

t J M I T H C H A R T E R S .

Paper Number(s): **E4.17**  
**AM1**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE  
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2000

MSc and EEE PART IV: M.Eng. and ACGI

**HIGH PERFORMANCE ANALOGUE ELECTRONICS**

Monday, 22 May 2000, 10:00 am

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks.

**Corrected Copy**

Q6(b) 10.00

Time allowed: 3:00 hours

Examiners: Dr A.J. Payne, Prof C. Toumazou

**Special instructions for invigilators:**

*(Several copies)*

A Smith Chart, labelled with seat and candidate number, should be placed on each desk.

Please remind candidates to tie this chart into their answer book if they have answered Question 2.

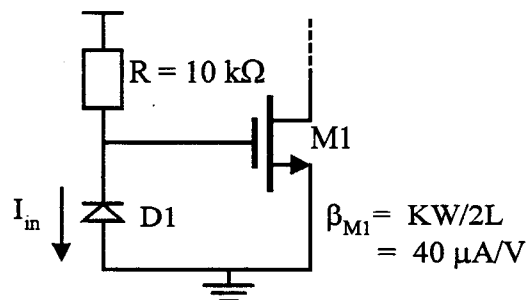
**Information for candidates:**

A labelled Smith Chart is provided. Please tie this into your answer book if you have answered Question 2.

1. (a) Sketch and label the small-signal equivalent circuit of an n-channel MOS transistor, including all major sources of noise within the device. Clearly name and label the noise sources and give expressions by which their mean square values may be calculated. By referring all noise sources to the input of the device, derive expressions for the equivalent input mean square noise voltage ( $v_n^2$ ) and noise current ( $i_n^2$ ). 10 marks

(b) *Figure 1* shows a MOSFET M1 used to amplify an input signal from a pn junction photodiode D1, where R is an input current load resistor. When the photodiode is illuminated, an input current  $I_{in} = 10 \mu A$  is generated by the diode. By using the small-signal equivalent noise model developed in part (a), derive an expression for the total equivalent input-referred mean square noise current  $i_{eq}^2$  generated when the photodiode is illuminated. You may assume that the output impedance of the photodiode can be neglected, and the MOSFET gate leakage current is negligible. From this expression for  $i_{eq}^2$  derive an expression for the noise figure NF of this circuit (you may assume that the input noise source is the noise generated by the diode). Hence calculate the minimum drain current of M1 necessary to ensure that the midband NF < 4 dB, given that  $k = 1.38 \times 10^{-23} \text{ J/K}$ ,  $T = 290 \text{ K}$ ,  $q = 1.6 \times 10^{-19} \text{ C}$ .

15 marks



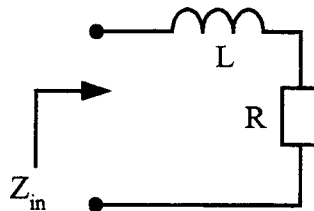
*Figure 1*

2. (a) The input impedance ( $Z_{in}$ ) of a particular integrated circuit (IC) can be modelled as shown in *Figure 2*, where  $R = 9 \Omega$  and  $L = 3 \text{ nH}$ . This IC is to be connected to an antenna with an impedance of  $50 \Omega$ . By using a Smith Chart, design a passive matching network to maximise the power transfer from the antenna into the IC at an operating frequency  $\omega = 12.5 \times 10^9 \text{ rad/s}$ :
- if the antenna should be a.c. coupled to the IC
  - if the antenna should be d.c. coupled to the IC.

22 marks

- (b) In practice, the antenna is found to be equivalent to a  $50 \Omega$  resistor in series with a  $3 \text{ nH}$  inductance. How should the matching network designed in (ii) be modified to ensure correct matching?

3 marks



*Figure 2*

3. (a) *Figure 3* shows the basic topology of a superheterodyne television receiver. Briefly describe the function of each of the shaded blocks.

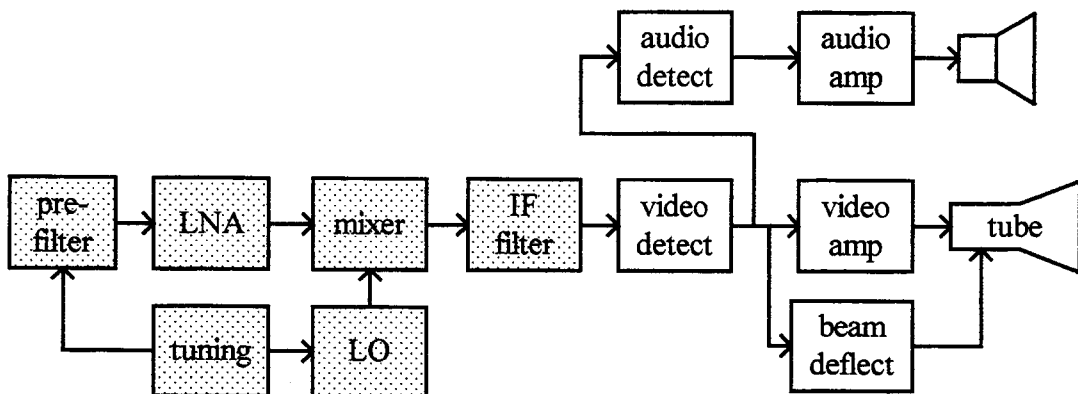
9 marks

- (b) In the single conversion architecture of *Figure 3*, outline the tuning requirements of the pre-filter and local oscillator (LO) if the input signal carrier frequency is in the range 50 - 890 MHz and the intermediate frequency (IF) is 40 MHz. Show with the aid of a diagram how this architecture could be modified to relax the tuning requirements of the image filter and LO.

8 marks

- (c) A television designed for receiving PAL signals has 625 horizontal and 833 vertical lines and a refresh rate of 25 Hz. Calculate the maximum signal bandwidth required to transmit luminance information. Explain how chrominance information is also transmitted within the same bandwidth.

8 marks



*Figure 3*

4. (a) Explain why direct conversion is an attractive architecture for the implementation of a fully-integrated wireless receiver. Give two advantages and two disadvantages of direct conversion when compared to a superheterodyne approach.

6 marks

- (b) An alternative architecture for increasing integration levels is an image reject architecture such as the Hartley receiver shown in *Figure 4a*. Show by calculation that this receiver architecture will correctly receive the wanted channel and reject the image channel if  $\phi_1 = \phi_2 = 90^\circ$ . Show also that distortion will arise if there is a phase error  $\phi_2 = 90(1 + \epsilon)^\circ$ , and discuss briefly the nature of this distortion.

12 marks

- (c) *Figure 4b* shows the front-end of a wireless receiver. Define receiver sensitivity and give an expression by which the sensitivity of the receiver can be calculated. Hence calculate the low noise amplifier (LNA) noise figure (NF) required to achieve a sensitivity of -145 dB, given that the input stage is designed for maximum power transfer, the equivalent noise bandwidth (NBW) is 28 kHz,  $k = 1.38 \times 10^{-23}$  J/K,  $T = 300$  K.

8 marks

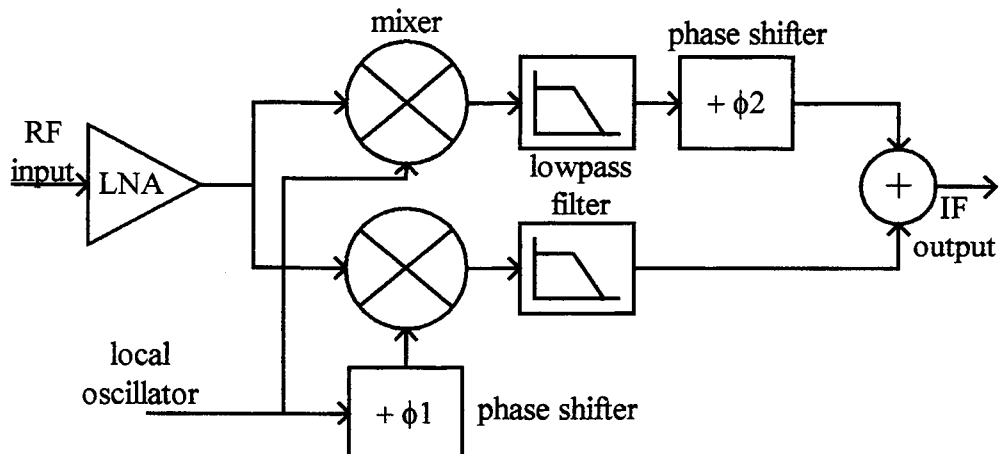


Figure 4a

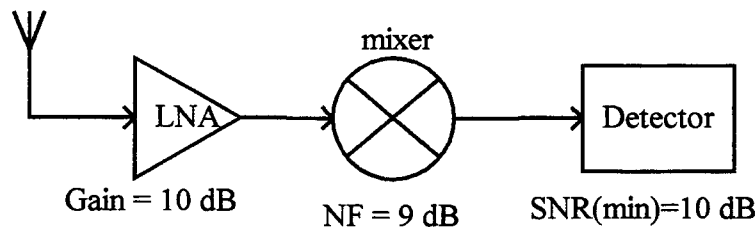


Figure 4b

5. (a) What is meant by the term 'phase noise' of an oscillator, and why is it important to implement low phase noise oscillators in a wireless transceiver architecture.

4 marks

- (b) A recent technique for understanding the phase noise response of an oscillator is based on the use of 'Impulse Sensitivity Functions' (ISF). What is meant by the ISF of an oscillator? Show that by considering the ISF of an oscillator, the excess phase can be calculated as:

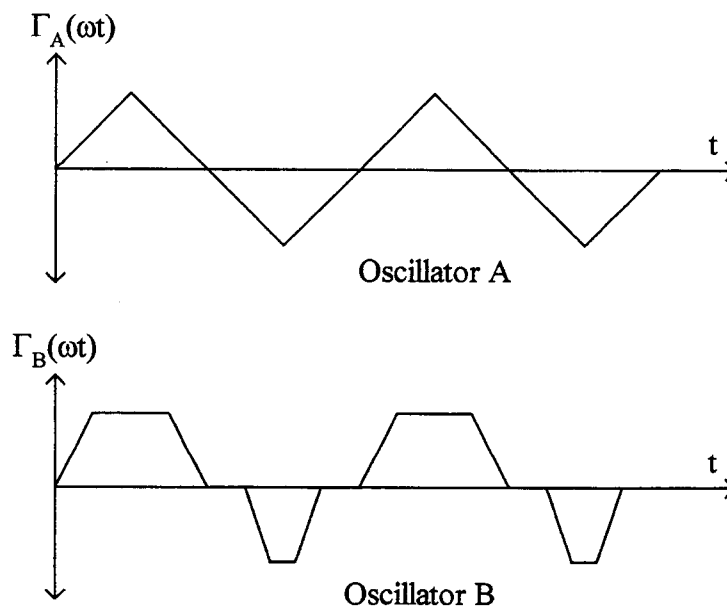
$$\phi(t) = \frac{1}{Q_{\max}} \left[ \frac{c_0}{2} \int_{-\infty}^t i(\tau) d\tau + \sum_{n=1}^{\infty} c_n \int_{-\infty}^t i(\tau) \cos(n\omega_0 \tau) d\tau \right]$$

explaining the meaning of each of the terms in the above equation.

15 marks

- (c) *Figure 5* shows the ISFs for two different oscillators. Explain giving your reasons which oscillator you would expect to exhibit the lowest phase noise, given that the actual noise sources in the two oscillators are identical.

6 marks



*Figure 5*

6. (a) *Figure 6a* shows the small-signal hybrid- $\pi$  model of a bipolar junction transistor (BJT). Explain why the Miller approximation applied to this model proves inadequate at frequencies approaching the  $f_T$  of the device, when the device is configured as a simple common-emitter (CE) amplifier stage. Sketch an alternative 'RF hybrid- $\pi$ ' model which does not suffer this limitation. What are the major differences in the transfer functions predicted by the Miller and RF models when applied to a simple CE amplifier?

9 marks

- (b) A BJT is configured as a common-collector (CC) stage as shown in *Figure 6b*, to implement a voltage buffer between a load (not shown) and an inductive source impedance. By using a simple hybrid- $\pi$  model which neglects  $r_{ce}$ ,  $r_b$  and  $C_{\mu}$ , derive an expression for the output impedance of the device. Hence explain why this CC stage should be used with caution.

10 marks

- (c) A CE amplifier stage with high gain has a limited bandwidth due to the Miller multiplication of  $C_{\mu}$ . Briefly outline a method suitable for neutralising the effect of  $C_{\mu}$  (i) for discrete designs, (ii) in an integrated circuit, illustrating your answer with a diagram in each case.

6 marks

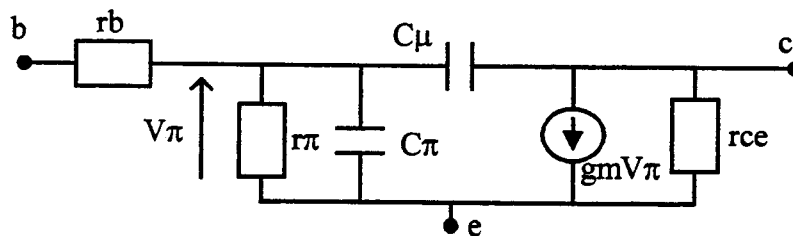


Figure 6a

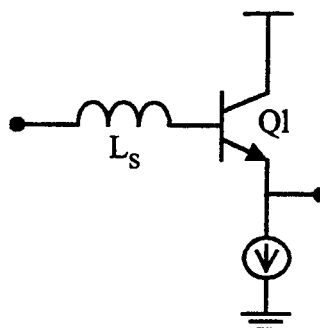
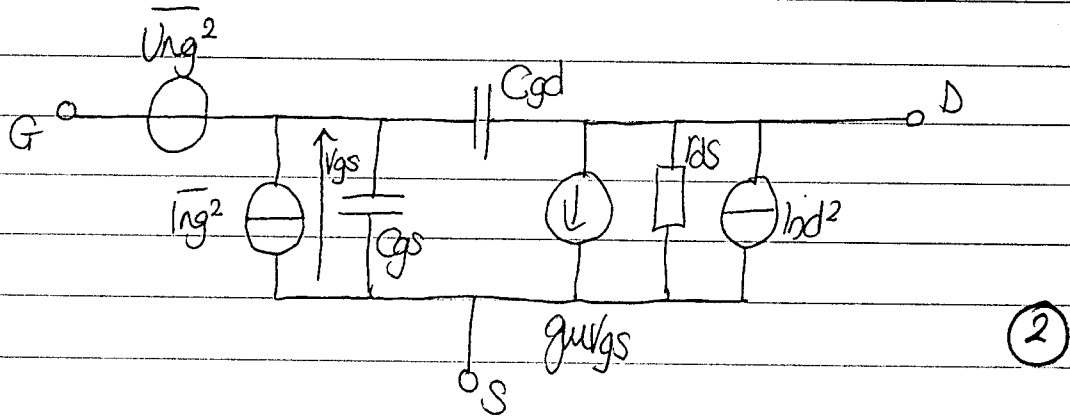


Figure 6b





(1) (2)



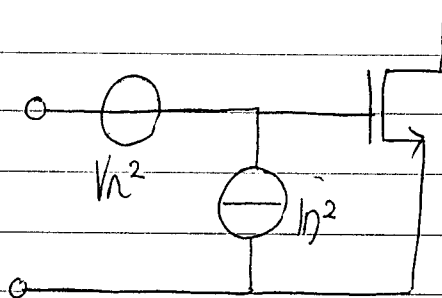
$V_{ng}^2$  = flicker (1/f) noise in series with the gate  
 $V_{ng}^2(f) = \frac{K_f}{C_{ox} W \cdot L \cdot f} V^2 / Hz$

$I_{ng}^2$  = shot noise due to gate-source leakage current  
 $= 2q I_g A^2 / Hz$

$I_{nd}^2$  = thermal noise due to channel resistance  
 $= \frac{8KTg_m}{3} A^2 / Hz$

(6)

equivalent noise model:

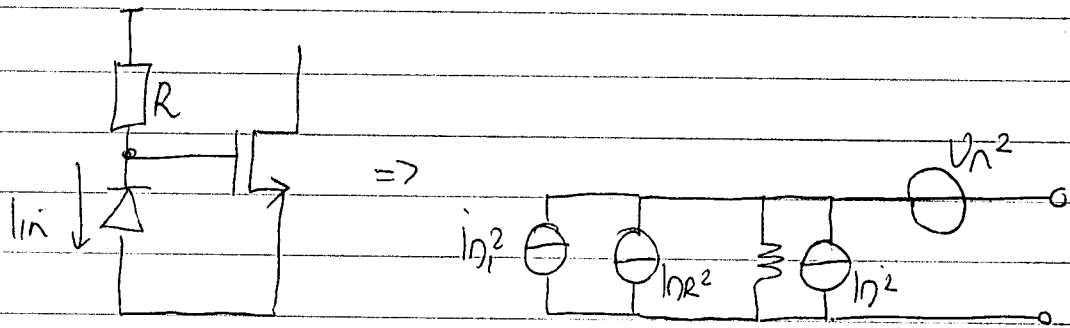


$V_n^2 = \frac{8KT}{3g_m} + \frac{K_f}{C_{ox} \cdot W \cdot L \cdot f} V^2 / Hz = \frac{8KT}{3g_m} (1 + f_c) \frac{V^2}{Hz}$

$I_n^2 = I_{ng}^2 = 2q I_g A^2 / Hz$

(2)

(b)



$$I_{eq}^2 = I_{n1}^2 + I_{nR^2} + I_n^2 + \frac{V_n^2}{R^2}$$

$$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & \uparrow \\ 2qI_{in} & \frac{4KT}{R} & \approx \emptyset & \frac{8KT}{3g_m R^2} \end{array}$$

(5)

$$F = \frac{I_{eq}^2}{I_n^2} = 1 + \frac{4KT/R}{2qI_{in}} + \frac{8KT/3g_m R^2}{2qI_{in}}$$

$$NF = 10 \log F \text{ dB}$$

(4)

$$F = 1 + \frac{2V_T}{R \cdot I_{in}} + \frac{4V_T}{3g_m R^2 I_{in}}$$

$$V_T = KT/q = 25 \text{ mV}$$

Require \$NF < 4 \text{ dB}\$ i.e. \$F < 2.5\$

$$2.5 > 1 + \frac{2V_T}{R \cdot I_{in}} + \frac{4V_T}{3g_m R^2 I_{in}} \quad \begin{array}{l} I_{in} = 10 \mu \\ R = 10 \text{ k} \end{array}$$

$$2.5 > 1 + 0.5 + \frac{33 \mu\text{s}}{g_m}$$

$$\text{Thus } g_m > 33 \mu\text{s}.$$

$$g_m = \frac{2\sqrt{\beta I_d}}{\beta} \quad \beta = 40 \mu \quad \therefore I_d = 0.1 \text{ mA}$$

(5)

$$\textcircled{2} \text{ Input impedance } Z_{in} = 9 + j(125 \times 10^9)(3 \times 10^{-9}) \\ = 9 + j37.5$$

$$\text{Normalised to } 50\Omega = 0.18 + j0.75$$

$\textcircled{2}$

(i) From point A ( $0.18 + j0.75$ ) move to point B ( $0.18 + j0.38$ )  
Thus we add  $0.37\Omega$  of normalised series capacitance  
(hence ac coupled).

$$\frac{1}{\omega C_1} = 0.37 \times 50$$

$$C_1 = 4.3 \text{ pF}$$

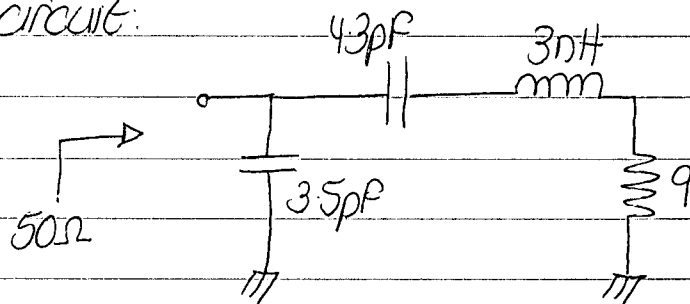
Reflect through centre of chart to point C ( $1.0 - j2.2$ )  
 $\Rightarrow$  parallel admittance mode.

Move to centre of chart ( $1.0 + j0$ ), thus add  $50/2.2 \Omega$   
of parallel capacitance

$$\frac{1}{\omega C_2} = 22.7$$

$$\therefore C_2 = 3.5 \text{ pF}$$

Final circuit:



$\textcircled{10}$

(ii) From point A ( $0.18 + j0.75$ ) we reflect through the centre  
to B ( $0.3 - j1.275$ )  $\Rightarrow$  parallel admittance mode  
Move to point C ( $0.3 + j0.46$ ), thus add  
 $50/1.735 \Omega$  of parallel capacitance

$$\frac{1}{\omega C_1} = 28.8$$

$$C_1 = 2.8 \text{ pF}$$

2 (continued)

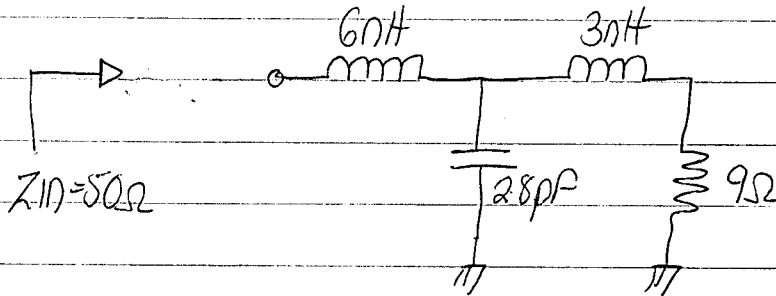
Reflect through the centre to D (1.0 - j1.5)

=> series impedance mode.

Move to centre by adding  $50 \times 1.5 \Omega$  of series inductance (thus dc coupled).

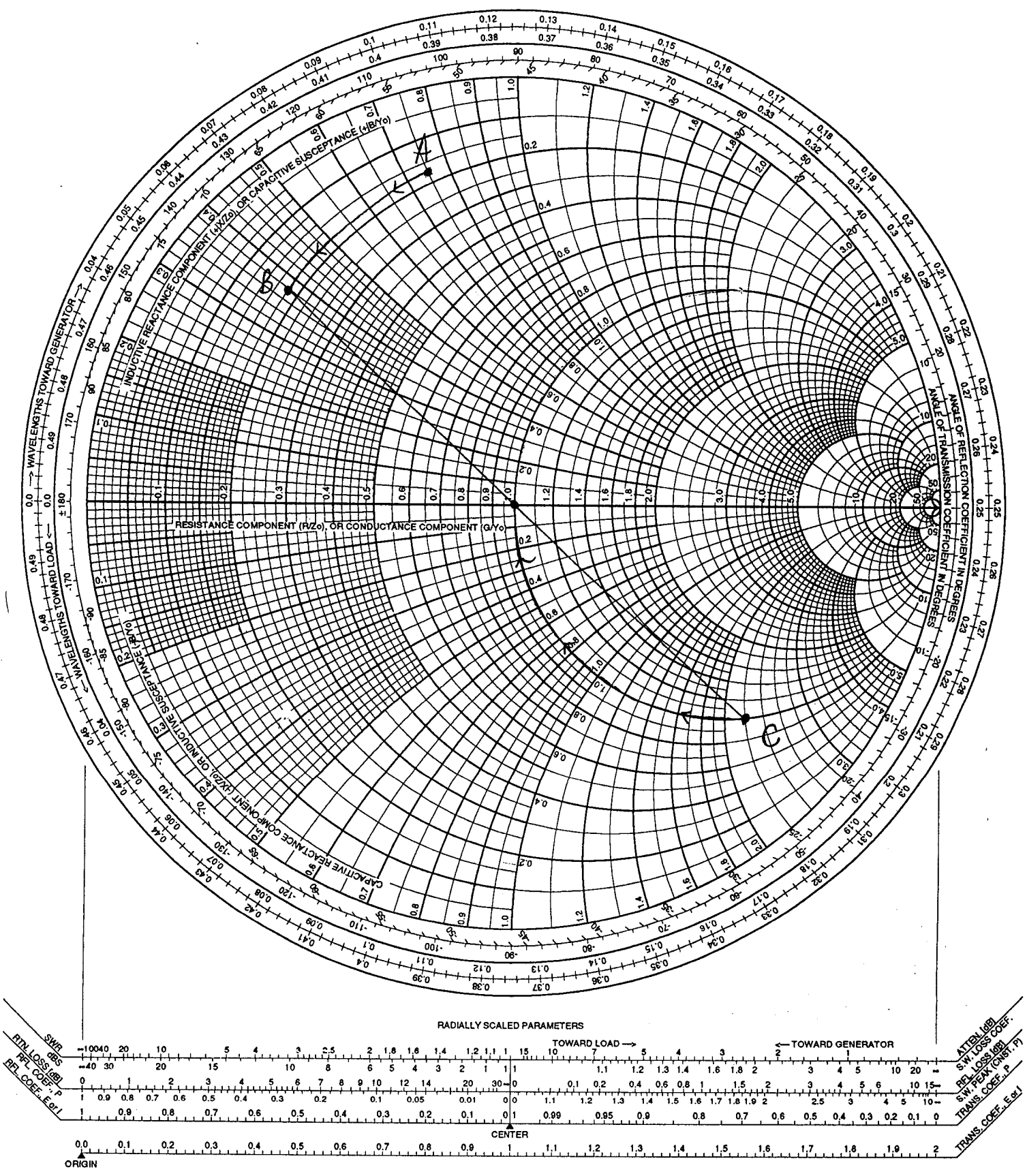
$$\omega L_2 = 75 \Omega \quad L_2 = 6 \text{ nH}$$

Final circuit:

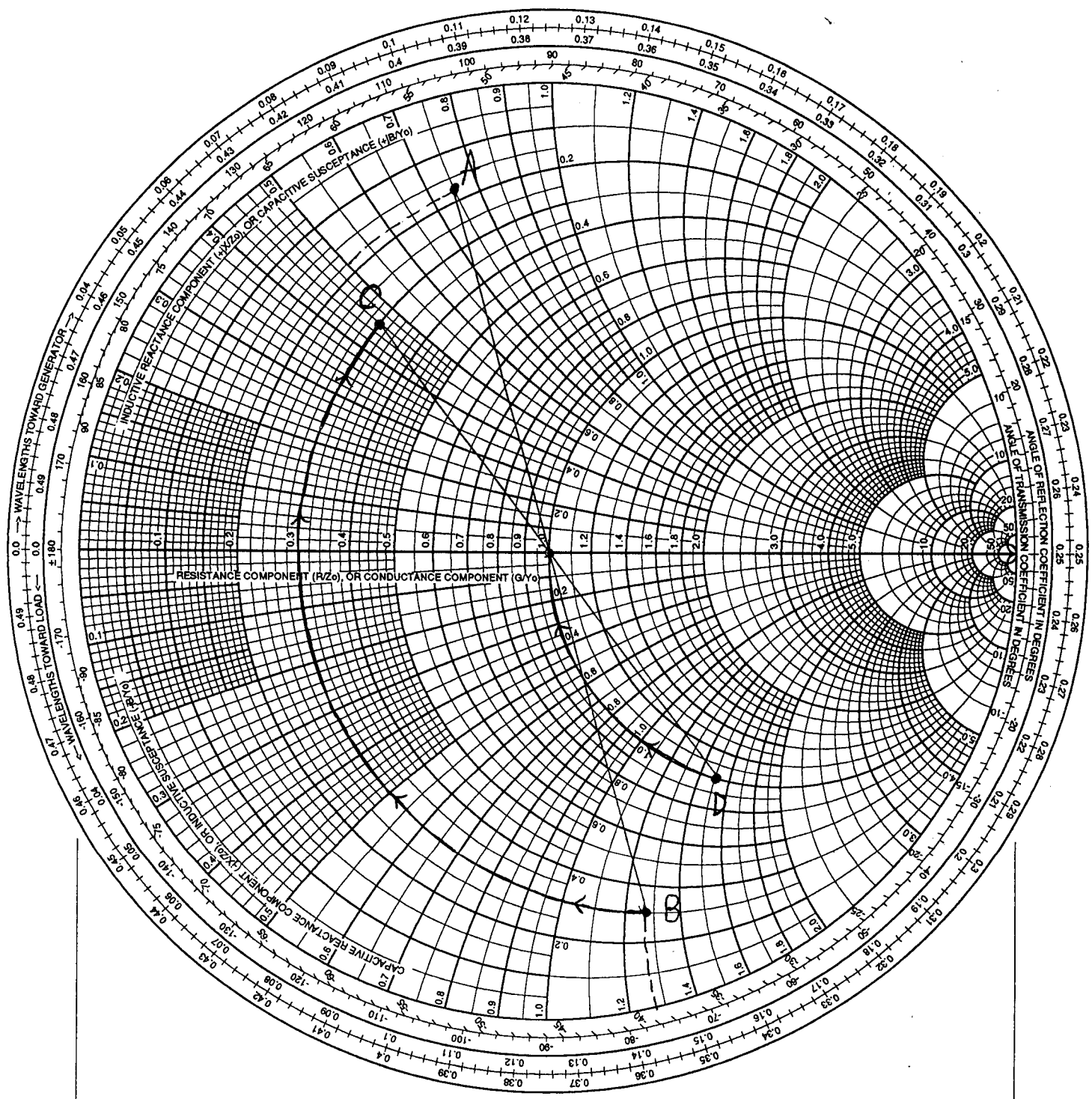


If antenna =  $50 \Omega$  in series with  $3 \text{ nH}$ , simply reduce the  $6 \text{ nH}$  inductance ( $L_2$ ) to  $3 \text{ nH}$ .

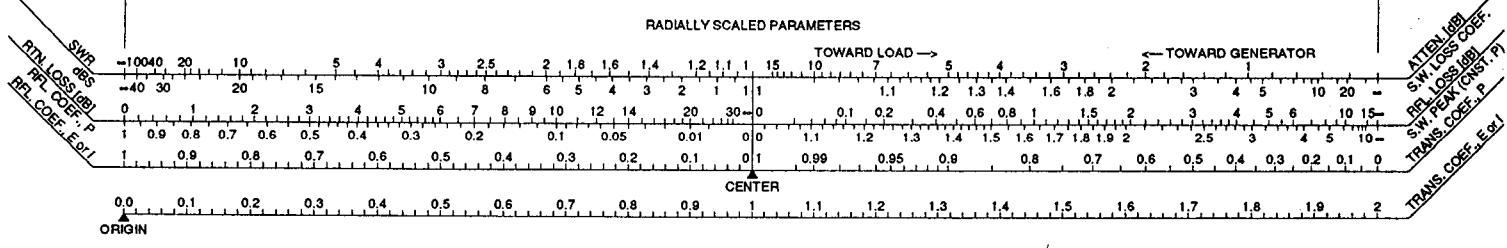
Q2 Part (i)



Q2 part (ii)



RADIALLY SCALED PARAMETERS



③

Pre-filter: rejects any signals at the image frequency  
 $f_{im} = f_{io} + f_{IF}$  (for high side injection)  
 Signal at the image frequency which reach the mixer will also be converted down to the IF a hence corrupt the wanted channel.

LNA: increases the receiver sensitivity by reducing the effect of the high mixer NF

Mixer: Performs frequency downconversion by multiplying input RF & LO signals:

$$A \cos \omega_{RF} t \cdot B \cos \omega_{LO} t = \frac{AB}{2} [\cos(\omega_{LO} - \omega_{RF})t + \cos(\omega_{LO} + \omega_{RF})t]$$

The input RF signal is thus shifted down to the IF.

IF filter: narrowband bandpass filter which selects the wanted channel while rejecting out of band signals.

Local Oscillator: Generates a reference signal at IF above the required RF channel, which is fed into the mixer to perform the required frequency conversion. Low phase noise required!

Tuning circuitry: The LO frequency must be variable to allow different RF channels to mix down to RF. The tuning circuitry controls the LO frequency & simultaneously adjusts the bandwidth of the pre-filter, which should pass the wanted signal (at  $\omega_{LO} - \omega_{RF}$ ) but reject the image (at  $\omega_{LO} + \omega_{RF}$ ).



3 (continued)

RF input = 50-890 MHz

IF = 40 MHz

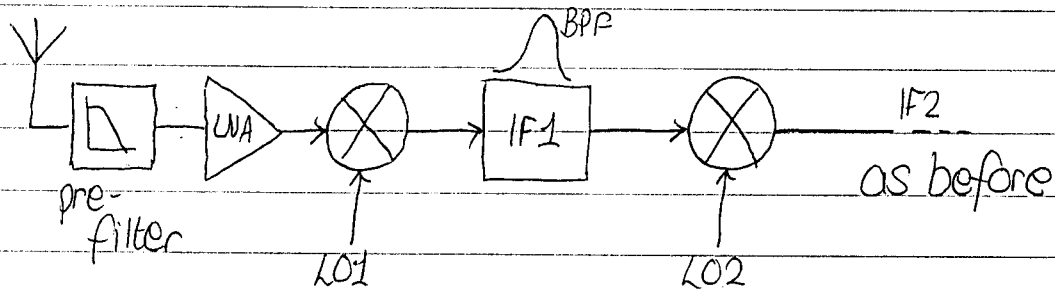
Thus LO tuning range required = 90 MHz  $\rightarrow$  930 MHz

More than a decade tuning range!

(Similarly for pre-filter)

(2)

This can be relaxed by having an additional first stage upconversion:



eg. suppose  $IF1 = 1\text{GHz}$ , then  $LO1$  tuning range  
 = 960 MHz  $\rightarrow$  1.89 GHz (less than 1 octave)

(2)

Bandpass/lowpass filter at  $IF1$  suppresses the image for the 2nd stage of mixing (downconversion).

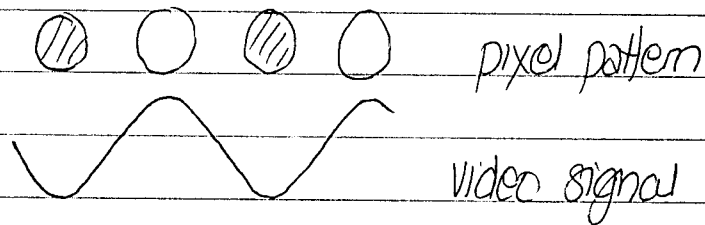
Since  $IF1$  &  $IF2$  are both fixed,  $LO2$  does not have to be tuneable (nor the bandpass filter at  $IF1$ )

Since the first stage uses upconversion, there will be no image. Instead the first filter must reject very high frequencies (1.96 GHz & above) whose difference frequencies with  $LO1$  could appear at  $IF1$ . Hence this filter can also be fixed.

(2)

3 (continued)

Maximum frequency signal will occur if pixels are alternately b & w:



Thus  $f_{\max} = \frac{1}{T_p}$ , where  $T_p = \frac{\text{time required to scan one pixel}}{2}$ .

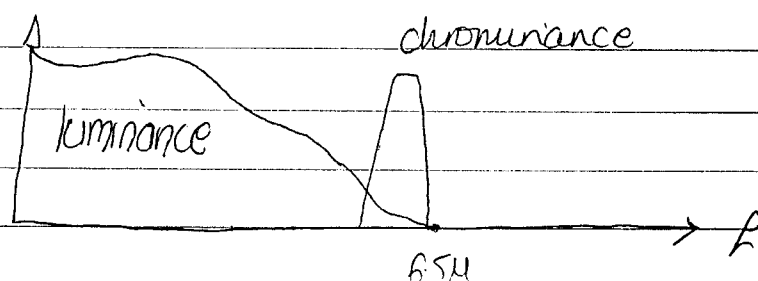
$$\text{Line scan time, } T_L = \frac{T_p}{2} \times 833$$

$$\text{Frame scan time, } T_F = \frac{T_p}{2} \times 833 \times 625$$

$$T_F = \frac{1}{25} \text{ thus } T_p = 154 \text{ ns, } f_{\max} = 6.5 \text{ MHz. } \textcircled{4}$$

Luminance (brightness) data is concentrated towards the lower end of the 6.5 MHz band, since low frequency information will relate to large 'blocks' on the screen. The eye is fairly insensitive to fine detail as the eye tends to average out high frequency luminance variations.

Thus high frequency info can be omitted with little reduction in perceived picture quality. This high end of the band is used to transmit chrominance info which typically requires a 1 MHz bandwidth. R, G & B signals are combined to produce 2 vector signals which modulate the amplitude & phase of a chrominance subcarrier.



④

(4) The wanted channel is converted straight to baseband, i.e.  $f_{LO} = f_{RF}$ . Thus there will be no image signal, & no image filter is required. Since  $f_{IF} = 0\text{Hz}$ , channel select filtering is done at baseband, which means that the filters can be implemented on chip. The relaxed filter requirements mean that no off-chip filtering is required. (2)

Advantages:

- No image signal thus no tuneable image filter reqd
- $f_{IF} = 0\text{Hz}$  so channel select filter can be lowpass

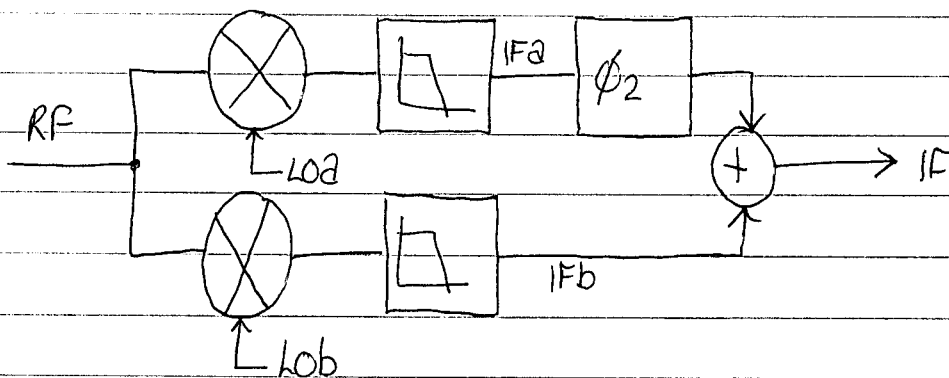
(2)

Disadvantages:

- front-end must have high-signal handling as less (or no) front-end filtering is done
- LO-reradiation can cause dc offsets which saturate the receiver
- Conversion straight to baseband means that 1/f noise is a problem

(2)

Image reject receiver:



4 continued

Let the RF signal consist of a wanted channel & an image:

$$RF = 2A \cos(\omega_{RF}t + \theta_A) + 2B \cos(\omega_{IM}t + \theta_B)$$

$$\omega_{RF} = \omega_{LO} - \omega_{IF} \quad \omega_{IM} = \omega_{LO} + \omega_{IF}$$

After the mixer & lowpass filter (which removes sum components):

$$\begin{aligned} IFA &= A \cos([\omega_{LO} - \omega_{RF}]t - \theta_A) + B \cos([\omega_{LO} - \omega_{IM}]t - \theta_B) \\ &= A \cos(\omega_{IF}t - \theta_A) + B \cos(\omega_{IF}t + \theta_B) \end{aligned}$$

$$\begin{aligned} IFB &= A \cos([\omega_{LO} - \omega_{RF}]t + \theta_1 - \theta_A) + B \cos([\omega_{LO} - \omega_{IM}]t + \theta_1 - \theta_B) \\ &= A \cos(\omega_{IF}t + \theta_1 - \theta_A) + B \cos(\omega_{IF}t - \theta_1 + \theta_B) \end{aligned}$$

After phase shift  $\theta_2$ :

$$IFA = A \cos(\omega_{IF}t + \theta_2 - \theta_A) + B \cos(\omega_{IF}t + \theta_2 + \theta_B)$$

$$IFB = A \cos(\omega_{IF}t + \theta_1 - \theta_A) + B \cos(\omega_{IF}t - \theta_1 + \theta_B)$$

$$\text{If } \theta_1 = \theta_2 = 90$$

$$\begin{aligned} IFA &= A \cos(\omega_{IF}t + 90 - \theta_A) + B \cos(\omega_{IF}t + 90 + \theta_B) \\ &= -A \sin(\omega_{IF}t - \theta_A) - B \sin(\omega_{IF}t + \theta_B) \end{aligned}$$

$$\begin{aligned} IFB &= A \cos(\omega_{IF}t + 90 - \theta_A) + B \cos(\omega_{IF}t - 90 + \theta_B) \\ &= -A \sin(\omega_{IF}t - \theta_A) + B \sin(\omega_{IF}t + \theta_B) \end{aligned}$$

$$IF = IF_a + IF_b = -2A \sin(\omega_{IF}t - \theta_A)$$

The wanted RF signal is received while the image is rejected.

4 (continued)

$$I_{FA} = A \cos(\omega t + \phi_2 - \phi_A) + B \cos(\omega t + \phi_2 + \phi_B)$$

$$I_{FB} = A \cos(\omega t + \phi_1 - \phi_A) + B \cos(\omega t - \phi_1 + \phi_B)$$

$$\text{Let } \phi_1 = 90 \text{ \& } \phi_2 = 90(1 + \epsilon) = 90 + \delta$$

$$\begin{aligned} I_{FB} &= A \cos(\omega t + 90 - \phi_A) + B \cos(\omega t - 90 + \phi_B) \\ &= -A \sin(\omega t - \phi_A) + B \sin(\omega t + \phi_B) \end{aligned}$$

$$\begin{aligned} I_{FA} &= A \cos(\omega t + 90 + \delta - \phi_A) + B \cos(\omega t + 90 + \delta + \phi_B) \\ &= -A \sin(\omega t + \delta - \phi_A) - B \sin(\omega t + \delta + \phi_B) \end{aligned}$$

The wanted channel in each path is slightly phase shifted, which will cause distortion.

The image channel is phase shifted in the two paths & thus will not cancel properly. This will lead to 'crosstalk' between wanted & image channels (6)

Sensitivity is the minimum input signal which can be successfully detected by a system.

$$S = P_{ni}(\text{dB}) + NF + SNR_{\text{det}}(\text{dB}) \quad (2)$$

$P_{ni}$  = received (input) noise

$$= kT \text{ V}^2/\text{Hz} \text{ for a matched system}$$

$$= 4.14 \times 10^{-21} \times 28 \times 10^3 \text{ for } 28\text{kHz b/w}$$

$$= 1.159 \times 10^{-16} \text{ V}^2$$

$$= -159 \text{ dB} \quad (2)$$

$$\text{Thus } -145 = -159 + NF + 10$$

$$NF = 4 \text{ dB} \quad (2)$$

Require overall NF = 4dB  
F = 2.51

$$F = F_1 + \frac{(F_2 - 1)}{G_1}$$

$$NF_2 = 9\text{dB} \therefore F_2 = 9.9$$

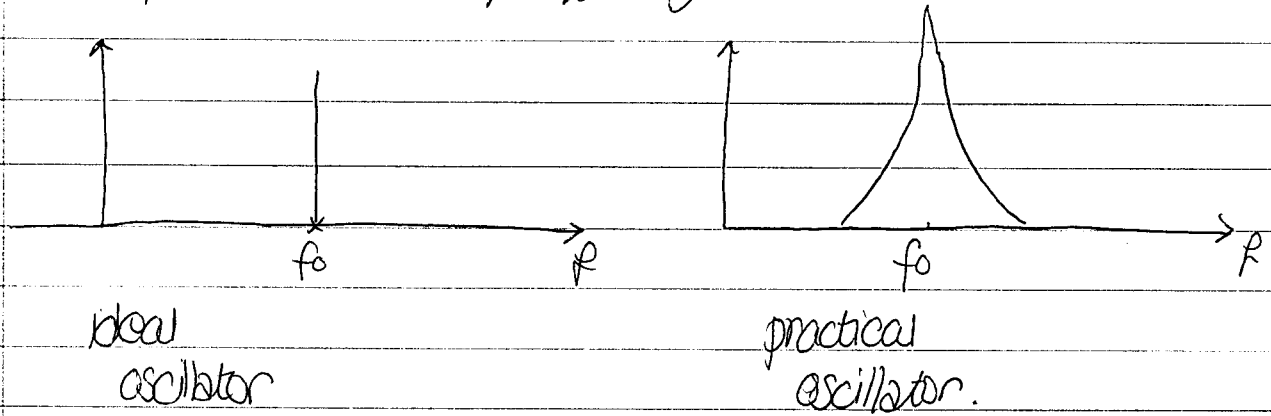
$$G_1 = 10\text{dB} = 10$$

$$\therefore 2.51 = F_1 + \frac{6.9}{10}$$

$$F_1 = 1.8$$

$$NF_1 = 2.6\text{dB} \quad (2)$$

⑤ An ideal oscillator should possess a frequency spectrum that is a single impulse. In practice this is impossible to achieve, and all oscillators exhibit some fluctuation in frequency.



This frequency fluctuation can be seen as a 'spreading out' of the oscillator frequency spectrum, & is referred to as phase noise.

Phase noise is defined as the 'noise power in a 1Hz bandwidth at a given offset frequency from the carrier, divided by the power of the carrier' i.e.

$$L(\Delta f) = \frac{P(f_0 + \Delta f)}{P(f_0)} \text{ dBc.} \quad \textcircled{2}$$

- In a receiver, phase noise can cause strong adjacent channel signals to be downconverted to the IF, thus corrupting the wanted signal.

- In a transmitter, an oscillator with high phase noise will cause the transmitted data to spread to adjacent channels.

5 (continued)

The Impulse Sensitivity Function (ISF) of an oscillator describes the excess phase (ie phase error) which will result if a unit impulse current is injected into a given node of the oscillator at time  $t = \tau$ .

The magnitude of the resulting phase error  $\phi(\tau)$  will depend on the point in the oscillator cycle at which we injected the noise impulse.

Measurements have shown there is a linear relationship between injected current impulse  $i(\tau)$  and resulting phase error  $\phi(\tau)$ :

$$\phi(\tau) = \frac{\Gamma(\omega_0 \tau) i(\tau)}{q_{\max}}$$

$\Gamma(\omega_0 \tau)$  is the oscillator ISF which will be periodic, with period  $T_0 = 2\pi/\omega_0$  ( $\omega_0 =$  oscillation frequency).  $q_{\max}$  is the maximum charge stored at that node of the oscillator (eg.  $q_{\max} = C \cdot V_{\max}$ ).

The total phase error resulting from all noise current impulses can be found by superposition:

$$\phi(t) = \frac{1}{q_{\max}} \int_{-\infty}^t \Gamma(\omega_0 \tau) i(\tau) d\tau$$



5 (continued)

Since  $\Gamma(\omega_0 z)$  is periodic it can be represented by a Fourier Series expansion:

$$\Gamma(\omega_0 z) = \frac{C_0}{2} + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 z)$$

$$\text{Thus } \phi(t) = \frac{1}{q_{\max}} \left\{ \frac{C_0}{2} \int_{-\infty}^t i(z) dz + \sum_{n=1}^{\infty} C_n \int_{-\infty}^t i(z) \cos(n\omega_0 z) dz \right\} \quad (11)$$

The above expression shows that only dc/low frequency noise currents  $i(z)$  will cause significant phase error due to multiplication with the ISF dc component  $C_0/2$ . All other freqs will be attenuated by the averaging nature of the integration.

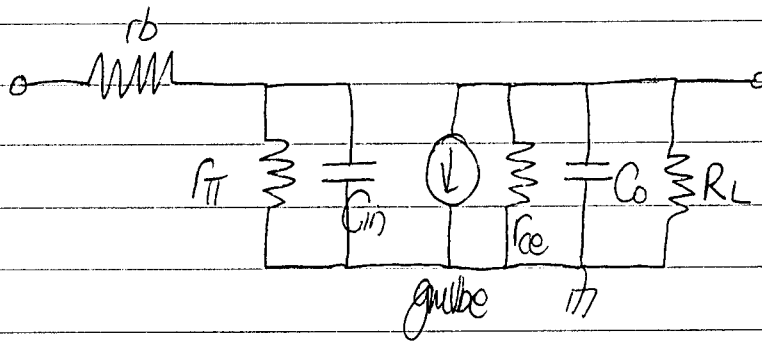
Similarly, only noise currents of frequency close to  $n\omega_0$  will cause significant phase error due to multiplication by the ISF component  $C_n \cos n\omega_0 z$ . All other frequencies will tend to average to zero. (4)

Thus oscillator A would be expected to exhibit the lowest phase noise, since

(i) ISF<sub>A</sub> is triangular while ISF<sub>B</sub> is more square. ISF<sub>B</sub> will thus contain higher amplitude harmonics ( $C_n$ ), leading to higher phase error. (3)

(ii) ISF<sub>B</sub> has a significant dc component, i.e.  $C_0$  is non-zero. This term will multiply with low frequency noise currents, which are generally higher due to  $1/f$  noise. ISF<sub>A</sub> has no such dc component. (3)

⑥ Miller approximation:



$$C_{in} = C_{\pi} + C_u(1 + g_m R_L')$$

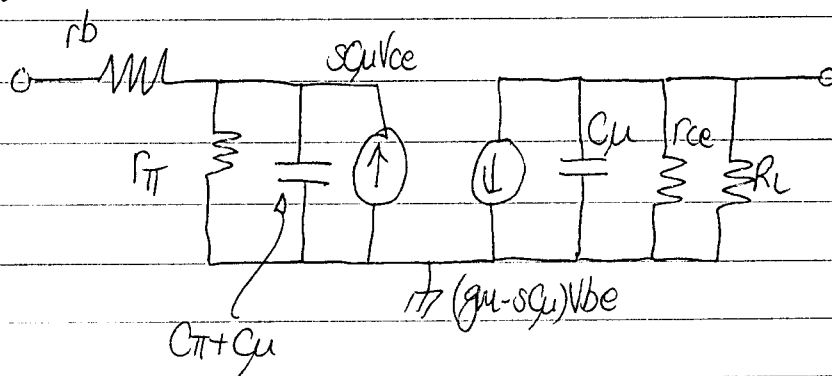
$$C_o \approx C_u$$

$$R_L' = R_L || r_{oe}$$

③

Once we are above the dominant pole of the amplifier (the input pole), the Miller gain is no longer equal to  $-g_m R_L'$ , but has rolled off. Thus the simple Miller model is not valid at frequencies approaching  $f_T$  where we are above this dominant pole.

RF hybrid- $\pi$  model:



Input current source  $sC_u v_{oe}$  shows feedback of output signals at high frequency via  $C_u$ .

Output current source  $(g_m - sC_u) v_{be}$  is modified to show feedforward of signals through  $C_u$ .

③

B (continued)

The Miller model predicts a dominant pole at

$$\omega_1 = \frac{1}{(r_b \| r_{\pi})(C_{\pi} + C_u(1 + g_m r_o))}$$

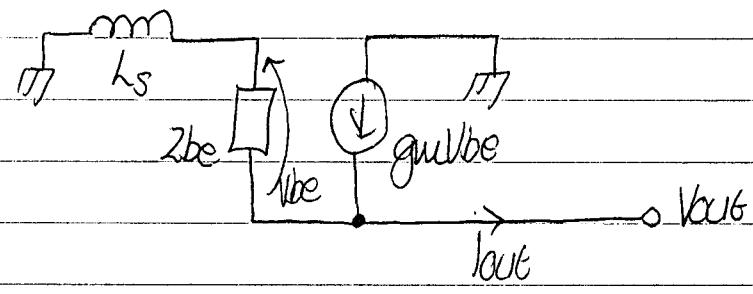
& a non-dominant pole at  $\omega_2 \approx \frac{1}{R_i' C_u}$

The RF model predicts the same location for the dominant pole, but a higher frequency for the non-dominant pole. In addition, a rhp zero exists at  $s = -g_m / C_u$

(3)

B (continued)

Simplified circuit:



(2)

$$I_{out} = g_m U_{be} + U_{be}/z_{be} = U_{be}(g_m + 1/z_{be})$$

$$V_{out} = I_{out}(sL_s + z_{be}) = \frac{U_{be}(sL_s + z_{be})}{z_{be}}$$

$$V_{out} = \frac{I_{out}}{(g_m + 1/z_{be})} \cdot \frac{(sL_s + z_{be})}{z_{be}}$$

$$\frac{V_{out}}{I_{out}} = \frac{sL_s + z_{be}}{1 + g_m z_{be}}$$

(2)

At  $\omega \rightarrow 0$ ,  $z_{be} \rightarrow r_{be}$  &  $sL_s \rightarrow \emptyset$

$$\frac{V_{out}}{I_{out}} = \frac{r_{be}}{1 + g_m r_{be}}$$

(2)

As  $\omega \rightarrow \infty$ ,  $z_{be} \rightarrow 1/s c_{be}$

$$\frac{V_{out}}{I_{out}} = \frac{sL_s + 1/s c_{be}}{1 + g_m/s c_{be}} = \frac{1 + s^2 C_{be} L_s}{g_m + s c_{be}}$$

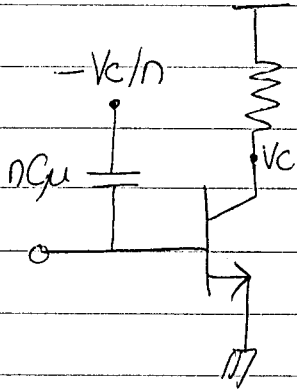
(2)

At high frequencies this could go negative & cause oscillation. At very high frequencies,  $Z_{out} = sL_s$  which could cause oscillation with a resistive load (2)

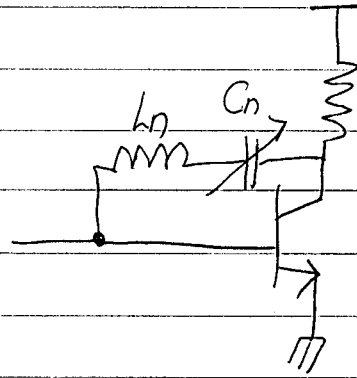
6 (continued)

Neutralisation of  $C_u$ :

(i) Discrete designs:



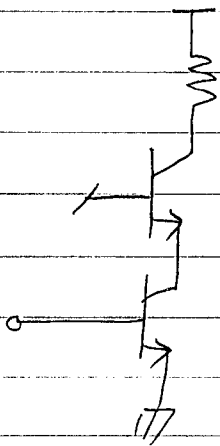
Inject a neutralising current



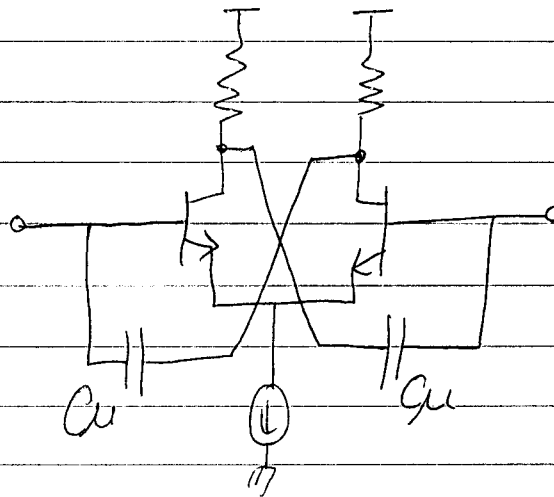
Use an inductor ( $L_{eq} = L_n - 1/\omega^2 C_n$ ) to resonate with  $C_u$ .

3

(ii) Integrated circuits



Cascode (also ok for discrete)



Cancellation using dummy capacitors.

3