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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2006

MSc and EEE PART IV: MEng and ACGI

HIGH PERFORMANCE ANALOGUE ELECTRONICS

Monday, 15 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

Corrected Copy

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible | First Marker(s) : E. Rodriguez-Villegas
Second Marker(s) : D.G. Haigh

1. (a) Draw the block diagram of a superheterodyne receiver and describe briefly the function of the different blocks.

[10]

(b) Figure 1.1 shows the spectrums of an RF signal and the image of that signal. Draw the spectrum of the IF signal and show the local oscillator signal.

[5]

(c) Figure 1.2 shows the block diagram of a double superheterodyne receiver. Describe the functionality of the first and second filters and the IF stages. Explain the advantages of the double superheterodyne receiver with respect to the superheterodyne receiver.

[5]

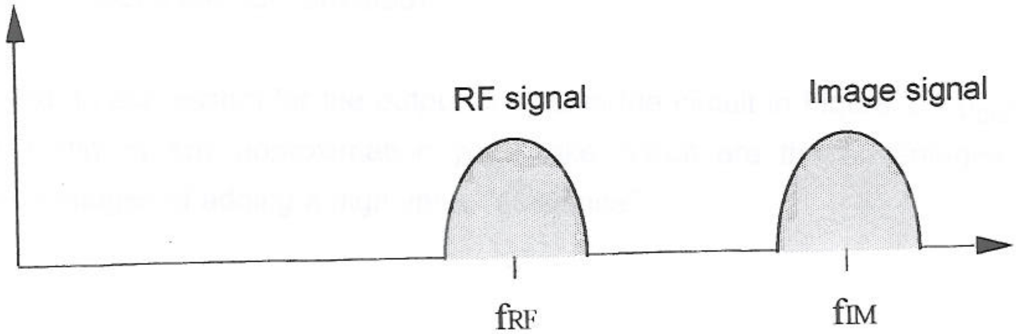

Figure 1.1

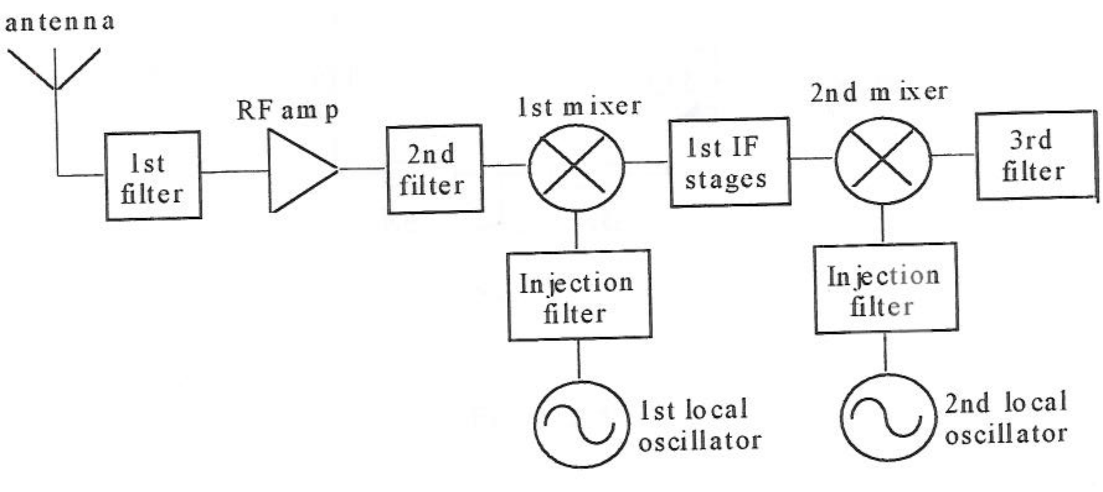

Figure 1.2

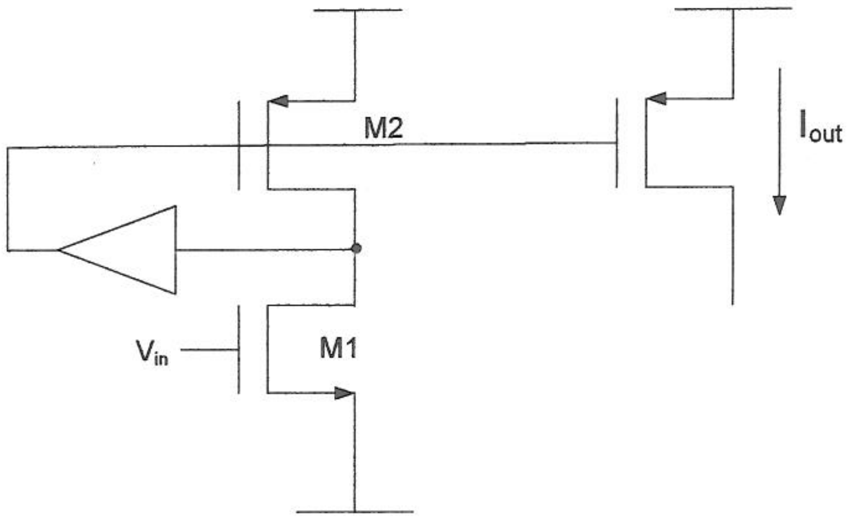

Figure 3.1

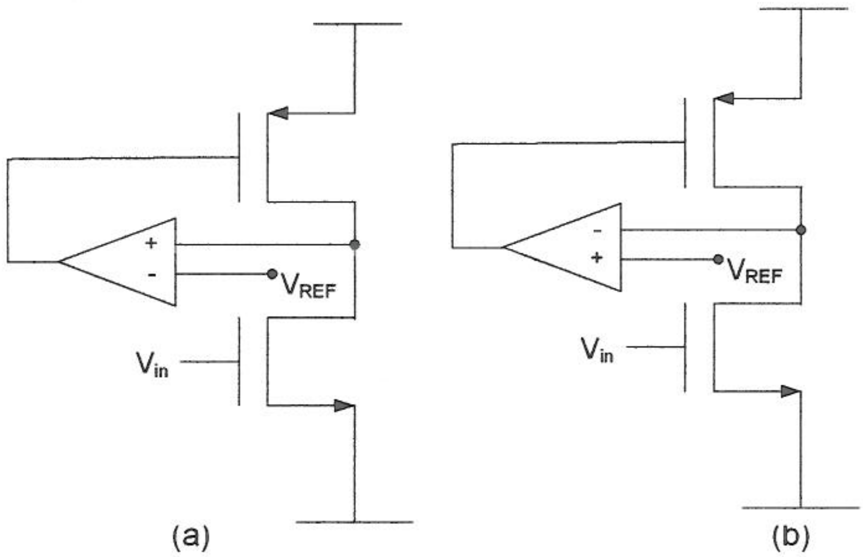

Figure 3.2

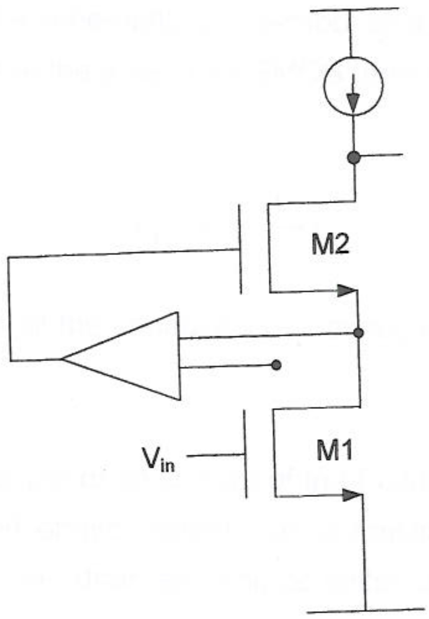

Figure 3.3

5. (a) Draw the schematic of a CMOS differential pair. Derive an expression for the output current, I_{out}, as a function of the differential input voltage, the tail current and β. Discuss the validity of this expression.

[5]

(b) The distortion terms at the output of the differential pair are going to depend on the coefficients of the nonlinear terms in a series expansion of the current equation. Derive expressions for the coefficients of the second and third terms in the series expansion of the output current.

[5]

(c) Draw a circuit that improves the performance in terms of linearity of a simple differential pair. Explain very briefly why this happens.

[5]

(d) Is there any real situation in which the even order harmonics of fully differential circuits would not cancel out? If there is, what is the reason for this?

[5]

6. (a) Figure 6.1 shows the schematic of a second order filter. Derive expressions for the transfer functions $\frac{V_{1p}-V_{1n}}{V_{inp}-V_{inn}}$ and $\frac{V_{2p}-V_{2n}}{V_{inp}-V_{inn}}$ as functions of G_{mi} (for i=[1,4]) and C. Which kind of filters do they correspond to?

[5]

(b) Derive expressions for the quality factor, centre frequency and gain of the lowpass filter.

[5]

(c) Imagine that this filter is used to process brain signals, that have very low amplitude (in the range [2μV to 0.5mV]), but you have a very tight power constraint. Propose an implementation for the transconductor and explain the reasons why you chose that topology.

[5]

(d) Signals generated by the brain are very low in frequency and in amplitude. Which is going to be the main concern when designing the transconductor? How would you choose the size of the transistors?

[5]

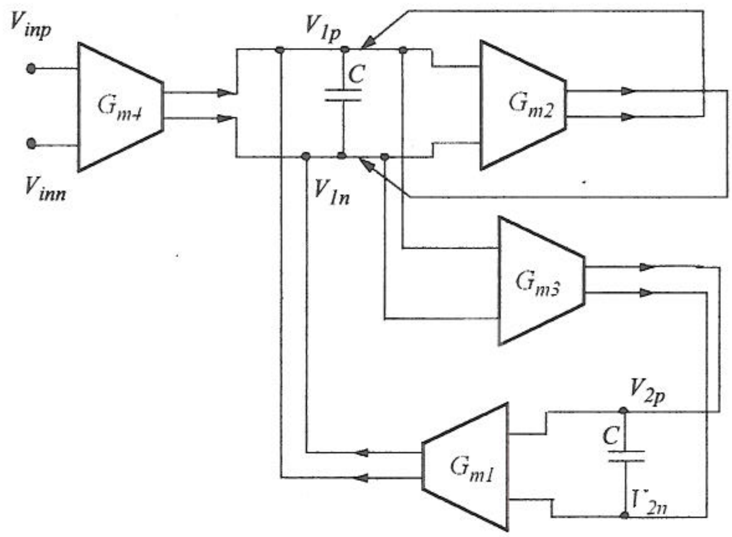

Figure 6.1

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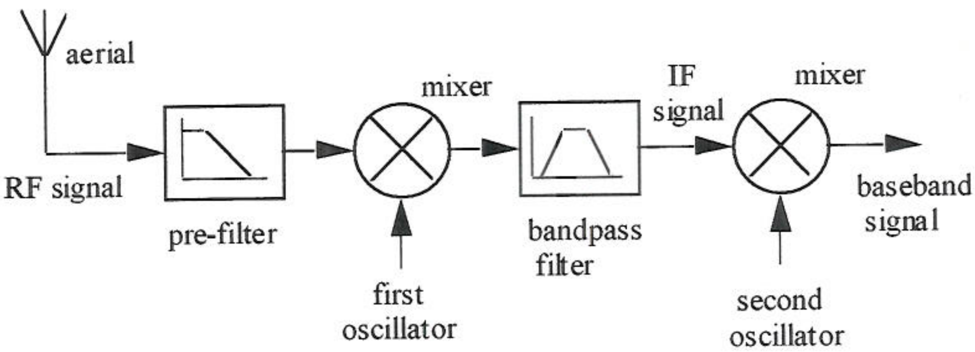
1.

(a) (theory)

The superheterodyne receiver downconverts the input signal to an intermediate frequency (IF), and a bandpass IF filter is then used to select the wanted signal. The design of the bandpass IF filter is eased since it doesn't have to be tuneable, and the IF centre frequency is much lower than the input RF signal.



The downconversion is performed by 'mixing' (multiplying) the RF input signal (f_{RF}) with a local oscillator signal (f_{LO}), such that the resulting output is at the required IF frequency (f_{IF}).

Received RF signal = $2A\cos[(f_{RF})t + ?]$ Local oscillator signal LO = $\cos(f_{LO})t$

Mixer output $= 2A\cos(f_{LO})t\cos[(f_{RF})t + ?]$

$= A\cos[(f_{LO} - f_{RF})t - ?] + A\cos[(f_{LO} + f_{RF}) + ?]$

i.e. sum and difference components

The sum components are at a very high frequency and are removed by filtering. The difference frequency component is a replica of the RF component in terms of amplitude and phase, but is shifted down to an intermediate frequency (IF):

$$f_{IF} = f_{LO} - f_{RF}$$

The oscillator frequency f_{LO} is often tuneable to ensure that a range of input RF frequencies can be selected.

[10]

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(b) (Application of theory)

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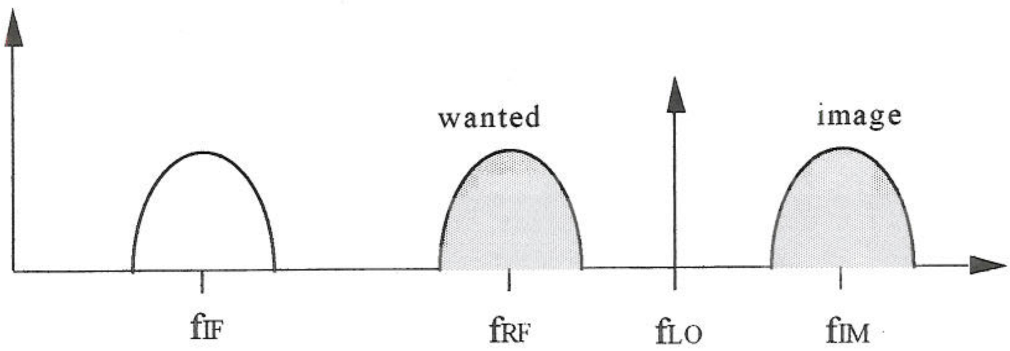

[5]

(c) (theory)

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• 1st filter (preselector): Limits bandwidth of spectrum reaching the RF amplifier (to reduce IM distortion); attenuates receiver spurious responses (esp. image frequency); attenuates oscillator and first IF re-radiation which may be picked up by the antenna.

• 2nd filter: Attenuates noise at the image frequency, and second harmonics which may originate in the RF amplifier.

• IF stages: The first IF filter determines adjacent channel selectivity. This is often a crystal filter (narrowband). Crystal filters are available only in certain centre frequencies, which will constrain the choice of IF. Often the second-image requirement is more stringent on the design of this filter. The IF amplifier is usually a high gain stage.

In a superhet receiver, the design of the prefilter is eased if the IF is high, while the design of the IF bandpass filter is eased if the IF is low. The double conversion superhet receiver avoids this conflict. The first IF is high which ensures that the image frequency is well separated from the wanted signal. The second IF is low, which enables the design of circuits with sharp selectivity and hence good adjacent channel rejection.

Oscillator frequency offsets: If the second local oscillator can be designed such that it tracks any frequency offsets or drift of the first LO, then the effects of these frequency offsets can be minimised.

$$f_{IF1} = (f_{LO1} + \delta f) - f_{RF}$$

$$\begin{aligned} f_{IF2} &= f'_{IF1} - (f_{LO2} + \delta f) \\ &= (f_{LO1} + \delta f) - f_{RF} - (f_{LO2} + \delta f) \\ &= f_{LO1} - f_{LO2} - f_{RF} \end{aligned}$$

[5]

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2.

(a) (theory)

In a MOSFET, there are three main sources of noise:

(i) *Thermal noise* due to the resistance of the channel:

$$ind^2 = \frac{8kTgm\Delta f}{3} \ A^2$$

This noise source can also be represented by an equivalent channel resistance rd = 3/2gm.

(ii) *Flicker (1/f) noise* in series with the gate:

$$vng^2 = \frac{k_f \Delta f}{CoxWLf} \ V^2$$

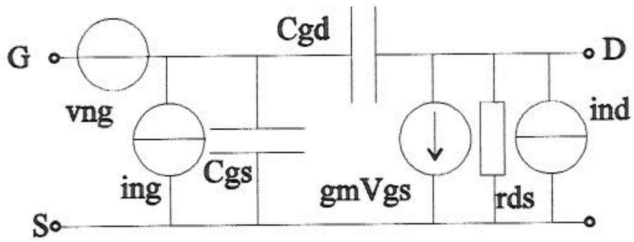
k_f is a flicker noise coefficient which is process dependent. Note that the 1/f noise is inversely proportional to gate area, thus bigger devices are less noisy.

(iii) *Shot noise* due to the gate-source leakage current:

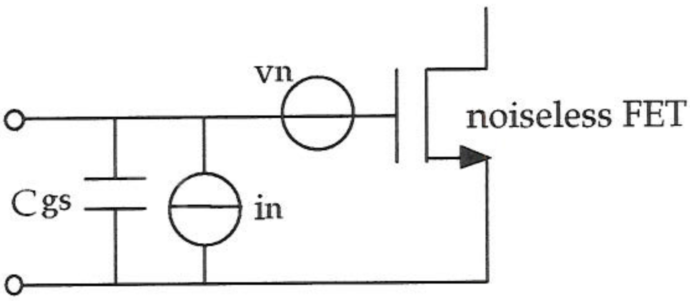
$ing^2 = 2qIg\Delta f \ A^2$ (often neglected since negligible)

(b) (theory)

Equivalent noise model:



Referring all noise sources to the input (assuming low to mid frequencies, i.e. neglecting Cgd)



$$vn^2 = \frac{ind^2}{gm^2} + vng^2 = \frac{8kT}{3gm} + \frac{k_f}{WLCox\,f}\ V^2$$

$$in^2 = ing^2 = 2qIg\Delta f\ \ A^2$$

(c) (application of theory)

As large as possible to minimize Flicker noise. As wide and short as possible to minimize thermal. It ultimately depends on the range of frequencies.

(d) (application of theory)

$$\text{Noisevoltage} = \frac{8kT}{3g_m}(10^3 - 0.01) + \frac{5k_f}{WLC_{ox}}\ln(10)$$

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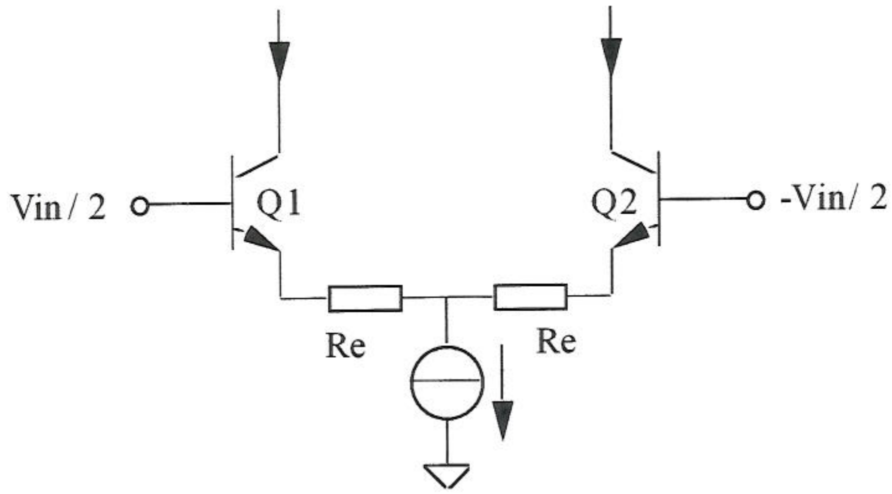
$$\text{Noisecurrent} = 2qI_g(10^3 - 0.01)$$

(e) (theory)



$$\begin{aligned} Vin &= Vbe1 + ReIc1 - Vbe2 - ReIc2 \\ &= Ic1(re1 + Re) - Ic2(re2 + Re) \end{aligned}$$

where the values of re1 and re2 depend on the instantaneous value of Vin. If Re >> re1, re2 then the non-linear variation of gm with Vin is swamped:

$$Vin = Ic1Re - Ic2Re = (Ic1 - Ic2)\,Re$$

if Iout = (Ic1 – Ic2); Iout/Vin = 1/Re

Although the linear dynamic range is increased, the transconductance gain is greatly reduced. This linearisation method suffers a severe noise penalty; the thermal noise of Re is transferred directly to the input, and the reduced gm increases noise contributions from the following stages.

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3.

(a) (application of theory)

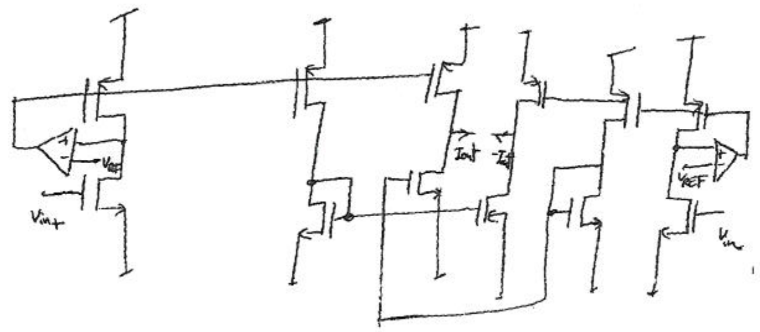
M1 ohmic, M2 saturation

$I_{out} = (2\beta V_c)Vin$ where Vc would be M1 drain voltage

(b) (new theory)

a is right. b is wrong because if M1 drain voltage increases, then M2 gate voltage decreases which would increase M2 drain current. In order to balance the current with M1, the drain voltage would have to increase. This is positive feedback.

(c) (new theory)



Advantage: Even order harmonics cancel out. Improved THD.

Disadvantage: Increased area.

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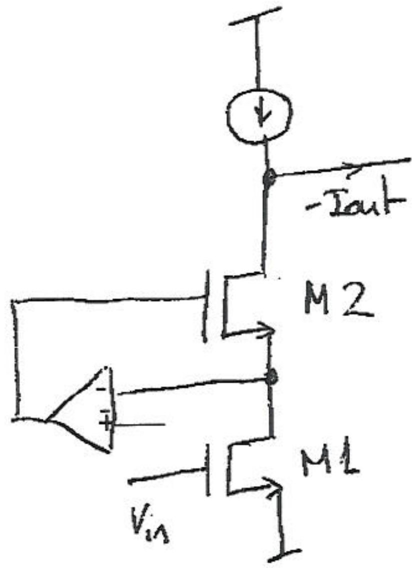
(d) (bookwork and application of theory)



Disadvantage: Higher power supply voltage is required.

(e) (application of theory)

Yes, the value for which M1 leaves the ohmic region.

V_{in} > Vbias +V_T

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4.

(a) (new theory)

Saturation: $$I_D = \frac{\mu C_{ox} W}{2L}\left(\sum_{i=1}^{N} \frac{C_i}{C_T} V_i - V_S - V_T\right)^2$$

Ohmic: $$I_D = \frac{\mu C_{ox} W}{L}\left[\left(\sum_{i=1}^{N} \frac{C_i}{C_T} V_i - V_S - V_T\right) V_{DS} - V_{DS}^2\right]$$

(b) (new theory)

$$I_{out} = I_B - I_2 = I_1 - I_B \Rightarrow I_{out} = \frac{I_1 - I_2}{2}$$

(c) (application of theory)

They are cascode transistors and are used to increase the output resistance of the transconductor and hence minimize the losses of the corresponding integrator

(d) (new theory)

$$I_1 = \beta\left[\left(\frac{V_{in1} + V_{b2}}{2} - V_T\right)^2 + \left(\frac{V_{in2} + V_{b1}}{2} - V_T\right)^2\right]$$

$$I_2 = \beta\left[\left(\frac{V_{in2} + V_{b2}}{2} - V_T\right)^2 + \left(\frac{V_{in1} + V_{b1}}{2} - V_T\right)^2\right]$$

(e) (new theory)

$$I_{out} = \frac{I_1 - I_2}{2} = \frac{\beta(V_{b2} - V_{b1})}{8} \cdot (V_{in1} - V_{in2})$$

(f) (application of theory)

Mismatch would cause even order harmonics to appear.

Finite output resistance.

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Signal dependent mobility.

(g) (application of theory)

$$V_{out} = \frac{\beta(V_{b2} - V_{b1})}{8sC} \cdot (V_{in1} - V_{in2})$$

5.

(a) (bookwork)

$$I_1 = \beta\left(\frac{V_{in}}{2} - V_s - V_T\right)^2$$

$$I_2 = \beta\left(\frac{-V_{in}}{2} - V_s - V_T\right)^2$$

$$I_{out} = I_1 - I_2 = -2\beta V_{in}(V_s + V_T)$$

$$I_B = 2\beta\left[\frac{V_{in}^2}{4} + (V_s + V_T)^2\right]$$

$$(V_s + V_T) = \sqrt{\frac{I_B}{2\beta} - \frac{V_{in}^2}{4}}$$

$$I_{out} = -2\beta V_{in}\sqrt{\frac{I_B}{2\beta} - \frac{V_{in}^2}{4}}$$

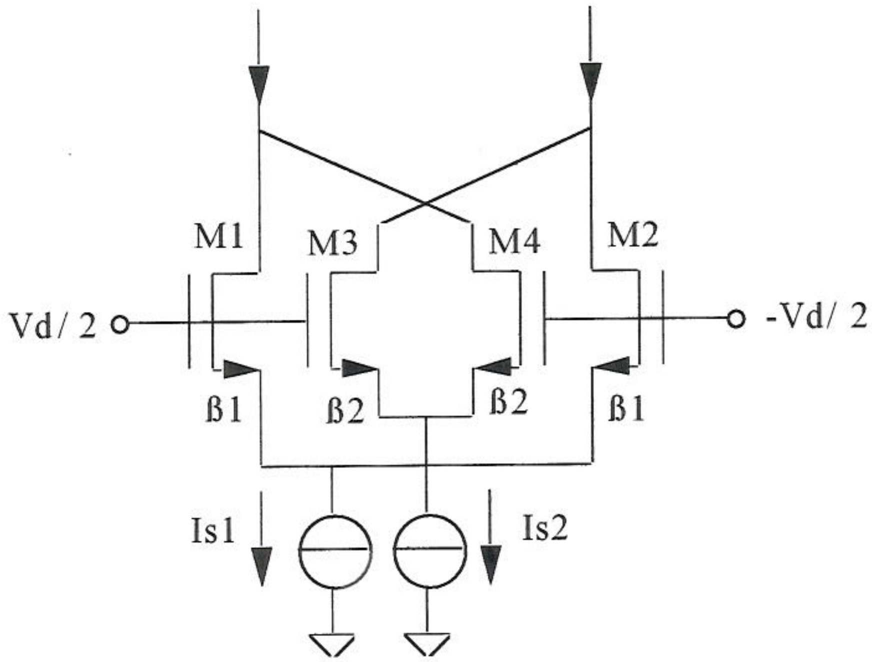
This expression is valid as long as the input transistors are in the strong inversion saturation region

(b) (new theory)

$$\text{Second}=\left.\frac{\partial^2 I_{out}}{\partial V_{in}^2}\right|_{V_{in}=0} = 0$$

$$\text{Third}=\left.\frac{\partial^3 I_{out}}{\partial V_{in}^3}\right|_{V_{in}=0} = \frac{\beta^{3/2}}{\sqrt{2I_B}}$$

(c) (theory)



The third order harmonic can be cancelled by adequately choosing the aspect ratios.

(d) (new theory)

Mismatch would generate even order harmonics.

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6.

(a) (bookwork) (1.1)

$$(V_{2p}-V_{2n})(s) = \frac{\omega_{o3}\omega_{o2}}{s^2+\omega_{o1}s+\omega_{o2}\omega_{o2}}(V_{inp}-V_{inn})(s)\text{. Lowpass}$$

$$(V_{1p}-V_{1n})(s) = \frac{s\omega_{o3}}{s^2+\omega_{o1}s+\omega_{o2}\omega_{o2}}(V_{inp}-V_{inn})(s)\text{. Bandpass}$$

where $\omega_{oi} = G_{mi}/C$.

(b) (bookwork)

$$\omega_o = \omega_{o2},\ \ Q = \frac{\omega_{o2}}{\omega_{o1}},\ \ H_{LP}(0) = \frac{\omega_{o3}}{\omega_{o2}},\ \ H_{BP}(\omega_o) = \frac{\omega_{o3}}{\omega_{o1}}\text{ where } \omega_{oi} = G_{mi}/C.$$

(c) (new theory)

A simple differential pair (with common source transistors as load). Since the signals are very small in amplitude linearity is not an issue. However, in order to keep the power down to a minimum the circuit must be as simple as possible.

(d) (new theory)

Flicker noise. The input transistors should be choosen to have as large area as possible. This would also improve matching. The parasitic capacitances will not be a problem for speed because of the low frequency bandwidth of the input signals.