[E417
AM1]

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
M.Sc. EXAMINATIONS 1999
PART IV: M.Eng. and ACGI EXAMINATIONS 1999

HIGH PERFORMANCE ANALOGUE ELECTRONICS

Thursday, 13 May: 10.00 - 13.00

There are SIX questions. ***Answer FOUR.***

All questions carry equal marks.

Corrected Copy

See Q. 5

Examiners responsible: A.J. Payne, C. Toumazou

Special instructions for invigilators:	**A Smith chart should be provided on each desk**
Information for candidates:	**A Smith chart is provided.**

1. *Figure 1a* shows a doubly-terminated passive LC-ladder lowpass filter. Give two advantages of the LC-ladder approach for implementing continuous-time filters. State two reasons why passive LC ladders are generally unsuitable for implementing fully-integrated filters. By constructing a signal flow graph of the ladder topology shown in *Figure 1a*, outline how this filter can be transformed into a topology suitable for integration, and sketch a block diagram of the resulting filter architecture. What is the advantage of transforming an LC ladder filter in this way rather than selecting an alternative architecture such as a cascade of biquads?

11 marks

Why is it preferable to avoid the use of floating capacitors in high frequency integrated circuit applications? Show how a transform of a section of the ladder filter topology shown in *Figure 1b* can eliminate the requirement for the floating capacitor C2, and construct a signal flow graph of this transformed section.

8 marks

Briefly describe the 'master-slave' approach for the continuous tuning of integrated filters.

6 marks

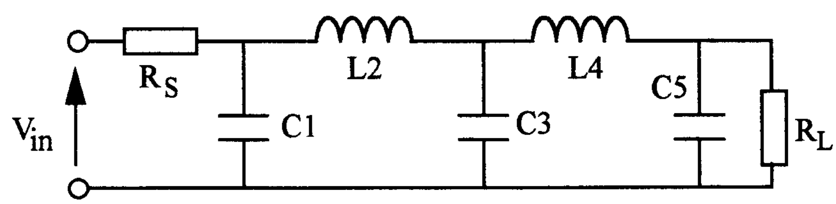

Figure 1a

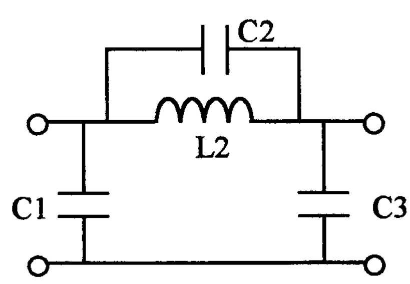

Figure 1b

2. Sketch and label the small-signal hybrid-π model of a bipolar junction transistor, including all major sources of noise within the device. Clearly name and label the noise sources and give expressions by which their mean square values may be calculated. By referring all noise sources to the input of the device, derive expressions for the equivalent input mean square noise voltage (v_n^2) and noise current (i_n^2). *10 marks*

Figure 2 shows a bipolar transistor Q1 which is used to amplify an input current, where components R_s and C_s represent the output impedance of the current source. Derive an expression for the total equivalent input-referred mean square noise current i_{eq}^2. Sketch the frequency response of i_{eq}^2, and calculate the midband noise figure given that $R_s = 300\ k\Omega$, $rb = 90\ \Omega$, $gm = 0.002$ S, $\beta = 100$. *13 marks*

The input signal is in the range 1 kHz – 70 kHz. Calculate the maximum knee frequency f_L and maximum source capacitance C_s to ensure that the total equivalent input noise current is minimised over this frequency range. *2 marks*

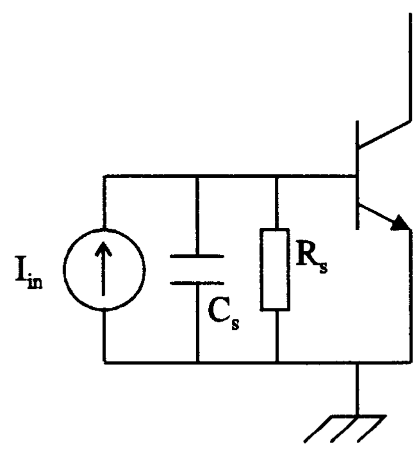

Figure 2

3. Briefly discuss the major difference between 'lumped' and 'distributed' RF circuit design. Outline why impedance matching networks are used in RF design, particularly when signals are routed on and off chip. *6 marks*

At an operating frequency of 915 MHz, the input impedance (Z_{in}) of a particular integrated circuit amplifier is as shown in *Figure 3*. By using a Smith Chart, design a passive matching network to maximise the power transfer into the amplifier from a source resistance of 50 Ω at this operating frequency. *13 marks*

The small-signal bandwidth of a common-emitter gain stage is limited by the 'Miller multiplication' of the base-collector junction capacitance C_μ. Describe two different methods for neutralising the effects of C_μ, one which is suitable for discrete circuit implementation and one which is suitable for integrated circuit implementation. *6 marks*

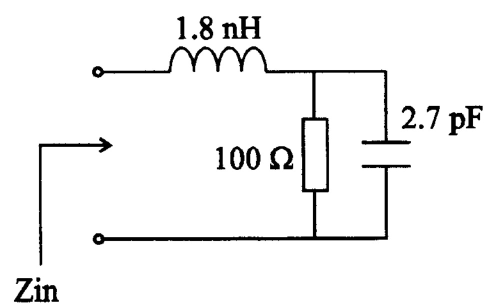

Figure 3

4. *Figure 4* shows a block diagram of a direct conversion paging receiver for detecting two-level frequency-shift keyed (FSK) modulated signals. Briefly explain the operation of this receiver, outlining the function of each of the blocks shown.

13 marks

The next generation of paging receivers is likely to use four-level FSK modulation. Outline the difference between two-level and four-level FSK, and sketch one possible block-diagram architecture for a four-level FSK paging receiver. *9 marks*

The mixers in the receiver of *Figure 4* are implemented as bipolar double-balanced mixers (Gilbert mixers). In a switching mixer, the local oscillator (LO) input is driven by a square wave rather than a sinusoidal input. Give two advantages and one disadvantage of a switching mixer compared to a mixer with a sinusoidal local oscillator signal. *3 marks*

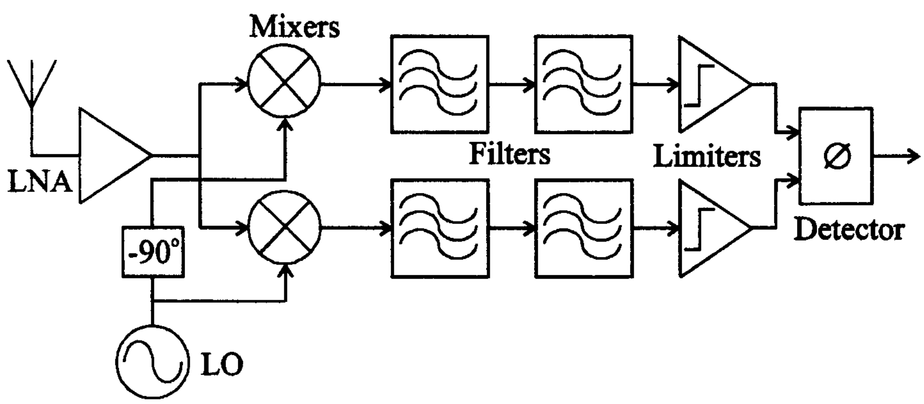

Figure 4

5. A terrestrial TV receiver is required to receive signals at carrier frequencies between 60 MHz and 900 MHz which must be converted down to an IF of 45 MHz. Two alternative architectures have been suggested to implement the receiver:

(i) single conversion

(ii) double conversion

With the aid of a block diagram, discuss the advantages and disadvantages associated with each of these approaches. Outline the specifications of critical components within each architecture in terms of frequency range, tuneability etc., and discuss the suitability of each of these possible architectures for implementing a fully-integrated front-end TV receiver. *17 marks*

A receiver with no front-end Low Noise Amplifier (LNA) is measured to have a sensitivity of -120.8 dBm. The input of the receiver is power-matched and the equivalent noise bandwidth of the receiver is 20 kHz. The required signal-to-noise ratio (SNR) at the intermediate frequency (IF) stage detector is 3 dB. Calculate the noise figure of this receiver with no LNA.

You wish to reduce the noise figure of this receiver by connecting a LNA at the front-end. Two different LNAs are available, with the following noise figure (NF) and power gain (G) values: (i) NF = 3 dB, G = 15 dB, (ii) NF = 2.2 dB, G = 9.5 dB. Which LNA should you choose? Show clearly the reasons for your choice. *8 marks*

Boltzmann's Constant $k = 1.3807 \times 10^{-23}$ (announced 12:30)

6. What is meant by the term oscillator 'phase noise', and how is phase noise formally defined? Give one reason in each case why oscillator phase noise is a problem for wireless communications (i) at the transmitter (ii) at the receiver. Given that phase noise generally has a $1/\omega^2$ characteristic, write down an expression for calculating the phase noise P_X at a frequency offset $\delta\omega_x$, given that the phase noise P_o at frequency offset $\delta\omega_o$ is already known. *7 marks*

A sinusoidal oscillator whose instantaneous frequency is modulated can be represented as $v(t) = V_m \sin(\theta(t) + \phi(t))$, where $\frac{d\theta(t)}{dt} = \omega_o$ is the unmodulated carrier signal frequency and $\frac{d\phi(t)}{dt} = \Delta\omega(t)$ represents the instantaneous fluctuation in frequency. For the simple case when $\Delta\omega(t) = \Delta\omega_X \sin \delta\omega_X t$, show that the resulting phase variation becomes translated to frequency sidebands, and derive an expression for the relative sideband to carrier power. *7 marks*

Using the results above, derive an expression for the total mean square residual FM of the carrier signal due to phase noise sidebands between ω_1 and ω_2. Hence calculate the rms residual modulation of an oscillator due to phase noise sidebands between 500 Hz and 2 kHz offset, given that the measured phase noise at 1 kHz offset is – 75 dBc. *11 marks*

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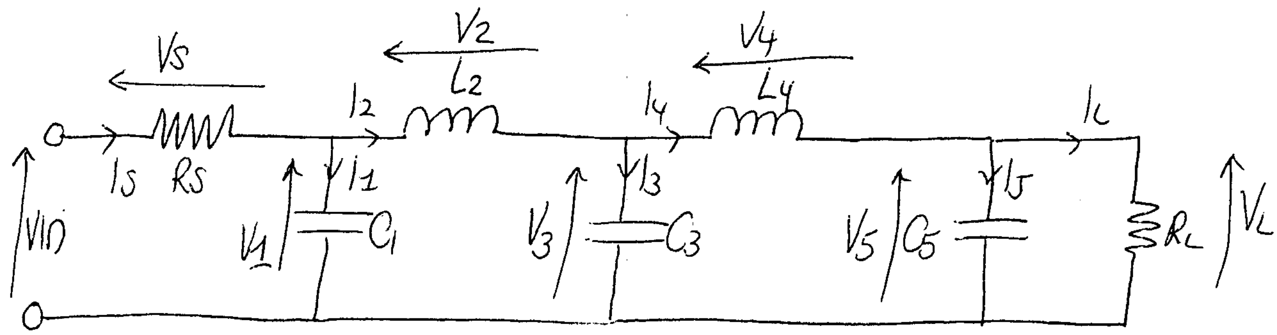
E4.17 High Performance Analog Electronics 1999

Solutions

(1)

- Low sensitivity to component tolerances
- Insensitive to parasitic capacitances (a design capacitor typically exists at each node)
- Component values for an extensive range of filter specifications are already tabulated. (2)

- Integrated inductors are not generally available
- A passive filter cannot be tuned to correct for component variations (tolerances). (2)



Ladder equations:

$V_S = V_{in} - V_1$ $\quad I_1 = I_S - I_2$ $\quad V_2 = V_1 - V_3$

$I_S = V_S / R_S$ $\quad V_1 = I_1 / sC_1$ $\quad I_2 = V_2 / sL_2$

$I_3 = I_2 - I_4$ $\quad V_4 = V_3 - V_5$ $\quad I_5 = I_4 - I_L$

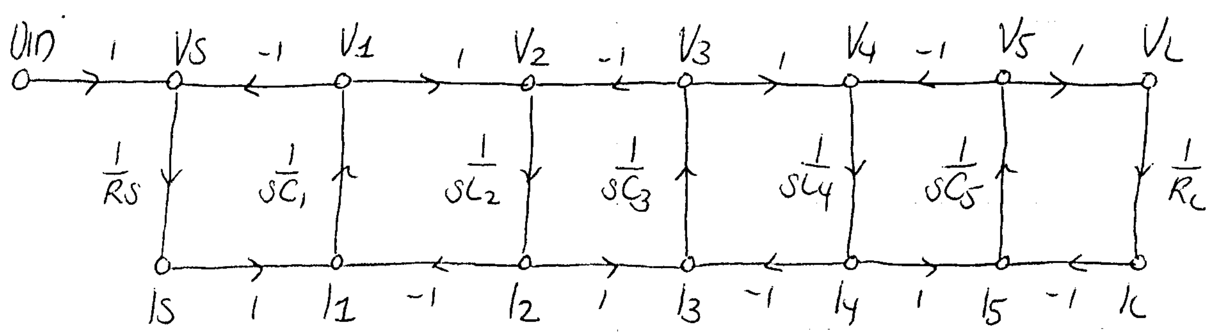
$V_3 = I_3 / sC_3$ $\quad I_4 = V_4 / sL_4$ $\quad V_5 = I_5 / sC_5$

$V_L = V_5$

$I_L = V_L / R_L$ (2)

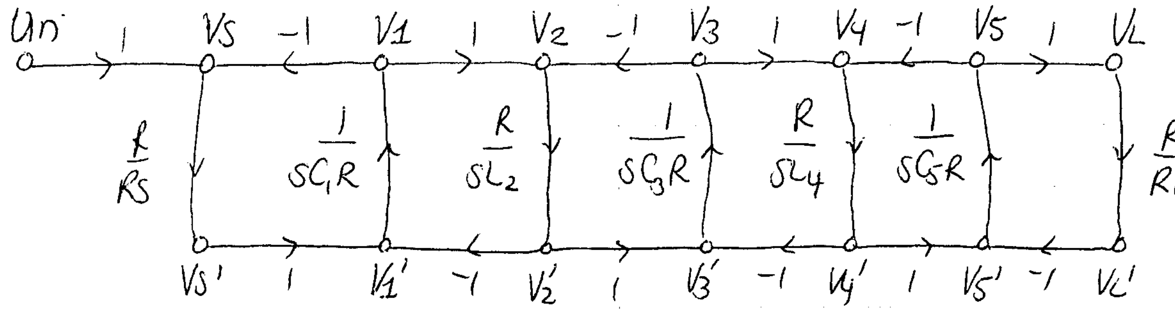
Q1 ---

Signal Flow Graph.



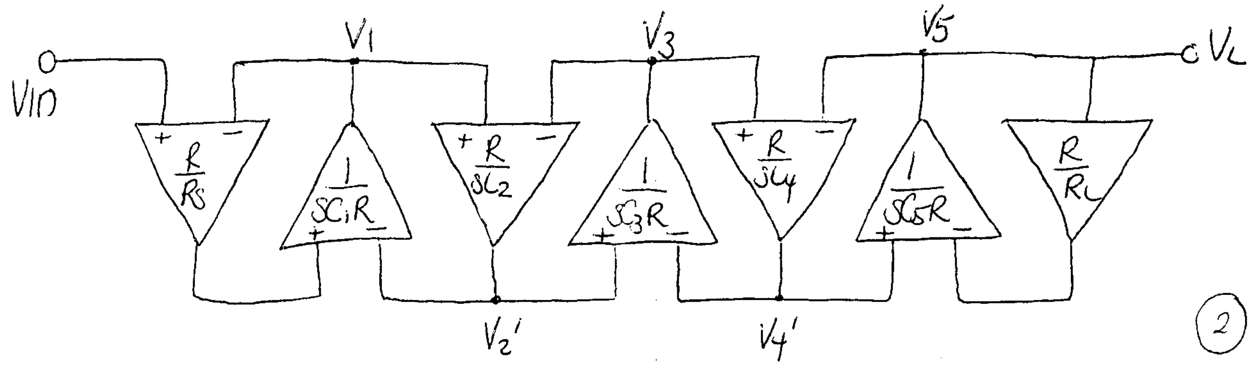
(1)

Scaled SFG:



(1)

Implementation using amplifiers & summing integrators:



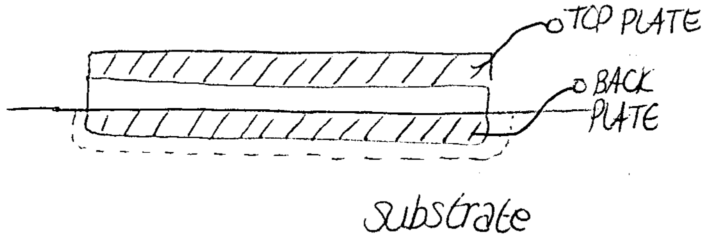
(2)

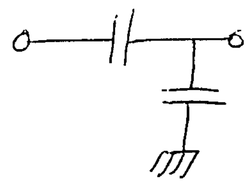
An active implementation of a passive doubly-terminated LC ladder retains the low sensitivity properties of the original prototype.

(1)

$\phi 1$ ---

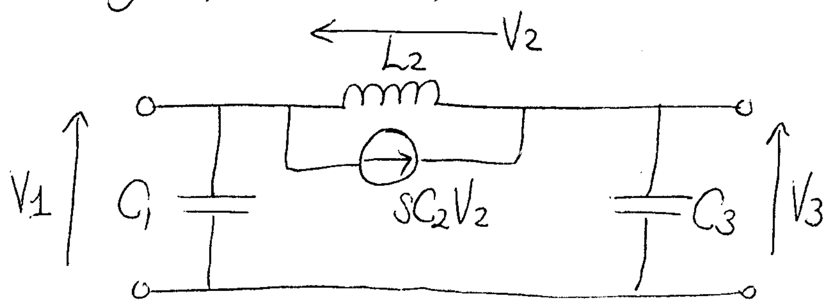
IC capacitors exhibit a parasitic capacitance from the back plate to the substrate, typically 10-15% of the design capacitor value. If the capacitor is used with one side connected to ac ground, then the effect of the back plate parasitic can be eliminated, provided that the back plate is grounded.
However if a floating capacitor is required then the parasitic cannot be shorted out.



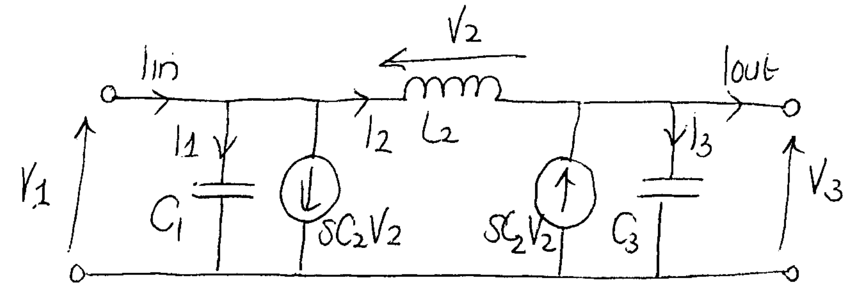


3

Floating capacitor can be eliminated as follows:



1



1

$I_1 = I_{in} - I_2 - sC_2V_2$ $\quad\quad$ $V_2 = V_1 - V_3$

$V_1 = I_1 / sC_1$ $\quad\quad$ $I_2 = V_2 / sL_2$

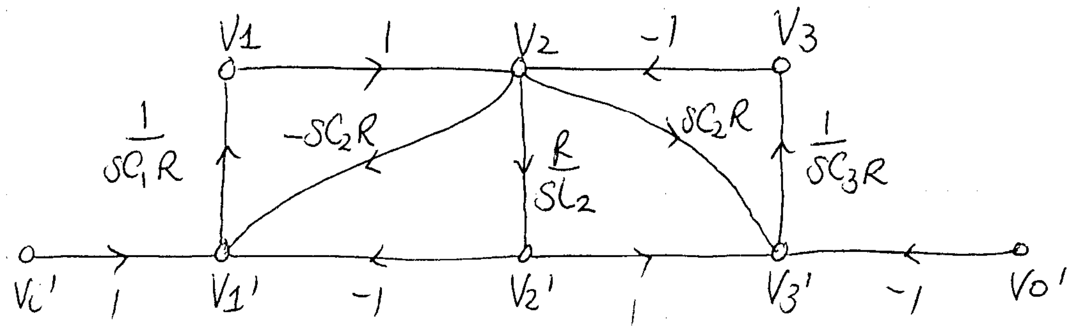
$I_3 = I_2 + sC_2V_2 - I_{out}$

$V_3 = I_3 / sC_3$

1

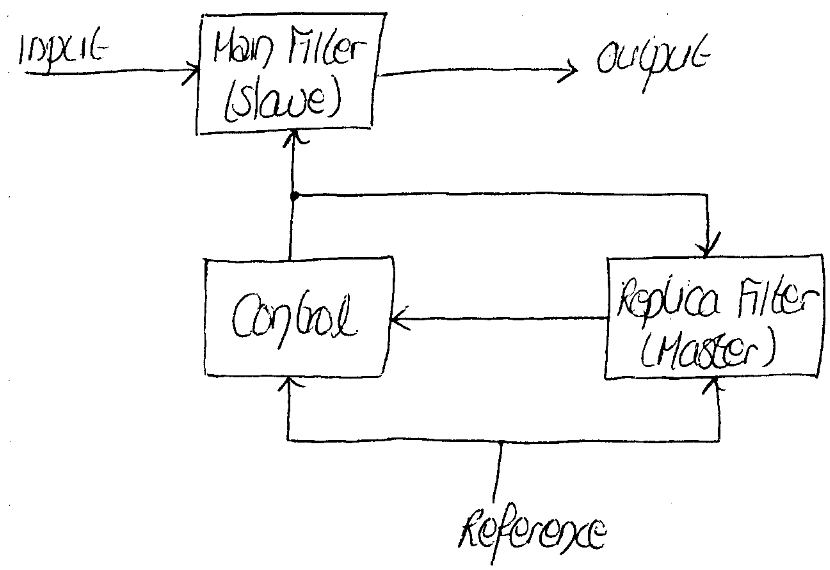
Q1 - -

Scaled & transformed SFG :



2

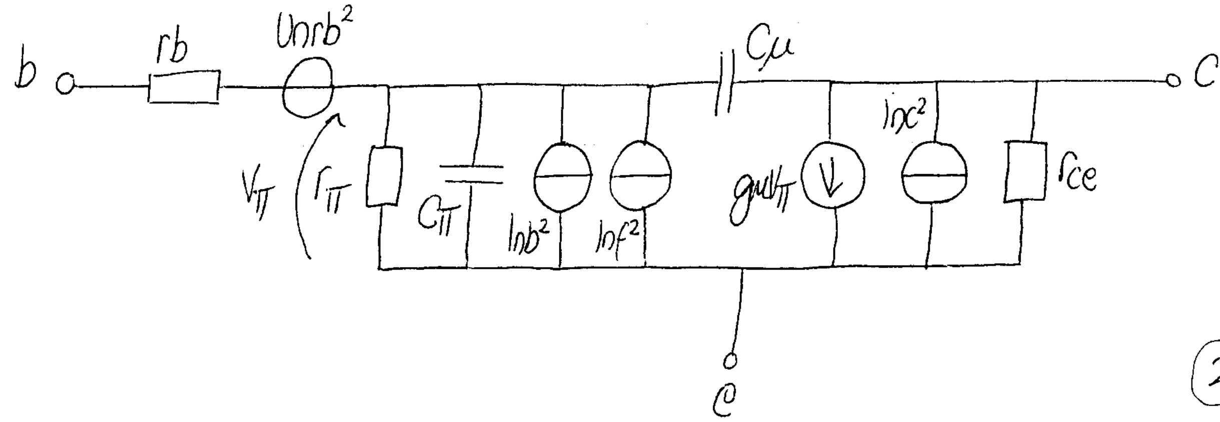
MASTER-SLAVE TUNING



2

Main filter (slave) performs the required signal processing. The master filter is a replica of all or part of the slave, sufficient to model the filter behaviour. A reference signal is applied to the master; a frequency control block detects any errors in the response of the master & generates a tuning signal which is applied simultaneously to the master & the slave. This tuning scheme thus relies on close matching between the master & slave circuits. 4

Q2.



(2)

$V_{nrb}^2 = 4kTr_b \ V^2/Hz$ Thermal noise due to series r_b

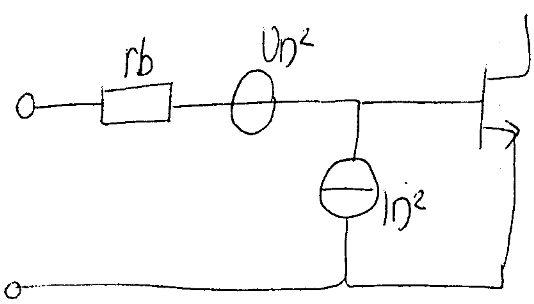
$I_{nb}^2 = 2qI_b \ A^2/Hz$ Base current shot noise

$I_{nf}^2 = 2qI_b \frac{f_L}{f} \ A^2/Hz$ Base current flicker noise

$I_{nc}^2 = 2qI_c \ A^2/Hz$ Collector current shot noise

(4)

Referring all noise sources to the input :



$$V_n^2 = V_{nrb}^2 + \frac{I_{nc}^2}{g_m^2} \quad V^2/Hz$$

$$= 4kTr_b + 2qI_c/g_m^2 = 4kT\left(r_b + \frac{r_e}{2}\right) \ V^2/Hz$$

since $g_m = \frac{I_c}{V_T}$

(2)

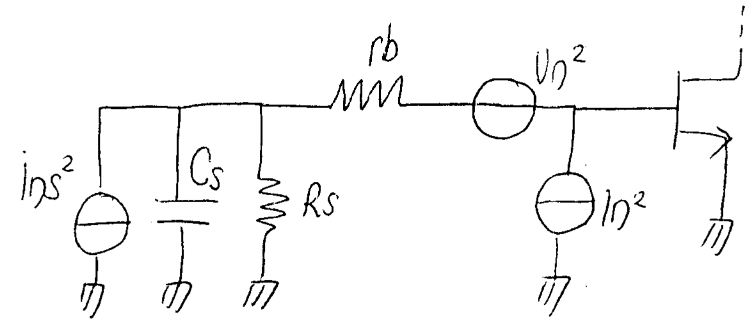
Q2 --

$$In^2 = Inb^2 + Inf^2 = 2q\,Ib(1 + f_L/f) \quad A^2/Hz$$

$$I_B = \frac{I_C}{\beta} = \frac{kT}{q\beta re}$$

$$\therefore \; in^2 = \frac{2kT}{\beta re}(1 + f_L/f) \quad A^2/Hz \qquad (2)$$

Equivalent circuit for noise calculation:



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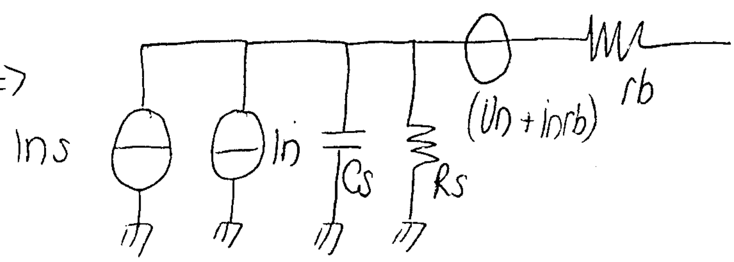


$$ieq = Ins + in + \frac{Vn}{Rs} + Vn\,j\omega Cs + \frac{Inrb}{Rs} + Inrb\,j\omega Cs$$

$$= Ins + in(1 + rb/Rs + j\omega Cs\,rb) + \frac{Vn}{Rs}(1 + j\omega Cs\,Rs)$$

$$ieq^2 = Ins^2 + In^2\left((1 + rb/Rs)^2 + (\omega Cs\,rb)^2\right) + \frac{Vn^2}{Rs^2}\left(1 + (\omega Cs Rs)^2\right)$$

Since $rb \ll Rs$, $(1 + rb/Rs)^2 \simeq 1$

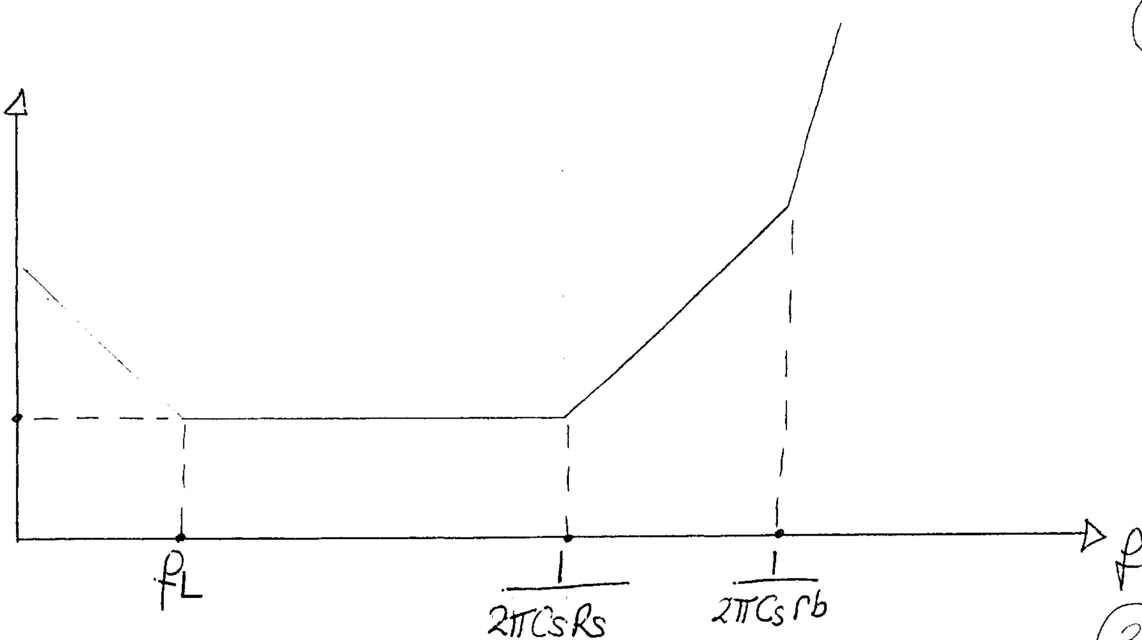
Q2 --

Thus

$$i_{eq}^2 = \frac{4KT}{R_S} + \frac{2KT}{\beta r_e}\left(1+\frac{f_L}{f}\right)\left(1+(\omega C_s r_b)^2\right) + \frac{4KT(r_b+r_e/2)\left(1+(\omega C_s R_s)^2\right)}{R_S^2}$$

$$= 4KT\left\{\frac{1}{R_S} + \frac{1}{2\beta r_e}\left(1+\frac{f_L}{f}\right)\left(1+(\omega C_s r_b)^2\right) + \frac{(r_b+r_e/2)\left(1+(\omega C_s R_s)^2\right)}{R_S^2}\right\}$$

A^2/Hz. (8)



(3)

Midband noise

$$i_{eq}^2 = 4KT\left\{\frac{1}{R_S} + \frac{1}{2\beta r_e} + \frac{(r_b+r_e/2)}{R_S^2}\right\} \quad A^2/Hz$$

NF :

$$\frac{i_{eq}^2}{i_{ns}^2} = \left(1 + \frac{R_s}{2\beta r_e} + \frac{r_b+r_e/2}{R_s}\right)$$

(2)

$$= \left(1 + \frac{300K(\cdot 0002)}{200} + \frac{90+500/2}{300K}\right) \simeq 4. = 6\,dB$$

Require $f_L < 1\,KHz$ & $1/2\pi C_s R_s > 70\,KHz$

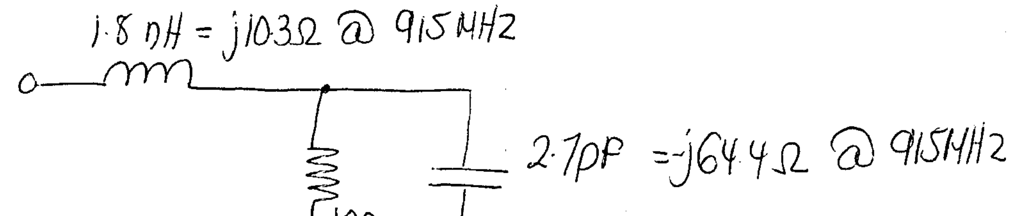
i.e $C_s < 7.5\,pF$ (2)

Q3.

'Distributed' designs are implemented when signal wavelengths are comparable to the dimensions of components in the circuit. Circuit components act as transmission lines & we must consider characteristic impedances.
'Lumped' design techniques are used when signal wavelengths are much greater than circuit/component dimensions. We can assume that voltages at all points along a wire are equal rather than being a travelling wave. (3)

When RF signals are routed on & off chip the package & pcb dimensions are likely to be comparable to the signal wavelength, ie distributed system. By using matching networks we ensure that impedance levels are matched, which thus avoids (travelling wave) reflections from occurring. Maximum signal power is thus transferred from one stage to the next. (3)

Matching network design:



$100\Omega \, // -j64.4 \Rightarrow$ normalised (1)

$Z_{in}' = 2\Omega \, // -j1.3$ (1)

$y_{in}' = 0.5 + j0.776$ (point A on Smith Chart). (1)

Q3 ---

Convert to series equivalent:

$Z_B = 0.6 - j0.9$ (1)

Add 1.8nH of series inductance = $j10.3\Omega$ = $j0.2\Omega$ normalised (1)

$Z_B' = 0.6 - j0.7$ (1)

Continue adding series inductance until point C

$Z_C = 0.6 + j0.5$ (1) i.e. add $j1.2\Omega$ normalised series L (1)

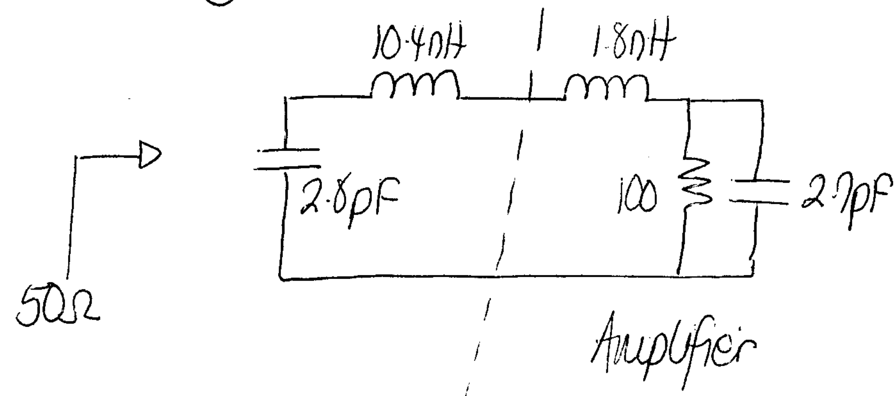
i.e. $L = 10.4\,nH$ (1)

Convert to parallel equivalent, $y_D = 1.0 - j0.8$ (1)

Add $j0.8\Omega$ normalised parallel capacitance (1)

i.e. $C_p = 2.8\,pF$ (1)

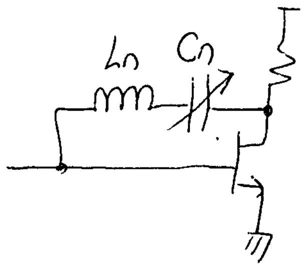
i.e. Matching network:



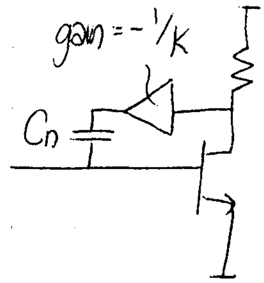
(1)

Q3 ---
Neutralisation of C_μ

(i) Discrete circuit implementation (3)



OR

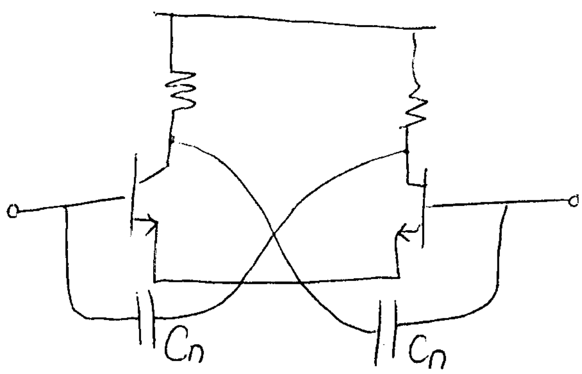


$C_n = kC_\mu$

$L_{eq} = L_n - 1/(\omega^2 C_n)$
Inductive neutralisation

Current injection

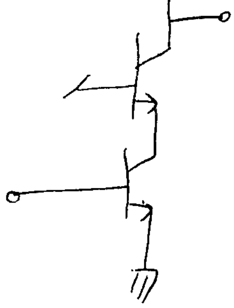
(ii) Integrated circuit implementation (3)



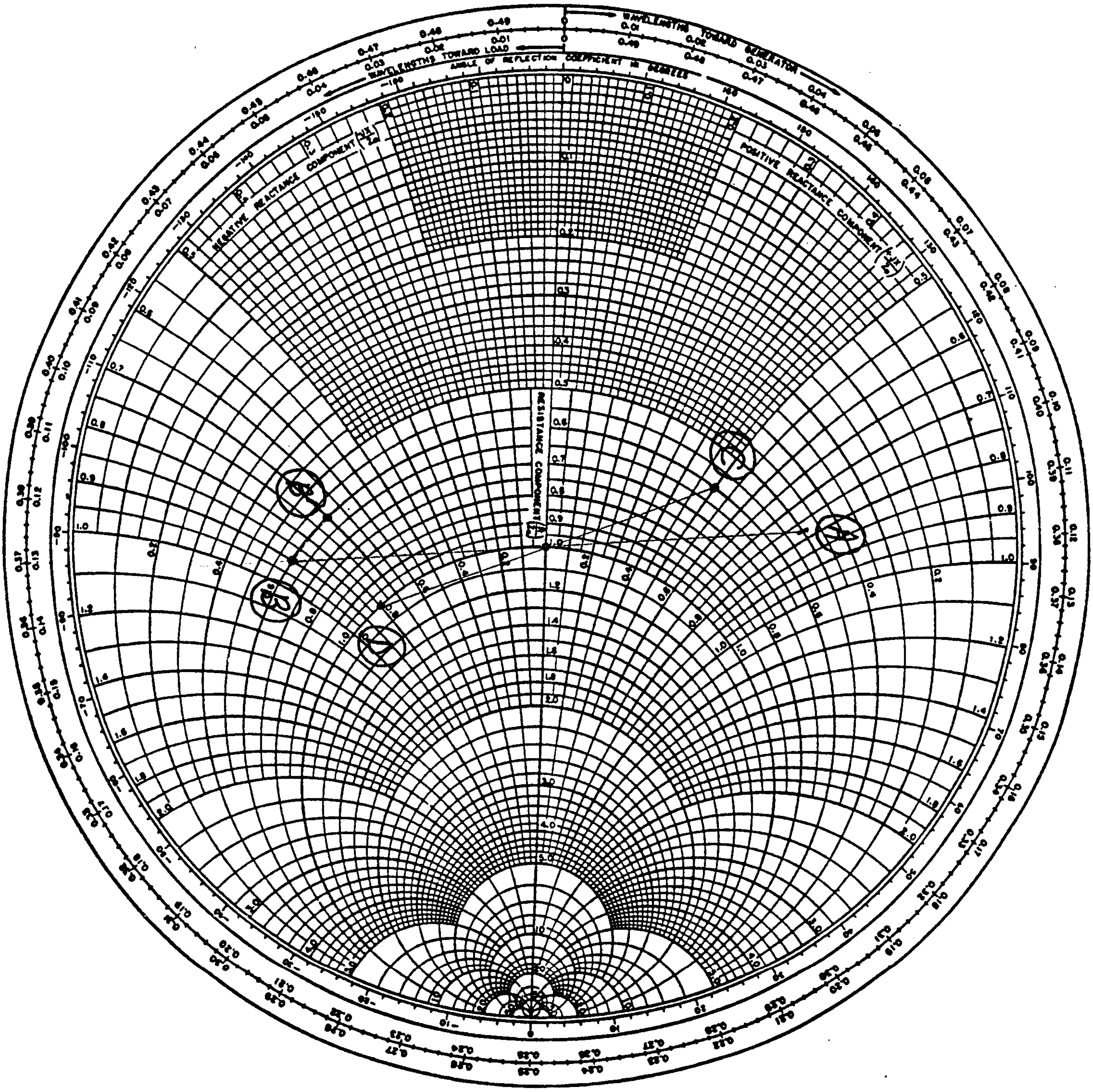
$C_n = C_\mu$ (dummy devices).

Neutralisation capacitors.

OR CASCODING (discrete or integrated)



Q3



Q4

2-level FSK

Data 1 = $f_c - \Delta f$ } f_c = RF carrier frequency
Data 0 = $f_c + \Delta f$ } Δf = deviation freq., typically 4-5 kHz.

- The LNA increases the sensitivity of the receiver by reducing the input-referred noise contribution from the mixers (1)

- The mixers multiply the RF signal with a locally-generated oscillator signal (LO). The two mixers have identical RF inputs but quadrature (90° phase shifted) LO inputs. (2) The LO operates at the RF carrier frequency f_c, thus the wanted RF signal is downconverted to $f_c - (f_c \pm \Delta f) = \pm \Delta f$. (1)

Signals at all other frequencies are due to adjacent channel transmission & are thus rejected by the filters. Since a fairly steep cut-off is required, a high-order lowpass filter is typically implemented by simulating an LC-ladder prototype. This channel-select filter is typically preceeded by an R-C filter with wider bandwidth but wide dynamic range, to prevent very high level signals from saturating the active filters. (2)

The limiters turn the sinusoidal signals into ~~a~~ square waves which are suitable for detection. (1)

4 ___

Received signal = data 1

$RF = 2\cos(f_c - \Delta f)t \quad LOI = \cos f_c t \quad LOQ = \cos(f_c t - 90°)$

After filtering:

I-channel = $\cos -(\Delta f t) = \cos \Delta f t$

Q-channel = $\cos -(\Delta f t - 90) = \cos(\Delta f t - 90)$

I leads Q by 90°

Received signal = data 0

$RF = 2\cos(f_c + \Delta f)t \quad LOI = \cos f_c t \quad LOQ = \cos(f_c t - 90°)$

After mixing & filtering:

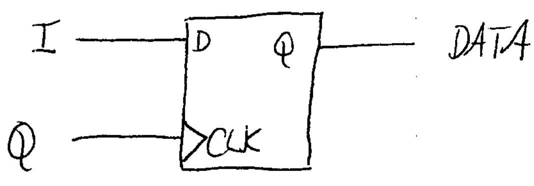
I-channel = $\cos(\Delta f t)$

Q-channel = $\cos(\Delta f t + 90°)$

Q leads I by 90°

Thus the 2-level FSK can be demodulated; the detector simply needs to detect whether the I channel is lagging or leading the Q channel. 3

Simple method:

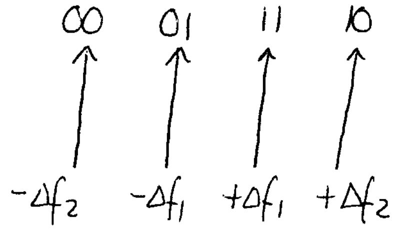


flip-flop.

This detector is not ideal because data is updated only on the rising edge of Q. It is better to combine a number of flip flops with logic to ensure that data is updated on the rising & falling edges of D & Q. 3

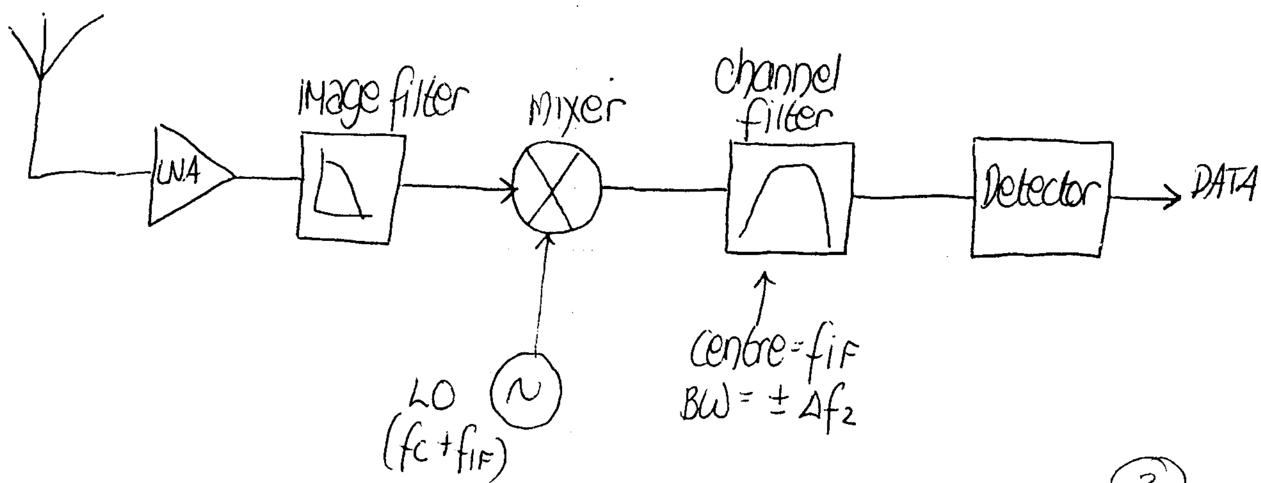
4 - - -

4-level FSK: Four frequencies are available ($f_c \pm \Delta f_1$, $f_c \pm \Delta f_2$). Each frequency represents 2 bits of data, ie.



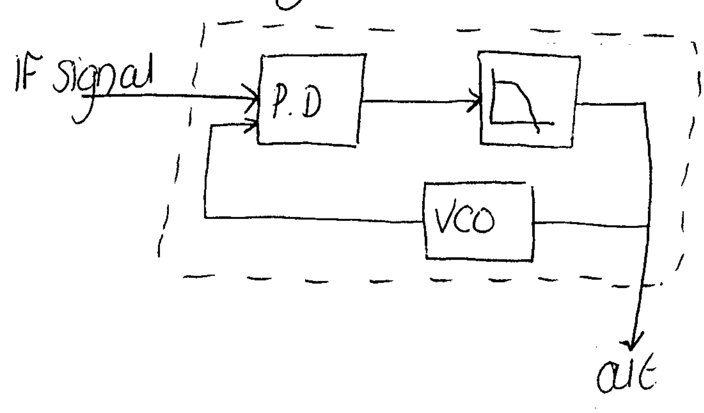
The simple quadrature direct conversion approach is no longer suitable, as the deviation frequency (& not just phase shift) is now important. (3)

Possible architecture:



(3)

Detector (frequency discriminator):



phase-locked loop.

(3)

Q4 ----

Switching mixer

Advantages

(i) Square waves are easier to generate than pure sinewaves (1)

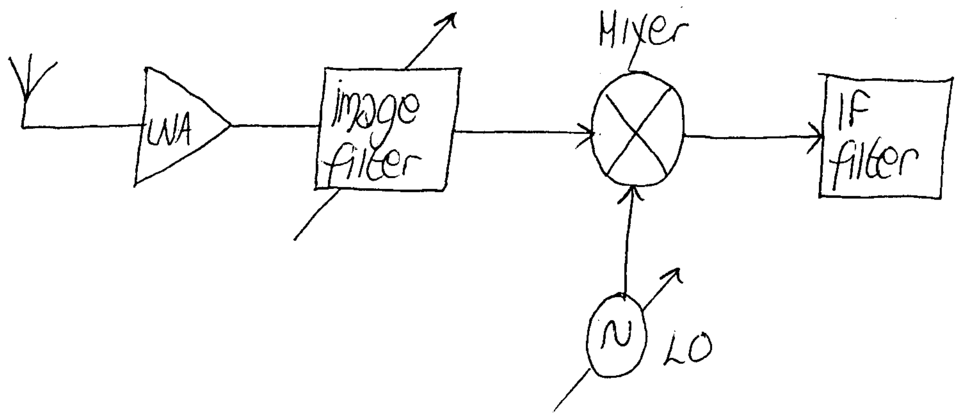
(ii) If the LO transistors are fully-switched on/off, they contribute little noise & thus the mixer noise figure is reduced. LO transistors will contribute significant noise only when they are switching over (1)

Disadvantage

(i) LO harmonics will mix with RF input creating many additional output signals which must be filtered out. (1)

Q5
TV signal, carrier frequencies 60M → 900MHz, IF = 45 MHz

(i) Single conversion



(2)

IF filter: bandpass filter (channel select) centered at 45 MHz.
LO: Must be tuneable from 105 MHz (60M RF) to 945 MHz (900M RF), i.e. 1→9 tuning range!
Image filter: Tuneable lowpass filter. Stopband edge tuneable from 150 MHz (60 M RF) to 990MHz (900 M RF)

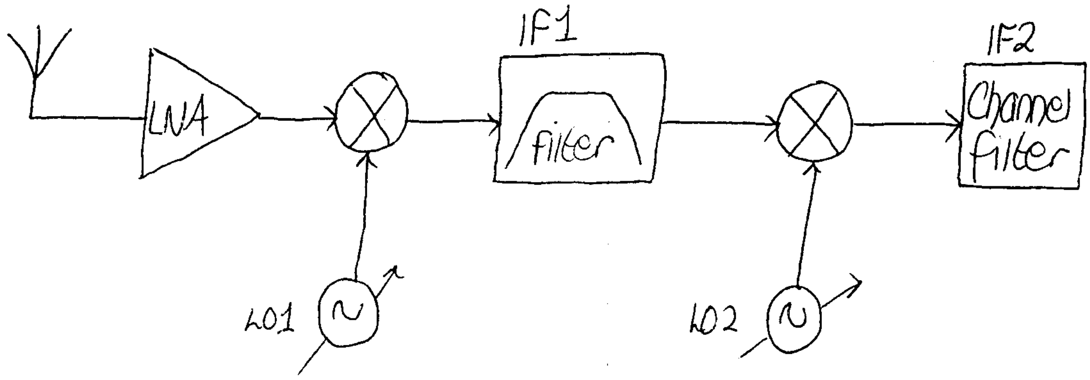
The tuning requirements for image filter & LO are impossible to achieve. Additionally, an LO with such wide tuning range is likely to exhibit poor phase noise, due to high gain (V → f) making the oscillator susceptible to noise on the control voltage.

One option is to split the signal into bands using a bank of filters. Each band is dealt with by a separate receiver section, eg 60-120M, 120-240M, 240-480M, 480-900M (4 bands). (4)

For integration: The mixers & perhaps oscillators could be fully integrated. Band-selective bandpass filters & tuneable image filters would be difficult to integrate. (2)

Q5 ---

(ii) Double conversion: The first conversion would have to be an upconversion, as the lower end of the RF band is already close to the final IF.
The first IF is not completely fixed, but varies over a lesser range than the input RF.



(2)

Choose a 1st IF of say 900M → 1.4GHz.

- LO1 must be tuneable from 840MHz (60M RF) to 500 MHz (900 M RF), i.e. less than 1–1.7 tuning range.

- Since the first mixing is an upconversion, there is no image signal, thus no pre-filter is required.

- LO2 must be tuneable from 945MHz (900M IF1) to 1.355 GHz (1.4 G IF1), i.e. less than 1→1.5 tuning range.

- Image signals from 2nd mixing process range from 855 MHz to 1.445GHz. Thus bandpass filter at IF1 can be fixed rather than tuneable. (5)

For integration: only fixed filters are required, which makes integration more feasible (though still difficult!)

However, the whole RF band is seen by the LNA & 1st mixers, which must thus have very high signal handling capability.

2

Sensitivity = P_{ni} (dB) + NF + SNR (det)

P_{ni} = − 174 dBm per Hz
= − 174 dBm + 10 log 20k
= − 131 dBm

Sensitivity = − 120.8 = −131 + NF + 3
NF = 7.2 dB

3

Total noise factor $F_T = F_1 + \frac{F_2 - 1}{G_1}$

1

F_2 = system noise factor = 7.2dB = 5.25

(i) NF = 3dB, F_1 = 2
G = 15dB = 31.62 F_T = 2.134 or 3.3dB

(ii) NF = 2.2dB F_1 = 1.66
G = 9.5dB = 8.91 F_T = 2.137 or 3.3dB

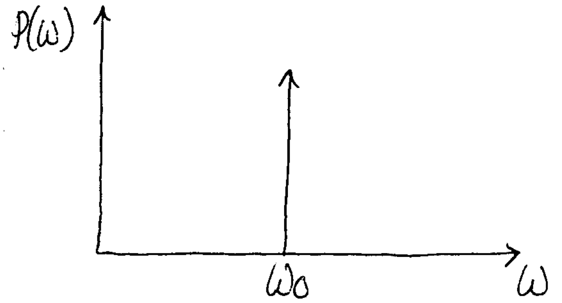
2

No difference!
I would choose either the cheapest, or choose (ii) since it has lower gain & thus will not saturate the mixers so quickly.

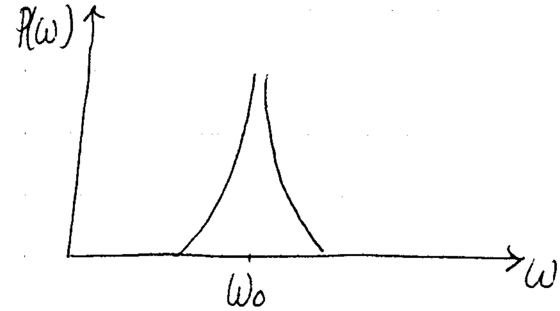
2

Q6.

The instantaneous frequency of any oscillator will fluctuate to some extent, and this can be seen as a 'spreading out' of the oscillator spectrum in the frequency domain:



ideal spectrum



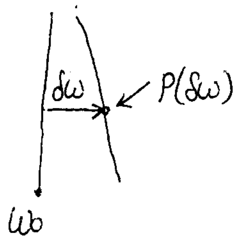
actual spectrum

Phase noise is formally defined as the noise power in a 1Hz bandwidth at an offset frequency $\delta\omega$, divided by the power of the carrier frequency ω_0, i.e

$$\mathcal{L}(\delta\omega) = \left\{ \frac{P(\omega_0 \pm \delta\omega)}{P(\omega_0)} \right\} \quad \text{dBc}$$ 3

Phase noise typically has a $1/\omega^2$ characteristic, i.e

$$P(\delta\omega) = \frac{K}{(\delta\omega)^2} \quad (K = \text{constant})$$



Thus $P_x(\delta\omega_x)^2 = P_0(\delta\omega_0)^2$

$$P_x = P_0(\delta\omega_0/\delta\omega_x)^2$$ 2

At the TX: Phase noise causes signals to spread to adjacent channels. At the RX: Phase noise causes adjacent channels to be downconverted to the IF also. 2

Q6

$$v(t) = V_m \sin\left(\theta(t) + \phi(t)\right)$$

$$\theta(t) = \omega_0 t \qquad \text{carrier}$$

$$\frac{d\phi(t)}{dt} = \Delta\omega_x \sin \delta\omega_x t \qquad \text{modulation}$$

$$\phi(t) = -\frac{\Delta\omega_x}{\delta\omega_x} \cos \delta\omega_x t$$

$$\therefore\ v(t) = V_m \sin\left(\omega_0 t - \frac{\Delta\omega_x}{\delta\omega_x} \cos\delta\omega_x t\right)$$

$$\sin(a-b) = \sin a \cos b + \cos a \sin b$$

$$\therefore\ v(t) = V_m \sin \omega_0 t \underbrace{\cos\left(\frac{\Delta\omega_x}{\delta\omega_x} \cos \delta\omega_x t\right)}_{\simeq 1 \ \left(\text{since } \frac{\Delta\omega_x}{\delta\omega_x} \ll 1\right)} + V_m \cos\omega_0 t \underbrace{\sin\left(\frac{\Delta\omega_x}{\delta\omega_x} \cos\delta\omega_x t\right)}_{\sin x \simeq x}$$

$$\therefore\ v(t) = V_m \sin\omega_0 t + V_m \cos\omega_0 t \frac{\Delta\omega_x}{\delta\omega_x} \cos\delta\omega_x t$$

$$= V_m \sin\omega_0 t + V_m \frac{\Delta\omega_x}{\delta\omega_x} \cos\omega_0 t \cos\delta\omega_x t$$

$$= \underbrace{V_m \sin\omega_0 t}_{\text{carrier}} + \underbrace{\frac{V_m}{2} \frac{\Delta\omega_x}{\delta\omega_x} \cos(\omega_0 t \pm \delta\omega_x t)}_{\text{sidebands}}$$

4

$$\text{Relative power} = \left(\frac{\Delta\omega_x}{2\delta\omega_x}\right)^2$$

Q6 ----

Relative sideband power due to a modulating signal of amplitude & frequency $\Delta\omega_x$, $\delta\omega_x$:

$$P_x = \left\{ \frac{\Delta\omega_x}{2\delta\omega_x} \right\}^2$$ (3)

Reversing this argument, if the relative sideband power P_x (ie phase noise) at an offset $\delta\omega_x$ is known, we can calculate the amplitude of a 'residual FM' signal of frequency $\delta\omega_x$:

$$(\Delta\omega_x)^2 = 4P_x(\delta\omega_x)^2$$ (3)

From 1st part of question, $P_x(\delta\omega_x)^2 = \text{constant} = P_o(\delta\omega_o)^2$

Thus $$(\Delta\omega_x)^2 = 4P_o(\delta\omega_o)^2$$ (2)

The total mean square residual FM of the carrier due to phase noise sidebands between $\delta\omega_1$ & $\delta\omega_2$:

$$\overline{(\Delta\omega_T)^2} = \int_{\delta\omega_1}^{\delta\omega_2} \frac{(\Delta\omega_x)^2}{2}\,\delta\omega = \int_{\delta\omega_1}^{\delta\omega_2} 2P_o(\delta\omega_o)^2\,\delta\omega$$

$$= 2P_o(\delta\omega_o)^2(\delta\omega_2 - \delta\omega_1)$$ (3)

or $$\overline{(\Delta f_T)^2} = 2P_o(\delta f_o)^2(\delta f_2 - \delta f_1)$$ $-75\text{dBc} = 3.2\times10^{-8}$

Thus $$\overline{(\Delta f_T)^2} = 2(3.2\times10^{-8})(1\times10^3)^2(2\times10^3 - 500)$$
$$= 94.86$$ (2)

$$\Delta f_T(\text{rms}) = 9.74\text{ Hz}.$$ (1)