulation we obtain

$$a_1 s(t) \cos^2 \omega_c t + a_2 s(t-\tau) \cos \omega_c t \cos(\omega_c t - \omega_c \tau)$$

$$= \frac{1}{2} a_1 s(t) + \frac{1}{2} a_1 s(t) \cos 2\omega_c t$$

$$+ \frac{1}{2} a_2 s(t-\tau) \cos \omega_c \tau + \frac{1}{2} a_2 s(t-\tau) \cos(2\omega_c t - \omega_c \tau)$$ 7

After low-pass filtering the resultant output from the system is

$$\boxed{\begin{aligned} S_{out} &= \frac{1}{2} a_1 s(t) + \frac{1}{2} a_2 s(t-\tau) \cdot \cos \omega_c \tau \\ &= \alpha_1 s(t) + \alpha_2 s(t-\tau) \end{aligned}}$$ 2

1 is a non-linear output

2 is linear. — this makes subsequent processing to remove echo's easier. 3

marks

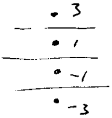
Since the modems are to use equal transmitter power it follows that

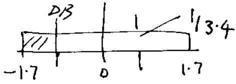
$$\frac{1}{4}\cdot 3^2+\frac{1}{4}\cdot 1^2+\frac{1}{4}(-1)^2+\frac{1}{4}(-3)^2 = 5 = 2x^2$$

$$\therefore x^2 = \frac{5}{2} \quad x = 1.58$$

Now let us calculate the probability of error in each case.

Consider modem 1





When 3 sent prob error $= (1.7 - 1) \times \frac{1}{3.4} = .21$

When 1 sent prob error $= 2 \times .21 = \underline{\underline{.42}}$

When −1 sent prob error $= .42$

When −3 sent prob error $= .21$

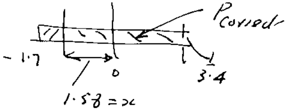
$\therefore$ Overall $P_{error} = \frac{1}{4}\{.21 + .42 + .42 + .21\}$

$= \underline{\underline{0.315}}$

(8)

Consider modem 2

Now for the symbols to be received correctly the noise must not be less than -1.58 volts in each dimension.



The probability of the noise being ~~too~~ such as not to cause error is 9

$$1 - \frac{(1.7 - 1.58)}{3.4} = \ldots = .96$$

$\therefore$ Prob of correct detection $= .96^2 = \underline{\underline{0.93}}$

Hence error probability $= 1 - .93 = \underline{\underline{0.07}}$

Comparison of the two results shows that the second modem makes much better use of the transmitted power. 3