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SO7

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2005

MSc and EEE PART IV: MEng and ACGI

Corrected Copy

TRAFFIC THEORY & QUEUEING SYSTEMS

Thursday, 28 April 10:00 am

Time allowed: 3:00 hours

There are FIVE questions on this paper.

Answer FOUR questions.

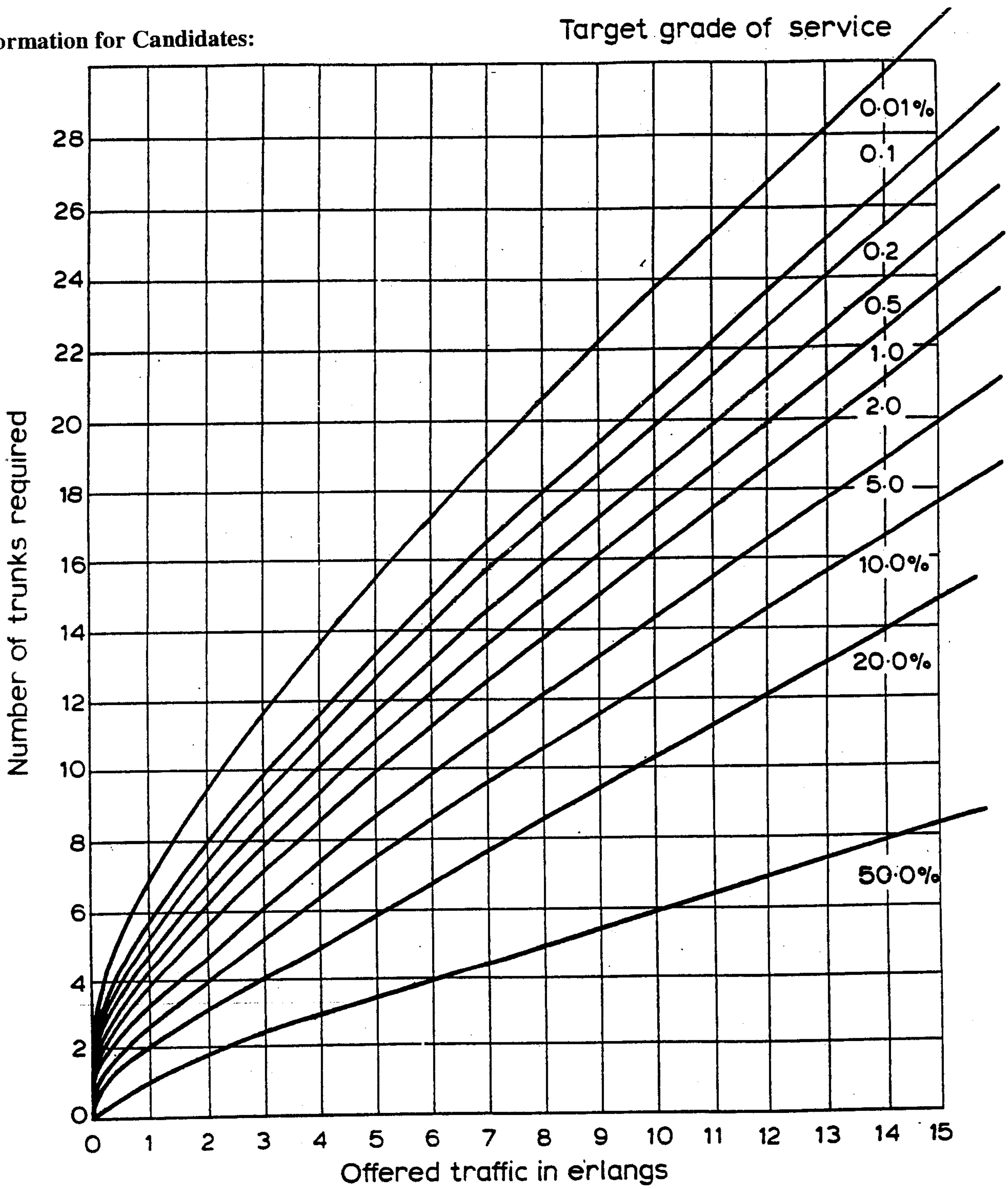
All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : J.A. Barria
Second Marker(s) : P. De Wilde

Especial Information for Invigilators: NIL

Information for Candidates:



Traffic capacity on basis of Erlang B. formula.

1.

a)

i) Describe and discuss underlying assumptions made when formulating the Erlang model.

ii) For the case of finite capacity system derive the probability of link saturation.

[10]

b) An N-channel link system is being offered ρ Erlangs of pure chance traffic.

i) Show that if the search for a free channel is always sequential (i.e. 1, 2, 3, …, N) the mean occupancy of channel number j is given by

$$\eta_j = \rho[E_{j-1}(\rho) - E_j(\rho)]$$

ii) Derive the average value of the channel occupancy in part (b) i). Discuss your results.

[10]

2

a)

i) Describe and discuss the usefulness of an Interrupted Poisson Process to describe an overflow link state.

ii) Define and discuss an overflow link traffic model using the on-off source model of Figure 2.1.

iii) Explain the meaning of α, β and λ.

iv) Derive the expressions of the global balance equations of an overflow link composed of one (1) channel.

[10]

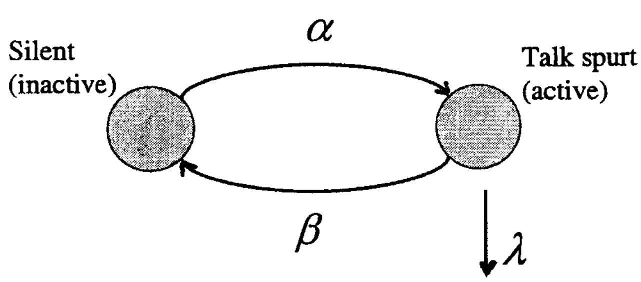


Figure 2.1.

b) A communications link with 8 channels is grouped so that half the incoming traffic have access to the first 4 channels and the other half have access to the other 4 channels. If the total offered traffic is 8 Erlangs of pure chance traffic

i) Determine the call congestion of the system.

ii) Determine the link occupancy distribution.

[10]

3.

a) Using mean-value analysis

i) Explain the meaning of and derive the expected residual time of an M/G/1 system.

ii) Derive the expression for the expected waiting time of an M/G/1 system.

State clearly all assumptions made.
Explain clearly all steps of your derivations.

[10]

b)

i) For a k-class priority queueing system, define and explain non-pre-emptive priority and pre-emptive priority mechanisms.

ii) Explain and derive an expression for the expected transit time in a pre-emptive priority system.

iii) Describe and discuss a priority mechanism that would reduce the system expected waiting time ($E(W)$).

[10]

4.

a)

i) Explain the importance of access control in ATM networks.

ii) The user parameter control technique proposed by the ATM forum has a number of equivalent representations. Explain the operation of one of such equivalent algorithms.

iii) Derive a simple approximation model of an access control algorithm known to you.

[10]

b) In an operation support system contact centre all incoming calls are handled on a delay basis by a group of 10 operators. Assume that incoming traffic is pure chance with a level of 8 Erlangs.

i) Determine the mean delay experienced by calls which are accepted but delayed for buffer capacity B = 5.

ii) Determine the mean delay experienced by calls which are accepted but delayed for buffer capacity B = 10.

[10]

5.

a)

i) Describe a multimedia traffic source model. Clearly define and describe all parameters of the model.

ii) Derive and depict a Markov model representation of a N on-off source multiplexor.

iii) Assuming that the service rate is ν and the arrival rate of cells is β cells/s. Derive the maximum number of multiplexed sources that the system can cope with.

[10]

b) For the fault tolerant system represented in Figure 5.1.

Assume that the repair time and time to failure are exponentially distributed and:

R_p; is the failure rate of each processor (assume it to be independent of the failure rate of the state of the other processors)

R_b; is the failure rate of the data base (assume that any failure in the data base results in the loss of the entire system) and

j/R_r; is the time to repair j faulty components

i) Obtain the Markov chain representing the fault tolerant system of Figure 5.1.

State clearly and discuss any assumptions made in your derivations.

[10]

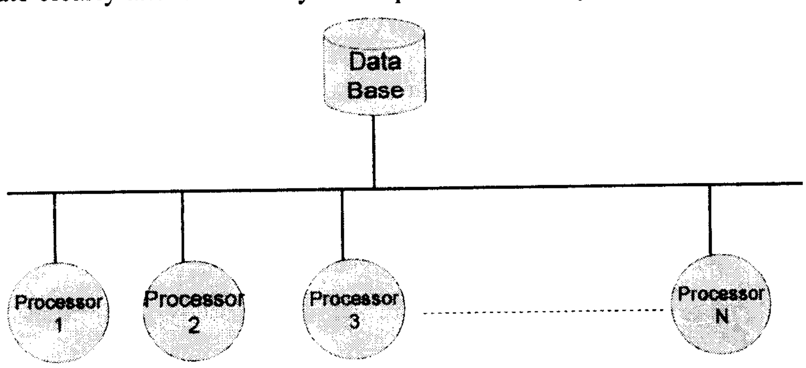


Figure 5.1:

Traffic Theory & Queueing Systems 2005

2004-5

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MODEL ANSWER and MARKING SCHEME		
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(a) Erlang Model Assumptions

i) The total arrival stream is a Poisson process with rate λ

ii) The channel holding times are independent exponential r.v with mean holding time of $1/\mu$

iii) The access switch gives full availability i.e each traffic source has access to all the channels on the link 5

ii) For $N < \infty$

$$= \frac{\rho^i}{i!}\pi_0 \qquad i = 0, 1, \ldots N$$

$$= \frac{1}{S}$$

$$\pi_i = \left[\frac{\frac{\rho^i}{i!}}{\sum_{j=0}^{N}\frac{\rho^j}{j!}}\right] \qquad i = 0, 1, \ldots N$$

$$P[\text{link Saturation}] = P[N_t = N]$$

$$= \pi_N$$

$$= \frac{(\rho^N/N!)}{S(N,\rho)} = E_N(\rho)$$ 5

$$S(N,\rho) = \sum_{j=0}^{N}\left(\frac{\rho^j}{j!}\right)$$

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1(a)

$c_1 \longrightarrow c_{j-1}$ = link of size $j-1$

overflow to $\{c_j, \ldots, c_N\} = \rho E_{j-1}(\rho)$

overflow to $\{c_{j+1}, \ldots, c_N\} = \rho E_j(\rho)$

Traffic carried by $c_j = \rho\,[E_{j-1}(\rho) - E_j(\rho)]$

$\Rightarrow \eta_j = \rho\,[E_{j-1}(\rho) - E_j(\rho)]$ 5

Average of η_j over all channels $c_1 \longrightarrow c_N$

$$\bar{\eta}_j = \frac{1}{N}\sum_{j=1}^{N} \eta_j = \frac{\rho}{N}\sum_{j=1}^{N}[E_{j-1} - E_j]$$

$$= \frac{\rho}{N}[E_0 - E_N]$$

$$= \frac{\rho}{N}[1 - B_c] = \eta$$

5

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2a IPP

i) In the Kosten model the arrival process of the overflow traffic consists of a Poisson arrival stream which is

ON when the first-choice link is saturated

OFF when the first-choice link is not saturated 2

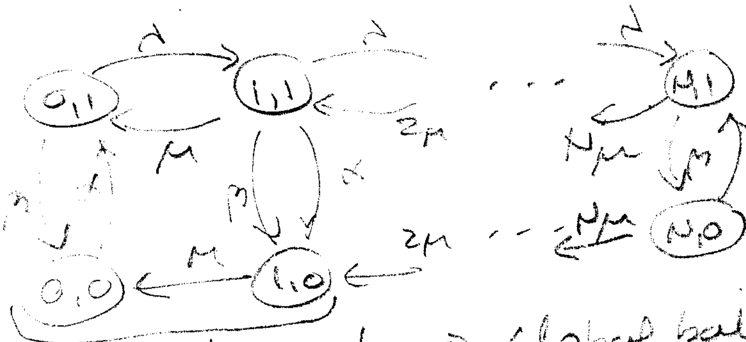
We can approximate this process to an ON/OFF switching process and can be represented by a 2-state Markov process (Figure 2.1)

ii) mean OFF period = $1/\alpha$

mean ON period = $1/\beta$ 2

- Poisson stream when system is ON

iii)



for a N channel overflow link 3

1 channel $\Rightarrow$ global balance equation

iv) $\alpha\pi_0 = \beta\sigma_0 + \mu\pi_1$

$\pi_0 = P(N_t = 0, Y_t = 0)$

$\sigma_0 = P(N_t = 0, Y_t = 1)$

...

$(\beta+\lambda)\sigma_0 = \alpha\pi_0 + \mu\sigma_1$ 3

$(\alpha+\mu)\pi_1 = \beta\sigma_1$

$[illegible]\,\sigma_1 = \alpha\pi_1 + \lambda\sigma_0$

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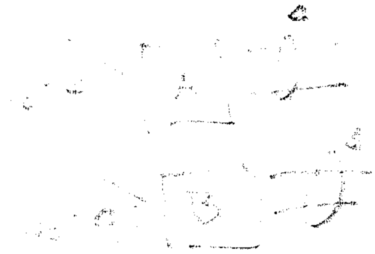
2(b)



$B_c = P[\text{arrival is blocked}]$

$= P[\text{arrival is on A}]\,P[\text{all A channel busy}]$

$+ P[\text{arrival is on B}]\,P[\text{all B channel busy}]$

$= \frac{1}{2}E_4(4) + \frac{1}{2}E_4(4) = 0.311$ 5

with occupancy distribution

$\pi_i' = P[\text{Total nr of busy channels} = i]$

$\pi_i = P[\text{Nr of busy A channels} = i]$

$= P[\text{Nr of busy B channels} = i]$

$\pi_0' = \pi_0^2$

$\pi_1' = 2\pi_0\pi_1$

$\pi_2' = (\pi_1)^2 + 2\pi_0\pi_2$

$\pi_3' = 2(\pi_0\pi_3 + \pi_1\pi_2)$

$\pi_4' = (\pi_2)^2 + 2(\pi_0\pi_4 + \pi_1\pi_3)$

$\pi_5' = 2(\pi_1\pi_4 + \pi_2\pi_3)$

$\pi_6' = (\pi_3)^2 + 2\pi_2\pi_4$ 5

$\pi_7' = 2\pi_3\pi_4$

$\pi_8' = (\pi_4)^2$

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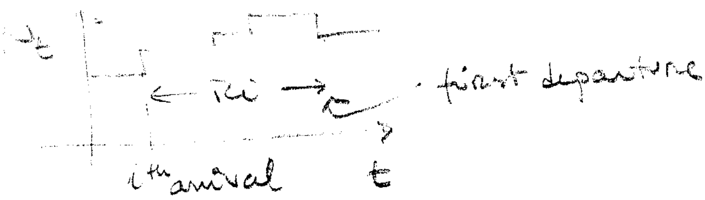
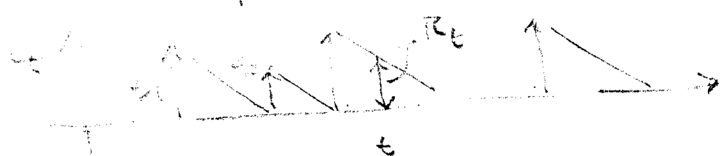
3a – = residual service time = time until the first departure seen by ith arrival



R_t = residual service time seen by a virtual arrival at time t

then at equilibrium $\{R_t\}$ is a continuous time stochastic process which look like



assuming that $\{R_t\}$ is Ergodic (in mean)

$$E(R_t) = \lim_{T\to\infty} \frac{1}{T}\int_0^T R_t\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\sum_{i=1}^{M_T}\left(\tfrac{1}{2} S_i^2\right)$$

M_T = no of completed services in $[0,T]$

$$E(R_t) = \lim_{T\to\infty} \frac{1}{2}\underbrace{\left(\frac{M_T}{T}\right)}_{\lambda}\underbrace{\left[\frac{1}{M_T}\sum_{i=1}^{M_T} S_i^2\right]}_{E(S^2)}$$

$$E(R_t) = \frac{1}{2}\lambda E(S^2)$$ 5

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3a)

S_i = service time
W_i = waiting time
Q_i = queue length found on arrival

ith arrival

$$W_i = \left[R_i + \sum_{j=1}^{Q_i} S_{i-j} \right]$$

$$E(W_i) = E(R_i) + E\left[\sum_{1}^{Q_i} S_{i-j} \right]$$

$$= E(R_i) + E\left[E\left(\sum_{1}^{Q_i} S_{i-j} \,\middle|\, Q_i \right) \right]$$

sum of Q_i iid RV

$$= E(R_i) + E\left[Q_i E(S) \right]$$

$$= E(R_i) + E(Q_i) E(S)$$

Since Poisson arrivals see unbiased sample of queue behaviour

$$E(W) = E(R) + E(Q)E(S)$$

by Little's

$$E(Q) = \lambda E(W)$$

$$E(W) = \frac{E(R)}{1-\rho}$$

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3. i) a) Non-pre-emptive priority : service of item may not be interrupted by a higher-priority arrival

b) Pre-emptive priority : service is interrupted by any higher-priority arrival.

- In this case there are 2 possibilities :
  Pre-emptive resume
  Pre-emptive re-start

3

ii)

$$T_K = W_K + \hat{S}_K$$

$$E(W_K) = \frac{E(R_K)}{(1-\sigma_{K-1})(1-\sigma_K)}$$

$$E(V_K) = \sum_{i=1}^{k-1} \lambda_i \, E(\hat{S}_K) \, E S_i$$

$$= \sigma_{K-1} E(\hat{S}_K)$$

$$E(\hat{S}_K) = E(S_K) + E(V_K)$$

$$= \frac{E(S_K)}{1-\sigma_{K-1}}$$

$$E(T_K) = E(W_K) + E(\hat{S}_K)$$

4

iii) In a non-pre-emptive priority give priority to items with shorter expected service times. For example a queue discipline like shortest job first.

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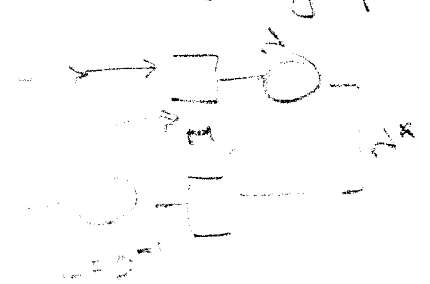
4 i) Assume a call has been admitted and the connection established, it is necessary to monitor and control the traffic actually generated by the call to ensure it conforms to the traffic description originally specified.

This procedure is referred to as user parameter control, access control, credit management or traffic policing.

The usage parameter control technique 5
proposed by the ATM forum, called the Generic Rate Algorithm has a number of equivalent implementations one of which is the leaky bucket technique. An interpretation of this algorithm involves the use of a "token pool" buffer. A cell must have a token waiting to be transmitted. Tokens are generated once per D sec, and wait in buffer until buffer full. At this time no further token is generated.

Average throughput λ^* differs from the load because of possible cell loss



$$\lambda^* = \lambda(1 - P_L)$$

$$P_L = \frac{\rho^M(1-\rho)}{1-\rho^{M+1}}$$

(M tokens in the system) 5

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4(b) $\rho = \frac{\lambda}{K\mu} = 0.8$ erlangs/channel $(K=10)$

$$P[\text{Loss}] = \left[\frac{(1-\rho)\rho^{Q} E_{K}(K\rho)}{(1-\rho) + \rho(1-\rho^{Q})E_{K}(K\rho)} \right]$$

$E_K(K\rho) = 0.122$

$P[\text{Loss}] = 0.030 \quad B=5$

$= 0.009 \quad B=10$ 5

$$P[Q_t = i] = \left(\frac{1-\rho}{1-\rho^{Q}}\right)\rho^{i} \qquad i = 0, 1, \ldots B-1$$

$E(Q_t \mid W>0) = 1.56 \quad B=5$

$2.80 \quad B=10$

using little

$\lambda_E = \lambda[1 - P[\text{Loss}]]$

$= 0.1035 \text{ sec}^{-1} \quad B=5$

$= 0.1057 \text{ sec}^{-1} \quad B=10$

$$E(W \mid \text{Delay}) = \frac{E(Q_t \mid \text{Delay})}{\lambda_E}$$

$= 15$ sec $\quad B=5$

$= 26.5$ sec $\quad B=10$ 5

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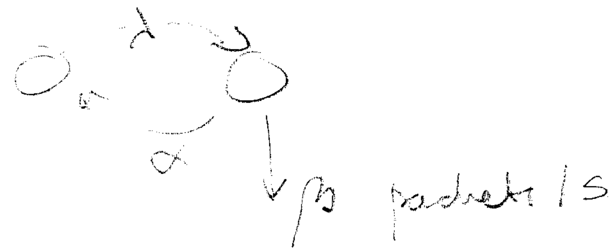
3a) i) ON-OFF source



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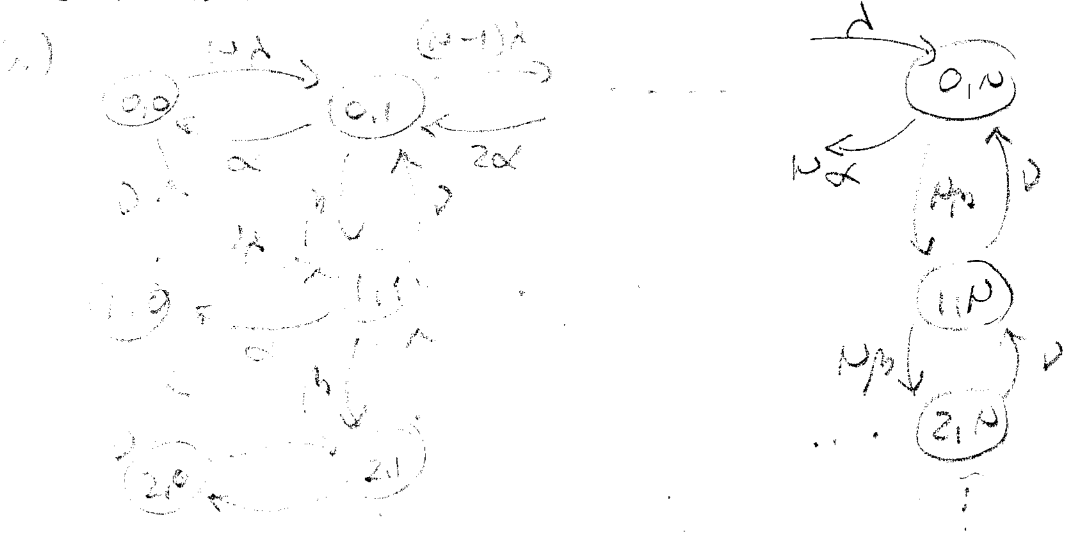
Discussion

ii)



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iii) Average cells/s per source $= \beta\frac{\lambda}{\alpha+\lambda}$

Average cells/s N multiplexed sources $= \frac{N\beta\lambda}{\alpha+\lambda}$

Capacity of system D =>

$N\beta\frac{\lambda}{\alpha+\lambda} < D$

$N < \frac{D(\alpha+\lambda)}{\lambda\beta}$

4

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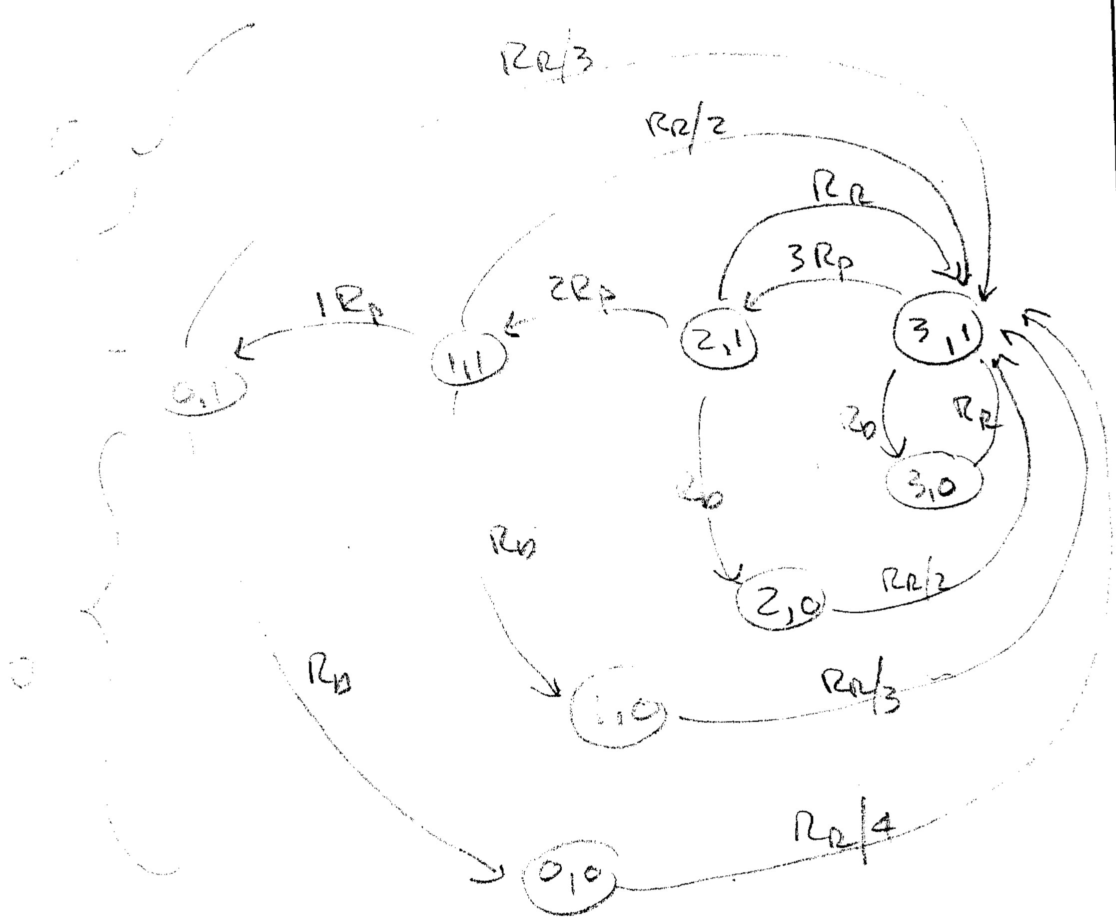
5b



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