Neural Network Models Exam Coursework 2001 Deadline 30 April 2001 at 10.00 am *

Download the paper *Stability of Asymmetric Hopfield Networks*, T. Chen and Shun Ichi Amari, IEEE Transactions on Neural Networks, Vol 12, No 1, January 2001, p. 159–163 from http://iel.ihs.com/ and choosing the option 'GO TO IEL'. If asked for a username and password, use your teaching system username and password. It is not necessary to read any other papers. The course textbook is sufficient.

This paper is about the stability of equilibria of analog neural networks. The authors derive stability results without using a Lyapunov function, different from Chapter 4 of the course text. By asymmetric is meant that the synapse matrix T does not necessarily have to be symmetric.

Skip the introduction, and start reading at section II: Some General Results. In Lemma 1, you can assume that the norm $\| w \|$ of the state vector w is just its length. More general norms are defined at the beginning of page 160. The notation diag $[C_1, \ldots, C_N]$ stands for a matrix with $C_1, \ldots, C_N$ on the diagonal, and zeroes elsewhere. Add the condition that $\eta > 0$, as in Proposition 1 on the next page. The authors use the terms Lemma, Proposition, Corollary, as well as Theorem. All these terms mean Theorem. Lemma 1 is referred to further in the paper as Lemma 2, and Lemma 2 is referred to as Lemma 3. This coursework description uses the correct numbers.

Lemma 1 is given without proof. Do not try to prove it. Check the proof of Lemma 2, and correct a small error. The Cauchy convergence principle states that a sequence $\{u_n\}$ converges if and only if for every $\epsilon > 0$ there exists a number M such that $\| u_p - u_q \| < \epsilon$ for all $p, q > M$. You can assume that this not only holds for integer n, but also for real n. Go through the proof of Proposition 1. In the second line before equation (6), the sum should be over i and j. The constant η_1 depends on η, ξ, and C. Give a short proof of Corollary 1 (3/20). Skip Propositions 2, 3, and their proofs.

*Hand it in at the General Office, Level 6

Section III discusses global stability of analog neural networks. Global stability means that all trajectories converge to a limit that is constant in time. In this paper, Corollary 1 is used to prove global stability. This coursework does not deal with local stability, and you should skip Section IV of the paper. Start reading Section III, keeping in mind that $w = \dot{u}$. Briefly indicate how you prove Proposition 4 A.1 and A.2 (2/20). Skip B.1, B.2, C.1, and C.2. Briefly indicate how you prove Theorem 1 A and B (2/20). Skip everything after formula 26. Skip Section IV.

This is the main part of your coursework. Program a numerical solver for the trajectories $u_i(t)$ of the neural network. Assume no external input, $I_i = 0$, and three neurons, $N = 3$. You can generalize the Runge-Kutta formulas (4.5) from the course textbook to three neurons, or use a pre-programmed numerical solver. Use your program and the results from Section III that you have proven to investigate the effect of the size of the diagonal elements T_{ii} of the synapse matrix on the convergence speed (8/20). As there are three diagonal elements, and the state space is three-dimensional, you will need to choose the variables in your graphs carefully.

To make sure that your results are different from those of your fellow students, you have to derive the variable C_1 from the CID number on your Imperial College swipe card in the following way: divide the CID number by the smallest power of 10 so that the result lies between 0 and 1. For example, if your CID number is 00000123456, $C_1 = 0.123456$.

The Challenge. If you want to get top marks, you have to do this challenge. However, you will get better marks for a good report without the challenge than for a mediocre report with the challenge solved. This challenge consists of two unrelated parts.

Part 1. As you can see from the proof of Proposition 1, the inequality only holds when $w_i(t) \neq 0$. What consequence does this have for the stability (2/20)?

Part 2. Using the solver you programmed for the main part of the coursework, make some numerical investigations to determine the capacity of the neural network (15) from the paper (3/20).

You could organize your work as follows.

day 1 Read the paper, looking up anything you don't understand in your lecture notes, especially in chapter 4. Plan what you are going to program.

day 2 Do the programming, and debug your program.

day 3 Run the simulations, and collect the results in a form that you can present in your report. Simulations can be in any programming language, on any machine. You can use Matlab or other software packages, but make sure that you have control over the parameters that you want to vary. If you are really desperate, you could use a calculator, or even pen and paper, but this will make this coursework difficult.

day 4 Write the report. It should be maximum six pages (single sided) a4, in a font not smaller than 10 point. You will not get marks for anything exceeding six pages, even if it is appendices. Font size in pictures should not be smaller than 10 point either. Describe the problem, and how you have solved it. Describe your simulations, but do not give programme listings. Do not give references to the literature. Make sure you do and answer everything that is asked for in the coursework. If you have problems with graphics or formulas in text processing, do them by hand, you will not loose marks. Do not bind the report but staple the pages together. Mention your name, and indicate for what degree you are studying.

day 5 Check the consistency and quality of your work. Make last minute changes if necessary. If you feel confident and have the time, tackle the challenge. Resist the temptation to spend more than five 8-hour days of intensive effort on your coursework. You will not be compensated for it in marks. Just as an exam paper requires a concentrated effort over a few hours, this coursework requires a concentrated effort over a few days.

Do not forget to attend on the "exam" day. Bring a copy of your report with you. You will be asked one or two questions based on what you have written in your report, to make sure that you have written it yourself. No preparation is necessary.

Good luck.

Dr. P. De Wilde