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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2007

EEE PART IV: MEng and ACGI

Corrected Copy

Q3, Q6

POWER SYSTEM ECONOMICS

Friday, 27 April 2:30 pm

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer TWO questions from Section A and TWO questions from Section B.

All questions carry equal marks.

Use separate answer books for Sections A and B.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : C.A. Hernandez-Aramburo, G. Strbac

Second Marker(s) : G. Strbac, C.A. Hernandez-Aramburo

Section A. Answer any two questions from this section.

Question 1

a) Explain what "services unbundling" means in the electricity supply industry, why it is necessary to carry it out to introduce competition in a monopoly, and list some of its characteristics.

[4]

b) Briefly describe the roles and objectives of the following bodies in a competitive electricity supply industry.

i) Independent System Operator

[4]

ii) Market Regulator

[4]

iii) Market Operator

[4]

iv) Distribution companies

[4]

Question 2

a) Consider a generic cost function C(q) to produce an output q of a certain product. Demonstrate that the marginal cost curve for this product intersects the average cost curve at the minimum value of the latter.

[6]

b) The inverse supply and demand functions of a certain product are modelled as follows:

Inverse demand function: $\pi = -0.1q + 400$ [£/unit]

Inverse supply function: $\pi = 0.0001q^2 - 0.05q + 100$ [£/unit]

i) Determine the price at which the social welfare is maximised.

[5]

ii) For market conditions of $\pi = 300$ [£/unit] and q=1000 [units]; calculate the following:

- Producers' revenues and profits

[3]

- Consumers' net and gross surplus

[3]

- Social welfare and welfare loss

[3]

Question 3

Consider the case of a certain company that owns two generating units, A and B. The cost functions of these generating units are as follows:

$C_A = 10+2.1P_A+0.04P^2_A$ [£/h]

$C_B = 20+1.8P_B+0.03P^2_B$ [£/h]

Where $P_{A,B}$ represents the power produced by units A and B, correspondingly.

The demand that this company must meet throughout this question is 300MW.

a) Find the optimal dispatch setting for each unit if the total cost is to be minimised and calculate the marginal cost of production.

[6]

b) Assume now that the generating company can buy and sell energy from and to the market. Currently, the sell price is π_{sell}=11.00 [£/MWh] and the buy-price is π_b =11.50 [£/MWh]. Calculate the optimal dispatch of the generating units, and the amount of power to be sold to (or bought from) the market. Briefly explain the reasoning behind your answer.

[6]

c) Consider now that the company is a position to sell energy to the market at a price of π_s. This time it must be taken into consideration that the maximum capacity for unit A is 160MW; and for unit B, 170MW. Using the Lagrange multipliers technique, do the following:

- State the constrained optimisation problem to be solved to ~~minimi~~se the company's ~~costs~~. maximise profits
- Form the Lagrangian function (i.e., state the un-constrained optimisation formulation).
- Find the optimality conditions.

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[6]

d) Assume that unit A develops a fault and must be taken off-line, and that unit B does not have the capacity to fulfil the company's generation commitment at a particular point in time. For a certain reason, during this time the company was unable to buy any energy from the market. Explain how the company will be penalised if it is taking part in the UK's BETTA.

[2]

Section B. Answer any two questions from this section.

Question 4

a) In a competitive electricity market, transmission is often separated from the other components of a traditional vertically integrated utility. List and briefly describe four characteristics transmission as a stand-alone business. [2]

b) The supply functions for the electricity markets in Borduria and Syldavia (see figure 4.1) are respectively:

$$\pi_B = MC_B = 6 + 0.011P_B$$

$$\pi_S = MC_S = 10 + 0.022P_S$$

i) Calculate locational marginal prices (LMPs) and congestion surplus for a transmission capacity of 150MW. [3]

ii) What do LMPs represent? [2]

iii) What is the meaning of the differences in LMPs across the two busbars? [2]

c) Assume that the existing transmission link between the two markets in part (b) is decommissioned and a merchant transmission developer is examining the economic viability of building a 750 km HVDC interconnection between Borduria and Syldavia power systems (see Figure 4.1). The pricing for transmission will based on its value, i.e. on the difference in short-run marginal energy prices between Syldavia and Borduria $[\pi_T(F)]$. Answer the following questions:

i) Show that the transmission demand function is given by:

$$F(\pi_T) = 988 - 30.3\pi_T$$

[3]

ii) Plot the transmission revenue as a function of the capacity built. At what capacity does the revenue reach its maximum? [2]

iii) Given that the annuitised cost of transmission investment can be approximated by:

$$C(F) = 25{,}000{,}000 + 90{,}000 \cdot F \quad \text{£/year}$$

determine the optimal capacity that should be built, by balancing the short term value of transmission with the long term cost of transmission. [4]

iv) If this capacity were to be built, how much revenue per annum would be generated? [2]

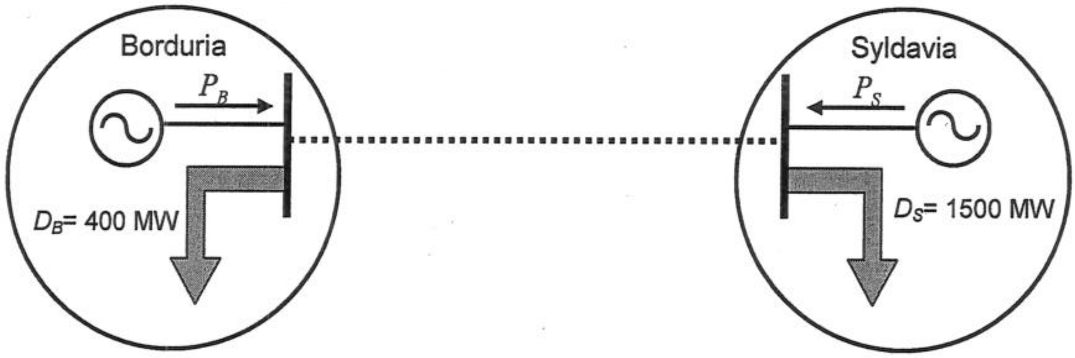

Figure 4.1. Model of the Borduria/Syldavia interconnection

Question 5

a) What are the objectives for pricing the use of transmission networks? [4]

b) Syldavia Power operates two regions of a small power system that are not connected. Generators 1 and 2 are located in the Northern Region while generators 3 and 4 are located in the Southern Region. The load in the Northern Region is 100 MW and the load in the Southern Region is 420 MW. The marginal cost of these generators are:

Northern Region

$$MC_1 = 3 + 0.02P_1 \quad [£/MWh]$$

$$MC_2 = 4 + 0.04P_2 \quad [£/MWh]$$

Southern Region

$$MC_3 = 3.6 + 0.025P_3 \quad [£/MWh]$$

$$MC_4 = 4.2 + 0.025P_4 \quad [£/MWh]$$

Calculate the marginal costs in both regions and the corresponding generation dispatches. [4]

c) Miranda Power is considering building a 450km long transmission link between the two regions described in part (b). The annuitsied investment cost of transmission (including the allowable profit) is 37£/MW.km.year. The local consultant has proposed two schemes to be considered: (a) 80 MW and (b) 150MW link. For both schemes calculate:

i) marginal prices in the Northern and the Southern region [3]

ii) generator payments, demand charges and congestion surplus [2]

iii) network revenues if the transmission company charges for the use of link on the basis of short-run marginal cost [3]

d) Calculate the optimal transmission capacity required to connect the two regions of part (b). In this calculation use the fact that at the optimum, the short-run marginal cost of transmission equals the long-run marginal cost of transmission. [4]

Question 6

a) Answer the following questions:

i) Why do short-term electricity prices vary with location? [2]

ii) What is the objective of point-to-point transmission congestion contracts? [2]

iii) How congestion surplus is allocated using among users of transmission network? [2]

b) For the system shown in Figure 6.1, and the companion data given in Tables 6.1, 6.2 and 6.3, answer the following questions:

i) Ignoring the impact of transmission constraints, calculate the minimum output of the generators the corresponding network flows, and show that branch 1-2 is overloaded. [2]

ii) What is the minimum increase in generation at 3 required to eliminate this overload? [3]

iii) Show that the marginal price at 3 is 11.25£/MWh. [3]

iv) In order to manage its exposure to price fluctuations, Load at bus 2 has:

- made a contract for difference with generation at 1 at a strike price of 9.5 £/MWh, and
- purchased a transmission congestion contract that gives the right to transport 60MW from bus 1 to bus 2.

Calculate the income from the transmission congestion contract received by the load at bus 2 and calculate the unit price that demand will pay for the energy consumed under this condition. [3]

v) Because of the maintenance on one of the lines of the transmission circuit between nodes 2 and 3, the flow through this circuit has to be restricted to 65MW. Show that the price at node 2 will drop to 5£/MWh. How much the demand at bus 2 will now pay for the electricity consumed, given the new conditions its contract portfolio? [3]

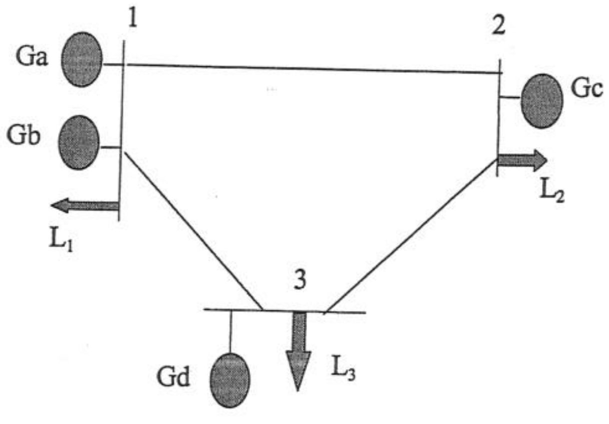

Figure 6.1: The system layout

Table 6.1 Load data

Node	Load (MW)
1	50
2	60
3	300

Table 6.2 Generator data

Generator	Capacity (MW)	Marginal cost (£/MWh)
Ga	120	7.5
Gb	285	6
Gc	90	14
Gd	85	10

Table 6.3 Line data

Line	Per unit reactance	Capacity (MW)
1-2	0.2	126
1-3	0.2	250
2-3	0.1	100

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Part A - model answers 2007

1. Answer the following questions in relation to the electricity supply industry.

[This material to answer this question is included in the students' hand-outs]

a) Explain what "services unbundling" means, why it is necessary to carry it out to introduce competition in a monopoly and list some of its characteristics.

Unbundling:

Generation, transmission, distribution and retail functions are separated from a single body and performed by different companies. Services unbundling is essential to make competition work

Characteristics:

- **One participant should not be able to prevent others from competing**
- **Management of the network or system should be done independently from sale of energy**
- **One company should not be able to prevent others from competing using congestion in the network**
- **"Open access" to the transmission network**
- **Separation of "energy businesses" from "wires businesses"**
- **Energy businesses become part of a competitive market**
- **Wire businesses remain monopolies**

[4 marks]

b) Briefly describe the roles and objectives of the following bodies in a competitive electricity supply industry.

i) Independent System Operator

Roles:

- **Maintains the security of the system**
- **Should be independent from other participants to ensure the fairness of the market**
- **Usually runs the market of last resort**
- **Balance the generation and load in real time**
- **Owns only computing and communication assets**

- **An Independent Transmission Company (ITC) is an ISO that also owns the transmission network**

Objectives:

- **Ensure the security of the system**
- **Maximize the use that other participants can make of the system**

[4 marks]

ii) Market Regulator

It is usually a Government body with the following roles:

- **Determines or approves market rules**
- **Investigates suspected abuses of market power**
- **Sets the prices for products and services provided by monopolies**

Objectives

- **Make sure that the electricity sector operates in an economically efficient manner**
- **Make sure that the quality of the supply is appropriate**

[4 marks]

iii) Market Operator

Roles:

- **Runs the computer system that matches bids and offers submitted by buyers and sellers of electrical energy**
- **Runs the market settlement system**
- **Monitors delivery of energy**
- **Forwards payments from buyers to sellers**
- **Market of last resort run by the System Operator**
- **Forward markets often run by private companies**

Objective:

- **Run an efficient market to encourage trading**

[4 marks]

iv) Distribution companies

Role

- **Owns and operates distribution network (referred also as "Distribution Network Operator" or DNO)**
- **Traditional environment:**
- **Monopoly for the sale of electricity to consumers in a given geographical area**
- **Competitive environment:**
- **Network operation and development function separated from sale of electrical energy**
- **Remains a regulated monopoly**

Objective:

- **Maximize regulated profit**

[4 marks]

Question 2.

[This question tests the understanding of annual average cost and marginal costs, and sets numerical examples of the theory covered during lectures]

a) Consider a generic cost function C(q) to produce an output q of a certain product. Demonstrate that the marginal cost curve for this product intersects the average cost curve at the minimum value of the latter.

The average cost curve $AC(q)$ is defined as $AC(q) = \frac{C(q)}{q}$

At its minimum point $\frac{d}{dq}(AC(q)) = 0$ [This assumes that it is a convex curve, which is a valid assumption in PSE]

Therefore, $\frac{d}{dq}\left[AC(q)\right] = \frac{d}{dq}\left[\frac{C(q)}{q}\right] = 0$

$$\frac{q\frac{d}{dq}C(q) - C(q)}{q^2} = 0$$

This leads to $\frac{d}{dq}C(q) = \frac{C(q)}{q}$

The marginal cost function is defined as $\frac{d}{dq}C(q)$, therefore:

$$MC(q) = AC(q)$$

This means that the marginal cost and the average cost have the same value (i.e., intersect) at the minimum point of the latter

[6 marks]

b) The inverse supply and demand functions of a certain product are modelled as follows:

Inverse demand function: $\pi = -0.1q + 400$ [£/unit]

Inverse supply function: $\pi = 0.0001q^2 - 0.05q + 100$ [£/unit]

i) Determine the price at which the social welfare is maximised.

Inverse supply and demand functions:

$$\begin{cases} \pi = -0.1q + 400 & \text{......(1)} \\ \pi = 0.0001q^2 - 0.05q + 100 & \text{...(2)} \end{cases}$$

The social welfare is maximum at the market equilibrium point.

From (1): $q = \dfrac{400-\pi}{0.1}$

Substitute this into (2):

$$\pi = 0.0001\left[\frac{400-\pi}{0.1}\right]^2 - 0.05\left[\frac{400-\pi}{0.1}\right] + 100$$

$$\pi = 0.01\left[160\,000 - 800\pi + \pi^2\right] - 200 + 0.5\pi + 100$$

$$\pi = 1600 - 8\pi + 0.01\pi^2 + 0.5\pi - 100$$

$$\Rightarrow \quad 0.01\pi^2 - 8.5\pi + 1500 = 0$$

$$\pi = \frac{8.5 \pm \sqrt{(-8.5)^2 - 4(0.01)(1500)}}{2(0.01)} = \frac{8.5 \pm 3.5}{0.02}$$

$\pi_1 = \dfrac{12}{0.02} = 600$

$\pi_2 = \dfrac{5}{0.02} = 250$

Only π_2 produces a positive q (according to (2))

$\therefore \pi_2 = 250$

and $q = \dfrac{250-400}{-0.1} = 1500$

[5 marks]

ii) For market conditions of $\pi = 300$ [£/unit] and q=1000 [units]; calculate the following:

- Producers' revenues and profits

Producers' revenue: (£3000/unit)(1000 units) = £ 300 000

Producers' costs:

$$= \int_0^{1000} (0.0001\, q^2 - 0.05 q + 100)\, dq = 0.0001 \frac{q^3}{3} - 0.05 \frac{q^2}{2} + 100 q \Big|_0^{1000}$$

= £ 108,333.33

Producers' profits = Revenues − Costs = £300 000 − £ 108333.33

= £ 191 666.67

[3 marks]

- Consumers' net and gross surplus

From the diagram

Consumers' net surplus:

= Area A

$$= \frac{(400-300)(1000)}{2}$$

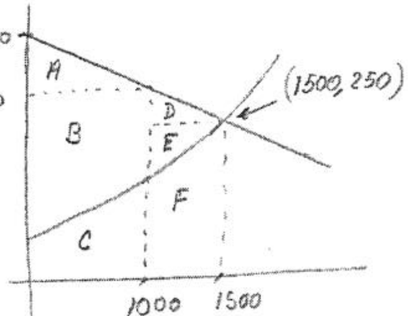
= £50000

Consumers' gross surplus:

= Areas A + B + C

= £50000 + (300)(1000)

= £ 350, 000



[3 marks]

- Social welfare and welfare loss

Social welfare = consumers' net surplus + producers' profit

= £50 000 + £191 666.67

= £241 666.67 ✓

From the diagram above:

Welfare loss = Areas (D + E)

Area (E + F) = (1500 − 1000)(250) = £125,000

$$\text{Area }(D) = \frac{(1500-1000)(300-250)}{2} = \frac{(500)(50)}{2} = £12{,}500$$

Area (F) =

$$= \int_{1000}^{1500} (0.0001q^2 - 0.05q + 100)\,dq = \left. 0.0001\frac{q^3}{3} - 0.05\frac{q^2}{2} + 100q \right|_{1000}^{1500}$$

= £97, 916.67

Welfare loss = D + (E+F) − F =

= 12 500 + 125 000 − 97 916.67 =

= £39 583.33 ✓

[3 marks]

Question 3

[This is a set of numerical problems modified from some examples covered in lectures]

Consider the case of a certain company that owns two generating units, A and B. The cost functions of these generating units are as follows:

$C_A = 10+2.1P_A+0.04P^2_A$ [£/h]

$C_B = 20+1.8P_B+0.03P^2_B$ [£/h]

Where $P_{A,B}$ represents the power produced by units A and B, correspondingly.

The demand that this company must meet throughout this question is 300MW.

a) Find the optimal dispatch setting for each unit if the total cost is to be minimised and calculate the marginal cost of production.

The marginal cost functions are

$MC_A = 2.1 + 0.08 P_A$ [£/MWh]

$MC_B = 1.8 + 0.06 P_B$ [£/MWh]

For optimality $MC_A = MC_B$ therefore

$2.1 + 0.08 P_A = 1.8 + 0.06 P_B$

$0.08 P_A - 0.06 P_B + 0.3 = 0 \quad \cdots\cdots (1)$

The constraint to meet the demand imposes $P_A + P_B = 300 \quad ..(2)$

$P_B = 300 - P_A$ into (1): $0.08 P_A - 0.06(300 - P_A) + 0.3 = 0$

$0.14 P_A - 18 + 0.3 = 0 \Rightarrow P_A = \frac{17.7}{0.14} = 126.429$ MW

$P_B = 300 - 126.429 = 173.571$ MW

$MC = 2.1 + (0.08)(126.429) = 12.21$ [£/MWh]

[6 marks]

b) Assume now that the generating company can buy and sell energy from and to the market. Currently, the sell price is π_{sell}=11.00 [£/MWh] and the buy-price is π_b =11.50 [£/MWh]. Calculate the optimal dispatch of the generating units, and the amount of power to be sold to, or bought from, the market. Briefly explain the reasoning behind your answer.

According to the current market prices and the required level of production, it is cheaper to buy some electricity from the market (at 11.50 £/MWh) than to produce it (at £12.21 £/MWh, calculated from part (a)). Therefore the company should reduce its current production level until the marginal cost is equal to the buy-price and buy its energy deficit from the market.

$$P_A = \frac{MC-2.1}{0.08} = \frac{11.5-2.1}{0.08} = 117.500 \text{ MW}$$

$$P_B = \frac{MC-1.8}{0.06} = \frac{11.5-1.8}{0.06} = 161.667 \text{ MW}$$

$$P_{deficit} = 300-(117.500+161.667) = 20.833 \text{ MW}$$

to be bought from the market

[6 marks]

c) Consider now that the company must meet the 300MW demand and it is in position to sell energy to the market at a price of π_s. This time it must be taken into consideration that the maximum capacity for unit A is 160MW; and for unit B, 170MW. Using the Lagrange multipliers technique, do the following:

- State the constrained optimisation problem to be solved to minimise the company's costs.

Objective function: $\text{Min}\{C_A + C_B - \pi_s P_s\}$

Subject to: $P_A + P_B - P_s = 300$

$P_A \leq 160$

$P_B \leq 170$

[2 marks]

- Form the Lagrangian function (i.e., state the un-constrained optimisation formulation).

The Lagrangian function may be defined as follows:

$$\ell(P_A, P_B; P_s, \lambda, \mu_1, \mu_2) = \left[C_A(P_A) + C_B(P_B) - P_s \pi_s\right] + \lambda(300 - P_A - P_B + P_s)$$
$$+ \mu_1 (P_A - 160)$$
$$+ \mu_2 (P_B - 170)$$

[2 marks]

- Find the optimality conditions.

Optimality conditions:

$$\frac{\partial \ell}{\partial P_A} = \frac{dC_A}{dP_A} - \lambda + \mu_1 = MC_A - \lambda + \mu_1 = 0$$

$$\frac{\partial \ell}{\partial P_B} = \frac{dC_B}{dP_B} - \lambda + \mu_2 = MC_B - \lambda + \mu_2 = 0$$

$$\frac{\partial \ell}{\partial P_S} = -\pi_S + \lambda = 0$$

$$\frac{\partial \ell}{\partial \lambda} = 300 - P_A - P_B + P_S = 0$$

$$\frac{\partial \ell}{\partial \mu_1} = P_A - 160 \leq 0$$

$$\frac{\partial \ell}{\partial \mu_2} = P_B - 170 \leq 0$$

[2 marks]

d) Assume that unit A develops a fault and must be taken off-line and that unit B does not have the capacity to fulfil the company's generation commitment at a particular point in time. For a certain reason, during this time the company was unable to buy any energy from the market. Explain how the company will be penalised if it is taking part in UK's BETTA.

Because of the fault, the company will fall short of generation. Under BETTA, the generator must pay the energy deficit at the system price (more specifically the system buy price), which is calculated according to the balance settlement code.

[2 marks]

Part B – Model answers

SOLUTION to Question 4:

a) For full mark, four of the following characteristics should be explained:

Transmission is a natural monopoly. In exchange for being granted a regional monopoly, a transmission company must accept that the regulatory authorities will determine its revenues. The biggest risk that these companies face is the regulatory risk, i.e. the risk that a change in regulatory principles or practices may decrease their allowed revenues.

Transmission is a capital-intensive business

The cost of these investments is high compared to the recurring cost of operating the system. Making good investment decisions is thus very important aspect of running a transmission company.

Transmission assets have a long life

Most transmission equipment is designed for an expected life ranging from twenty to forty years or even longer. Changes over such a long period are inevitable, including location and characteristics of generation plants due to changes in the cost of fuel or because of the emergence of a competitive technology. Furthermore, economic development may shift the geographical distribution of demand. A transmission line that was built on the basis of erroneous forecasts may therefore be used at only a fraction of its rating.

Transmission investments are irreversible

Once a transmission line has been built, it cannot generally be redeployed to another location The resale value of installed assets is very low.

Transmission investments are lumpy

Transmission equipment is design in a small number of standardised voltage and MVA ratings.

Economies of scale: transmission networks involve important economies of scale due to large fix cost component involved in building transmission circuits. [2]

b)

i) $\pi_B = 6 + 0.011 \text{x } (550) = 12.05$ £/MWh

$\pi_S = 10 + 0.022 \text{ x } (1350) = 39.7$ £/MWh

$E_{total} - R_{total} = (\pi_S - \pi_B) \text{ x } F_{BS} = 4147.5$ £/h [3]

ii) LMP - marginal cost depends on the location where the energy is produced or consumed. [2]

iii) The difference in LMP across two bus system represents the value of transmission [2] c)

i)

The demand function for transmission gives the value of transmission in terms of the amount of power F transmitted between Borduria and Syldavia:

$$\pi_T(F) = \pi_S(F) - \pi_B(F)$$

where $\pi_T(F)$ is the value of the transmission. The prices of electrical energy in Syldavia and Borduria, $\pi_S(F)$ and $\pi_B(F)$ are expressed in terms of the power transmitted:

$$\pi_T(F) = 10 + 0.022P_S(F) - 6 - 0.011P_B(F)$$

The production of the generators in Borduria and Syldavia can be expressed in terms of the flow on the interconnection and the local demands as follows:

$$P_B(F) = D_B + F$$

$$P_S(F) = D_S - F$$

Hence

$$\pi_T(F) = 32.6 - 0.033F$$

When the flow is equal to zero, the price of transmission is 32.6 £/MWh. The transmission price drops to zero when the flow reaches 988 MW, which is the value of the flow for which the prices of generation in Borduria and Syldavia are equal.

The above expression can be inverted to get the demand for transmission as a function of its price:

$$F(\pi_T) = 988 - 30.3\pi_T$$ [3]

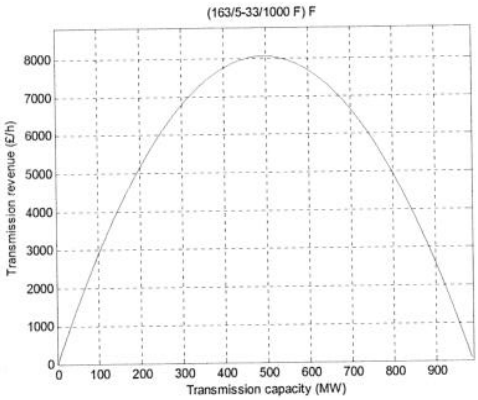
ii)

The revenue is given by the following expression:

$$R(F) = (32.6 - 0.033F) \cdot F$$

This is a quadratic function

Maximum revenue is achieved for F = 494MW.



[2]

iii)

Given the cost of transmission

$$C(F) = 25{,}000{,}000 + 90{,}000 \cdot F$$

Marginal transmission cost is

C'(F) = 90,000 £/a, or 10.27 £/h

Equating

$\pi_T(F) = 32.6 - 0.033F = 10.27 \Rightarrow F = 677\text{MW}$ [4]

iv) Revenue

Variable revenue Rv = 677x10.27x8760= 60,906,440

Cost = 25m+60.9m=85.9m

Hourly cost

C(h) = 9805.9£/h

Hourly revenue obtained:

$R(F) = (32.6 - 0.033F) \cdot F$ [2]

SOLUTION to Question 5

a) One of the consequences of the deregulation process has been separation of generation from transmission. This separation is indeed frequently considered indispensable to achieve open and nondiscriminatory access to the energy market. In this environment, pricing of transmission became the key to achieving both efficient operation and least-cost system development of the entire system. Coordination of investment in generation and transmission, which are now operated as separate entities, is to be achieved through efficient network pricing mechanisms. [4]

b) North region

$MC_1 = MC_2 = 4.67$ £/MWh

$P_1 = 83$ MW

P_2=17 MW

South region

MC_3=MC_4= 9.15 £/MWh

P_3= 222 MW

P_4= 198 MW [4]

c)

i) Case 80 MW

MC_1=MC_2= 5.73 £/MWh

MC_3=MC_4= 8.15 £/MWh

Case 150 MW

MC_1=MC_2= 6.67 £/MWh

MC_3=MC_4= 7.27 £/MWh [3]

ii) Generator payment (case 80 MW)

G1= P_1x MC_1= 783.5 £/h

G2= P_2 x MC_2= 248.4 £/h

G3= P_3 x MC_3= 1483.3 £/h

G4= P_4 x MC_4= 1288 £/h

Demand charges (case 80 MW)

D1= D_1 x MC1=573.3 £/h

D2= D_2 x MC_2 =3424 £/h

Congestion surplus (case 80 MW)

$$\begin{aligned}
&E_{TOTAL} - R_{TOTAL} = \pi_S \cdot D_S + \pi_N \cdot D_N - \pi_S \cdot P_S - \pi_N \cdot P_N \\
&= \pi_S \cdot (D_S - P_S) + \pi_N \cdot (D_N - P_N) \\
&= \pi_S \cdot F_{NS} + \pi_N \cdot (-F_{NS}) \\
&= (\pi_S - \pi_N) \cdot F_{NS} \\
&= (8.15 - 5.73) \cdot 80 = 193.3£ / h
\end{aligned}$$

Generator payment (case 150 MW)

G1= P_1x MC_1= 1222 £/h

G2= P_2 x MC_2= 444.4 £/h

G3= P_3 x MC_3= 1069 £/h

G4= P_4 x MC_4= 894.8 £/h

Demand charges (case 150 MW)

D1= D_1 x MC_1= 666,67 £/h

D2= D_2 x MC_2 = 3055 £/h

Congestion surplus (case 150 MW)

$$E_{TOTAL} - R_{TOTAL} = \pi_S \cdot D_S + \pi_N \cdot D_N - \pi_S \cdot P_S - \pi_N \cdot P_N$$
$$= \pi_S \cdot (D_S - P_S) + \pi_N \cdot (D_N - P_N)$$
$$= \pi_S \cdot F_{NS} + \pi_N \cdot (-F_{NS})$$
$$= (\pi_S - \pi_N) \cdot F_{NS}$$
$$= (7.27 - 6.67) \cdot 150 = 91.25 £ / h$$

[2]

iii) Network revenues

Case 80 MW

Network revenues = (MC_3-MC_1) x 80 = 193.3 £/h

Case 150 MW

Network revenues = (MC_3-MC_1) x 150 = 91.25 £/h [3]

d) Optimal transmission capacity

Long run transmission marginal cost

$$LRMC = \frac{450 \cdot 37}{8760} = 1.9 £ / MWh$$

LRMC for capacity 80 MW = 8.15-5.73=2.42 £/MWh
LRMC for capacity 150 MW = 7.27–6.67=0.6 £/MWh

It gives the optimal transmission capacity of 100 MW. The detailed calculation is given below. [4]

$\pi_T(F) = \pi_S(F) - \pi_N(F)$
$\pi_N(F) = 3 + 0.02 \cdot P_1$
$\pi_N(F) = 4 + 0.04 \cdot P_2$
$P_1 + P_2 = D_N + F = 100 + F$
$\pi_S(F) = 3.6 + 0.025 \cdot P_3$
$\pi_S(F) = 4.2 + 0.025 \cdot P_4$
$P_3 + P_4 = D_S - F = 420 - F$
$\pi_S(F) - \pi_N(F) = 1.9$
$\pi_S(F) = 1.9 + \pi_N(F)$
$1.9 + \pi_N(F) = 3.6 + 0.025 \cdot P_3$
$1.9 + \pi_N(F) = 4.2 + 0.025 \cdot P_4$
Solution:
$\pi_N(F) = 1.7 + 0.025 \cdot P_3$
$\pi_N(F) = 2.3 + 0.025 \cdot P_4$
$\pi_N(F) = 3 + 0.02 \cdot P_1$
$\pi_N(F) = 4 + 0.04 \cdot P_2$

$P_1 = 100 + F - P_2$
$\pi_N(F) = 3 + 0.02 \cdot (100 + F - P_2) = 5 + 0.02F - 0.02P_2$
from $\pi_N(F) = 4 + 0.04 \cdot P_2$ *we get* $P_2 = \dfrac{\pi_N(F) - 4}{0.04}$
$\pi_N(F) = 5 + 0.02F - 0.02(\dfrac{\pi_N(F) - 4}{0.04}) = 5 + 0.02F - 0.5\pi_N(F) + 2 = 7 + 0.02F - 0.5\pi_N(F)$
$1.5\pi_N(F) - 0.02F = 7$

from $P_3 + P_4 = D_S - F = 420 - F$ *we get* $P_3 = 420 - F - P_4$
and substitute *in* $\pi_N(F) = 1.7 + 0.025 \cdot P_3$ *so we get* $\pi_N(F) = 1.7 + 0.025 \cdot (420 - F - P_4) = 12.2 - 0.025F - 0.025P_4$
from equation $\pi_N(F) = 2.3 + 0.025 \cdot P_4$ *we get* $P_4 = \dfrac{\pi_N(F) - 2.3}{0.025}$
if we substitute *in* $\pi_N(F) = 12.2 - 0.025F - 0.025P_4 = 12.2 - 0.025F - 0.025(\dfrac{\pi_N(F) - 2.3}{0.025})$ *we get*
$2\pi_N(F) + 0.025F = 14.5$
We need to solve the following system equation:
$1.5\pi_N(F) - 0.02F = 7$
$2\pi_N(F) + 0.025F = 14.5$
From equation $1.5\pi_N(F) - 0.02F = 7$ *we express* $\pi_N(F) = \dfrac{7 + 0.02F}{1.5}$ *and* substitute *in* $2\pi_N(F) + 0.025F = 14.5$
we get $2 \cdot (\dfrac{7 + 0.02F}{1.5}) + 0.025 \cdot F = 14.5$. *or* $F \cdot (0.0775) = 7.75$
$F = 100MW$

SOLUTION to Question 6.

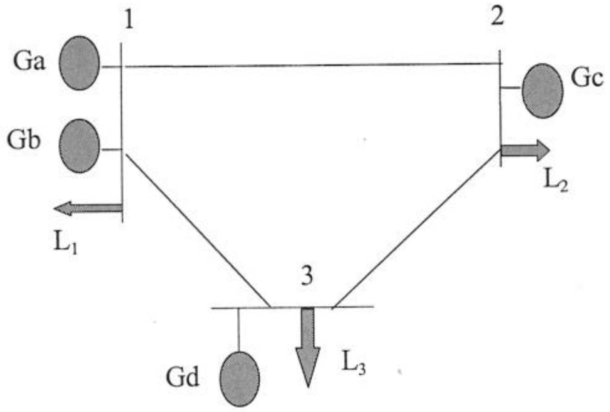


Figure 6.1: The system layout

(a)

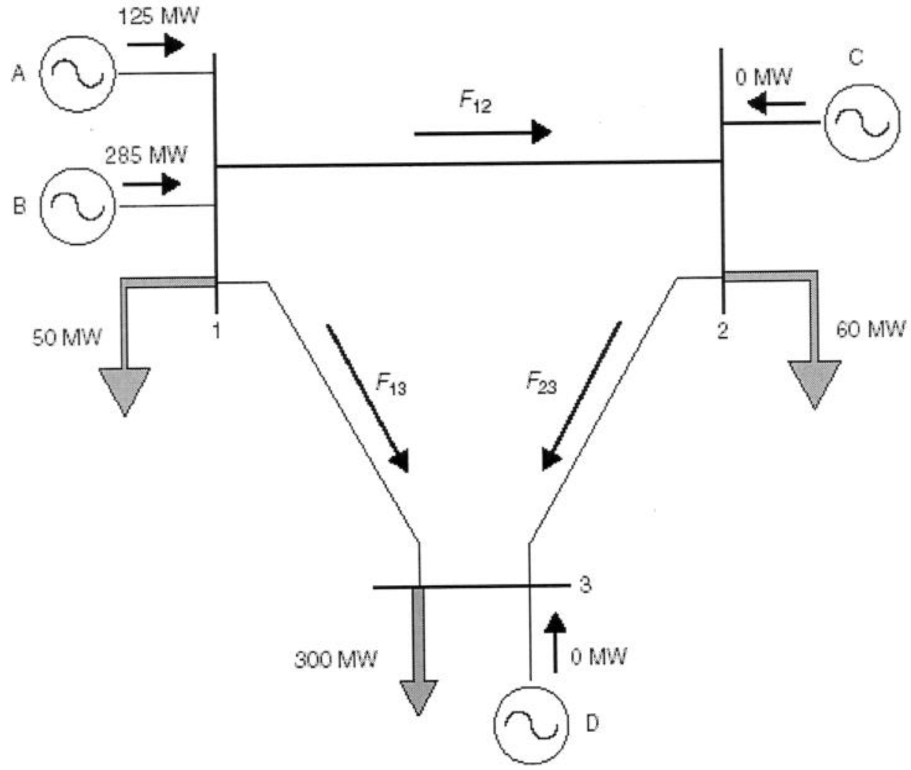
i) Presence of losses and transmission constraints make electricity prices across a transmission network location specific. [2]

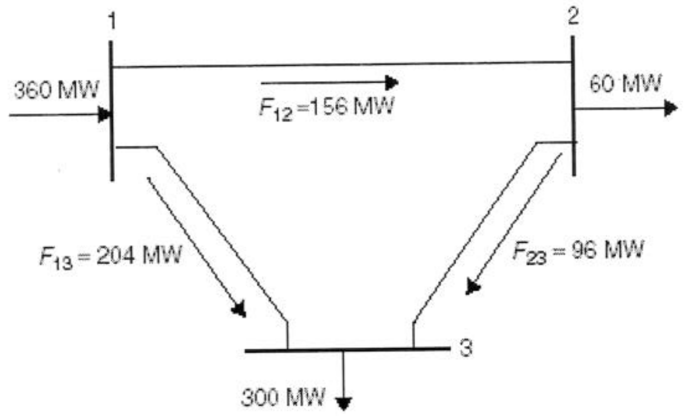
ii) The objective of the point to point TCC is to enable users of transmission network, that are located at various locations, to enter in the contracts for differences and protect them from the variations in energy prices caused by transmission congestions. [2]

iii) The congestion surplus allocated using among users of transmission network with respect of the value of their TCC (this is equal to the product for the difference in prices between two locations (direction must be respected) and the amount of power contracted for. [2]

(b)

i)





[2]

ii)

Given the impedances of the network, the sensitivity of flow 1-2 with respect to injection in 3 is 0.4. In other words, 1 MW injected at 3 and taken out from 1, will reduce the flow in 1-2 for 0.4 MW. Hence it is necessary to increase the output at 3 for 75 MW to reduce the flow in 1-2 for 30MW [3]

iii)

Price at 3: One more MW of consumption at 3 will be supplied from buses 1 and 3:

DP1+DP3 = 1

Respecting the sensitivities of the flow in 1-2 with respect to injections at 1 and 3 and imposing the requirements for no change in the flow:

0.6xDP1-0.2xDP3 = 0 (both injections increase the flow through 1-2)

Solving this we obtain:

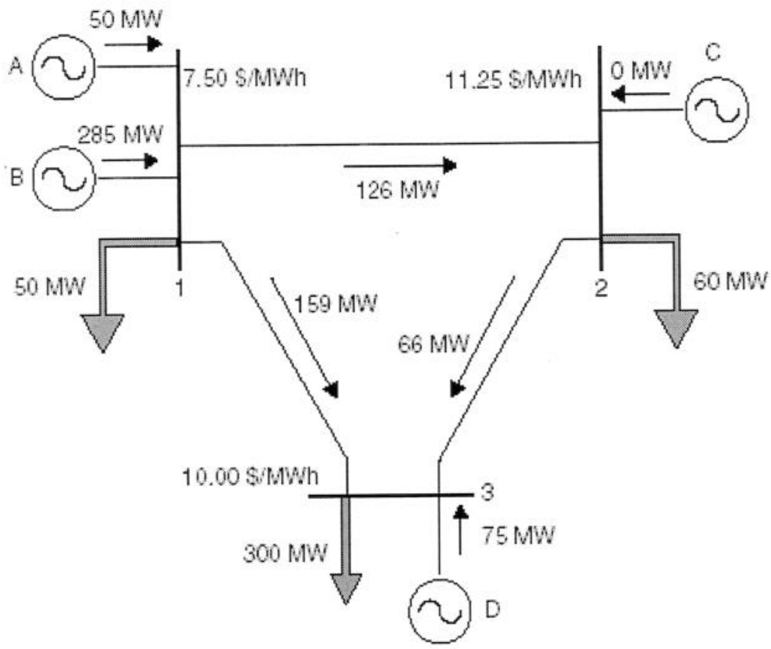
DP1 = - 0.5 MW

DP3 = 1.5 MW

Hence locational energy price at 2 is:

1.5x10-0.5x7.5 = 11.25£/MWh.

This is shown in Figure below:



[3]

iv) Demand pay to the pool 11.25£/MWh. Expenditure: 11.25x60 = 675£/h

Generator at 1 receives 7.5x60 = 450£/h

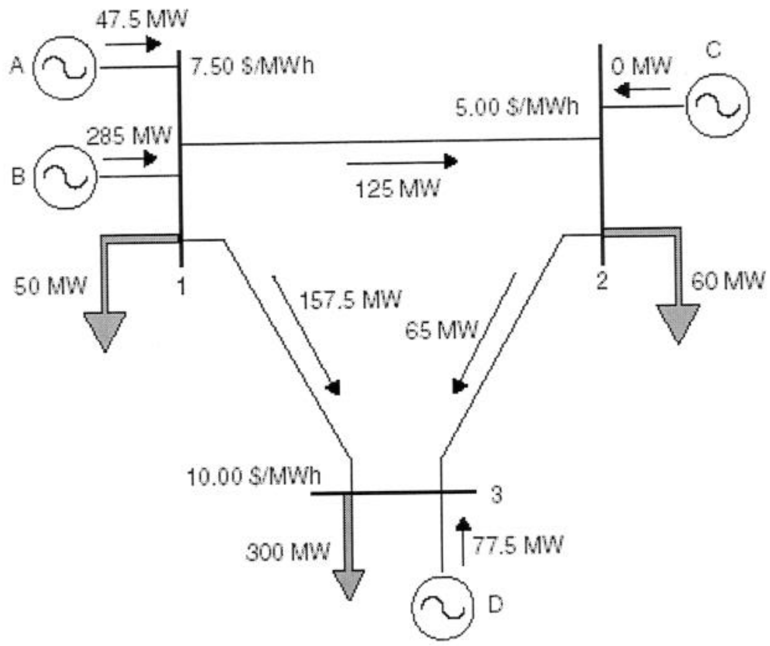
Demand expects to pay: 9.5x60 = 570£/h (demand short for 105£/h)

Generator expects to receive: 9.5x60 = 570£/h (short for 120£/h)

Total shortage is 105+120 = 225£/h

The TCC will generate: (11.25-7.5) x60 = 225 £/h, just enough to fund this shortfall. [3]

v) Line 2-3 constrained. Prices shown in figure below.



Demand pays to the pool 5£/MWh. Expenditure: 5x60 = 300 £/h
Generator at 1 receives 7.5x60 = 450£/h
Demand expects to pay: 9.5x60 = 570£/h (demand has a surplus of 270£/h)
Generator expects to receive: 9.5x60 = 570£/h (short for 120£/h)
Total surplus 270-120 = 150£/h
The TCC will generate <u>negative</u> income: (5-7.5)x60 = -150 £/h, just enough to und this shortfall.

[3]