E4.05
SO7

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2004

MSc and EEE PART IV: MEng and ACGI

TRAFFIC THEORY & QUEUEING SYSTEMS

Thursday, 6 May 10:00 am

Time allowed: 3:00 hours

There are FIVE questions on this paper.

Answer FOUR questions.

All questions carry equal marks

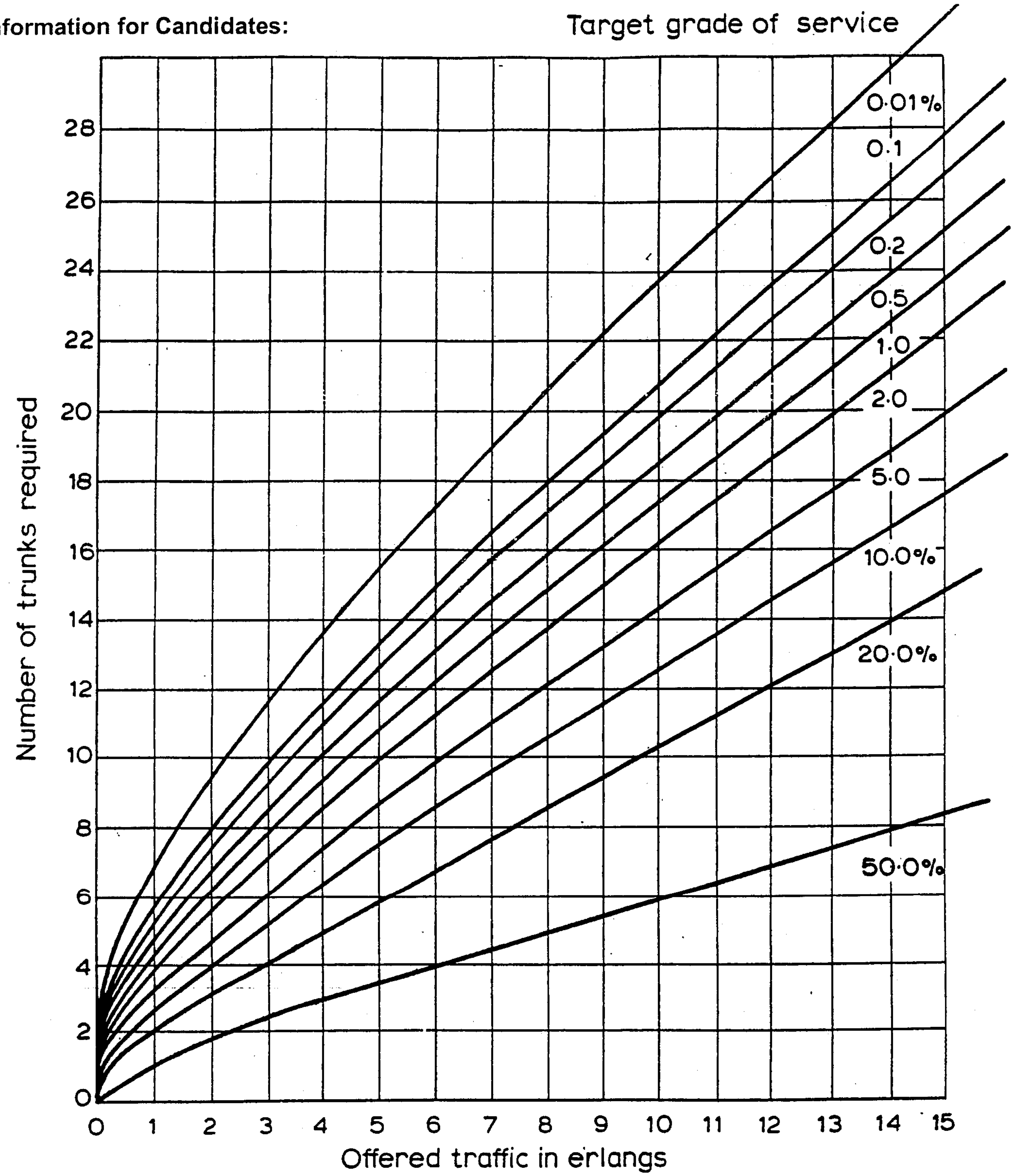
Corrected Copy

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : J.A. Barria
Second Marker(s) : P. De Wilde

Especial Information for Invigilators: NIL

Information for Candidates:



Traffic capacity on basis of Erlang B formula.

1.

a) Briefly indicate how the Erlang model should be modified in each one of the following situations:

i) When the full availability assumption is not valid (e.g. restricted access).

ii) When the number of users of the system is of the same order of magnitude as the number of channels in the link.

iii) When the link analysed is a second choice link (e.g. traffic overflow from first choice link).

iv) When there is coexistence of two types of calls with different arrival rate and different holding times.

[10]

b) If an N-channel link is fed by M sources (M > N), and if the offered traffic per free source is α , show that the total traffic offered to the link, ρ_0, is given by

$$\rho_0 = \frac{\alpha M}{1+\alpha(1-B_c)}$$

where, B_c is the call congestion.

[10]

2.

a) The stationary distribution of a Markov chain can be derived using balance equations.

Using a practical example, that is, by introducing a traffic model known to you:

i) Describe and discuss at least two balance equations known to you.
ii) Derive the stationary distribution of the model of your example using the two balance equations introduced in (i).

[10]

b) Consider the computer system shown in Figure 2.1 (three identical computers connected to each other by duplicated buses).

Assume that the computer system can be fully repaired only if it has failed (three computers in failure condition).

Also assume that the system can be represented by a Markov model, and that you know the following parameters:

hp(t) = failure rate of each computer,
hb(t) = failure rate of each bus,
r(t) = repair rate of the system.

i) Define the state (space) of the system.
ii) Derive the state-space transition diagram of this system
iii) Set up the equations to obtain the stationary probability distributions.
iv) Do the local balances equation hold in this case ?

[10]

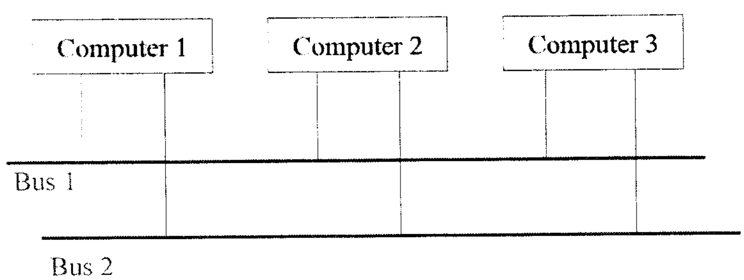


Figure 2.1

3.

a) If an M/M/K system is operating with a FIFO queue discipline, what can be said about:

   i) The queue length distribution seen by arrivals that find all K servers busy?
   ii) The unconditional queue length distribution ?
   iii) The waiting-time distribution for delayed arrivals (i.e. the ones that find K servers busy) ?
   iv) The waiting-time distribution for all arrivals (whether delayed or not)?

[10]

b) Determine the mean and variance of the carried traffic when ρ Erlangs of Poisson traffic is offered to a single communication channel via an infinite FIFO queue buffer.

[4]

Calculate the following performance parameters when $\rho = 0.95$ and the mean service time is $1/\mu = 0.25$ s.:

   i) Mean queue length,
   ii) Mean waiting time,
   iii) Mean transit time.

[6]

4.

a) Using a closed queuing network model as a simple approximation of a leaky bucket mechanism:

i) Discuss the underlying features and characteristics of the model,

ii) Derive the throughput λ^* of the modelled leaky bucket,

iii) State clearly and discuss any assumptions made in your derivations.

[10]

b)

i) Describe the stochastic Knapsack problem and give a practical system model example.

ii) Describe and discuss an equivalent capacity model known to you. State clearly and discuss any assumptions made.

[10]

5.

a) Show that for a fluid model approximation the stationary probability, $F_i(x)$, that the buffer occupancy is less or equal to x, given i sources in talkspurt can be obtained from the following equation:

$$(i-C)\alpha\frac{\partial F_i(x)}{\partial x} = [N-(i-1)]\lambda F_{i-1}(x) - [(N-i)\lambda + i\alpha]F_i(x) + (i+1)\alpha F_{i+1}(x)$$

[5]

i) Discuss when it would be reasonable to use a fluid flow approximation.
ii) Define and explain the relevance of C, α, x and λ.
iii) State clearly and discuss any assumptions made in your derivations.

[5]

b) How many channels would be needed for the following:

i) For 1 link to carry 15 Erlangs of pure chance traffic at a loss probability 0.005?
ii) For 3 separate links each carrying 5 Erlangs at a loss probability of 0.005?
iii) For 5 separate links each carrying 3 Erlangs at a loss probability of 0.005?

For each of the above cases determine the mean channel occupancy.

[6]

iv) Discuss which one of alternatives i), ii) or iii) you would choose if you need to take into account cost considerations,

v) Discuss which one of alternatives i), ii) or iii) you would choose if you need to take into account reliability aspects.

[4]

TRAFFIC THEORY + QUEUEING 2004

SYSTEMS **Examinations : ~~199~~ ~~9~~ Session** ***Confidential***

MODEL ANSWER and MARKING SCHEME

First Examiner	Dr Barria	Paper Code	E4.05-SO7
Second Examiner	Dr De Wilde	Question	Page 1 out of 13

Question labels in left margin — Marks allocations in right margin

Q1 a) Erlang model

i) Poisson arrival stream

ii) exponential holding time

iii) Full availability access

iv) No re-submission

Modifications

i) Restricted availability: use loss factor to represent the effect of the restriction 2

ii) Limited number of traffic sources: use the birth coefficients of the form $(M-i)\lambda$, where M is the number of sources and i is the state 2

iii) For overflow traffic: use birth coefficients of the form $(K+i)\lambda$ to generate traffic with variance > mean 2

iv) discuss knapsack problem 2

2004

Examinations : ~~199~~ 9 Session

Confidential

MODEL ANSWER and MARKING SCHEME

First Examiner

Second Examiner

Paper Code

Question

Question labels in left margin

Marks allocations in right margin

Q1 (b)

Offered traffic is

$$\rho_0 = [\text{offered traffic} \mid \text{free sources}] \times E(\text{number of free sources})$$ 4

$$= \alpha \times E(M - N_t)$$

$$= \alpha \times [M - E(N_t)]$$

$$= \alpha M - \alpha [(1-B_c)\rho_0]$$

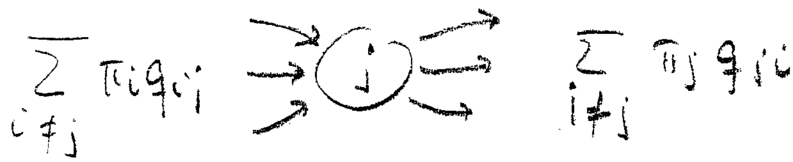
$$\Rightarrow \rho_0 = \frac{\alpha M}{1 + (1-B_c)\alpha}$$ 4

Then, total carried traffic $= (1-B_c)\rho_0$

4

Q2 a) i) Global balance equation

$$\sum_{i \neq j} \pi_i q_{ij} = \sum_{i \neq j} \pi_j q_{ji}$$



Local balance equation

$$\pi_i q_{ij} = \pi_j q_{ji}$$

(for each pair $i, j \neq i$)

i.e. flux balance between each pair of states — 5

ii) Engset Global

$$[(M-i)\lambda + i\mu]\pi_i = (M-i+1)\lambda\,\pi_{i-1} + (i+1)\mu\,\pi_{i+1}$$

M = no. source

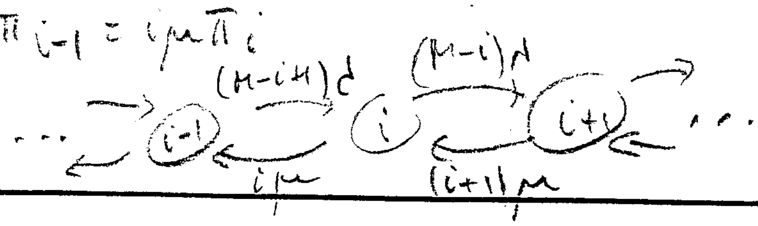
π_i = Eq. state probability in state i

λ = calling rate / free source

μ = release rate / channel

Engset local

$$(M-i+1)\lambda\,\pi_{i-1} = i\mu\,\pi_i$$



5

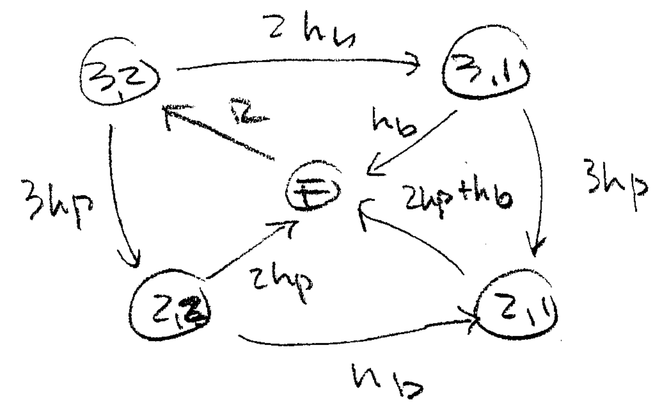
Q2b

i) E = (# operational computers, # operational buses)

At least E(2,1) to be operational therefore, all possible states

(3, 2) = fully operational system
(3, 1) = one bus down
(2, 2) = one computer down
(2, 1) = one computer and one bus down
(x, x) = system in failed condition 2

ii)



3

iii)

	3.2	3.1	2.2	2.1	F
3.2	−( )	2hb	3hp		
3.1		−( )		3hp	hb
2.2			−( )	hp	2hp
2.1				−( )	2hp+hb
F	R				−( )

= Q

2

$\pi^T Q = 0^T$, $\sum \pi_i = 1$ 3

iv) PO

2004

Examinations : · Session *Confidential*

MODEL ANSWER and MARKING SCHEME

First Examiner | Paper Code
Second Examiner | Question | Page 5 *out of* 13

Question labels in left margin | Marks allocations in right margin

Q3

a) i) $P[Q_t = i \mid \text{Delay}] = (1-\rho)\rho^i$

ii) $P[Q_t = i] = P[\text{Delay}]\, P[Q_t = i \mid \text{Delay}] + P[\text{No delay}]\, P[Q_t = i \mid \text{No delay}]$

$P[Q_t = i \mid \text{No delay}] = 1 \quad i = 0$

$= 0 \quad i > 0$

$$= D_K(A)\,(1-\rho)\rho^i \qquad i > 0$$

$$[1 - \rho D_K(A)] \qquad i = 0$$

3

$P[\text{Delay}] = D_K(A)$

$$D_K(A) = \frac{E_K(A)}{(1-\rho) + \rho E_K(A)}$$

$$E_K(A) = (A^K/K!) \Big/ \sum_{j=0}^{K} A^k/k!$$

3

iii) $P[W > z \mid Q_t = i] = P[\geq (i+1) \text{ departure in } (0, z)]$

$$= \sum_{j=0}^{i} \frac{(K\mu z)^j}{j!} e^{-K\mu z}$$

2

iv) $P[W \leq z \mid W > 0] = 1 - e^{-K\mu(1-\rho)z}$

2

2004

Examinations : ~~199~~ ~~.9~~ Session *Confidential*

MODEL ANSWER and MARKING SCHEME

First Examiner	Paper Code
Second Examiner	Question Page 6 out of 13

Question labels in left margin | Marks allocations in right margin

Q3 b) System is an M/M/1 queueing system and for equilibrium $\rho < 1$

Let X_t = no of busy channels at time t [$=0$ or $=1$]

Then,

$E[X_t] = 1 \cdot P[X_t = 1] + 0 \cdot P[X_t = 0]$

$= P[X_t = 1] = \rho$

similarly $E[X_t^2] = \rho$

so that $Var[X_t] = E[X_t^2] - [E(X_t)]^2 = \rho(1-\rho)$ 4

For $\rho = 0.95$, $1/\mu = 0.25$

Mean queue length $E(Q_t) = \dfrac{\rho^2}{1-\rho} = 18.05$ 2

Mean waiting time $E(W) = \left(\dfrac{\rho}{1-\rho}\right)\mu = 4.75$ sec 2

Mean transit time $E(T) = E(W) + E(S) = 5.0$ sec. 2

2004

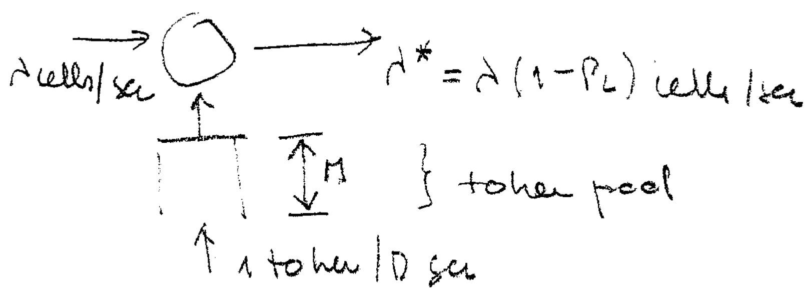
Examinations : ~~199 9~~ Session *Confidential*

MODEL ANSWER and MARKING SCHEME

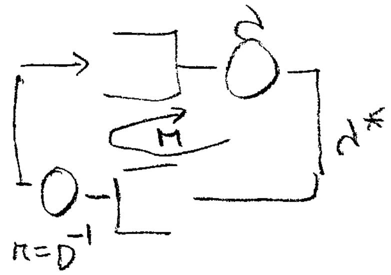
First Examiner	Paper Code
Second Examiner	Question Page 7 out of 13

Question labels in left margin | Marks allocations in right margin

4a) i)



ii)



3

i) In this algorithm a cell must have a token waiting to be transmitted. Tokens are generated once per D sec, and wait in the buffer, until buffer fills. At this time no further token is generated. In this case the average throughput λ^* differs from the load λ because of the possible cell loss (P_L represents the cell loss probability) 3

ii) P_L = prob "upper" queue is empty or the prob the "lower" queue is full

$$P_L = \frac{\rho^M(1-\rho)}{1-\rho^{M+1}}$$

$$\rho = \lambda/\mu = \lambda D$$

$$\lambda^* = \lambda\left[\frac{1-\rho^M}{1-\rho^{M+1}}\right]$$

4

2009

Examinations : ⟶ **Session** *Confidential*

MODEL ANSWER and MARKING SCHEME

First Examiner | Paper Code
Second Examiner | Question | Page 8 *out of* 13

Question labels in left margin | Marks allocations in right margin

Q4 (a) i) Stochastic knapsack

- C resource units
- Arrival from K classes
  - Poisson arrival : λ_k
  - Exponential Holding Time : $\frac{1}{\mu_k}$
  - hold b_k resource units 1
- Pure loss system

$n = (n_1, \dots, n_K)$ state of system

$b = (b_1, \dots, b_K)$

Admit class-k arrival iff $b_k < C - b \cdot n$ 2

Dynamic knapsack

$S = \{ n \in I^k : b \cdot n \le C \}$

$X(t) = (X_1(t), \dots, X_k(t))$ state at time t

aperiodic and irreducible Markov process over S

$S_k = \{ n \in S : b \cdot n \le C - b_k \}$

$B_k = 1 - \sum_{n \in S_k} \pi(n)$ 2

Q4 b ii) $C_L = \min [C_{LS}, C_{LF}]$

A: large number of sources multiplexed (C_{LS})

$N \gg 1$, $p \ll 1$

$\pi_i = \binom{N}{i} p^i (1-p)^{N-i}$ (binomial)

is approximated quite closely by the normal distribution ($m = Np$, $\sigma^2 = Np(1-p)$)

$$P_L = \frac{1}{m} \int_{J_0}^{\infty} \frac{e^{-(x-m)^2/2\sigma^2} (x-c)}{\sqrt{2\pi\sigma^2}} dx$$

$$\varepsilon = \int_{J_0}^{\infty} \frac{e^{-(x-m)^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}} dx$$

if $(c-m) > 3\sqrt{2}\sigma$

$$\varepsilon = \frac{\sigma e^{-(c-m)^2/2\sigma^2}}{\sqrt{2\pi}(c-m)} \quad ; \quad P_L = \frac{1-p}{c-m} \varepsilon$$

$$\ln(\sqrt{2\pi}\,\varepsilon) = \ln\left(\frac{\sigma}{c-m}\right) - \frac{(c-m)^2}{2\sigma^2}$$

$$C_{LS} = m R_p + \sigma \sqrt{-\ln(2\pi) - 2\ln\varepsilon}\; R_p$$ 5

B: Effect of the access Buffer

$G(x) \sim A_N \rho^N e^{-\beta_R x / R_p}$

(probability buffer occupancy $> x$)

2004

Examinations : . Session *Confidential*

MODEL ANSWER and MARKING SCHEME

First Examiner	Paper Code
Second Examiner	Question Page 10 *out of* 13

Question labels in left margin | Marks allocations in right margin

Q4 ii) ii) cont

$$R = (1-\rho)\left(1+\frac{\alpha}{\mu}\right)\Big/\left(1-\frac{C_L}{NR_p}\right)$$

$$\rho = \frac{N_p R_p}{C_L}$$

if $\rho \sim 1$ $AN\rho^N \sim 1$

$$P_L = e^{-\mu R_x / R_p}$$

$$\mu R_x / R_p = -\ln P_L$$

$$\frac{C_L}{R_p N} = \frac{1-k}{2} + \sqrt{\left(\frac{1-k}{2}\right)^2 + k\rho}$$

$$C_{LF} = R_p N\left(\frac{1-k}{2}\right) + R_p N\sqrt{\frac{1-k}{2}^2 + R_p}$$ 5

L

2004

Examinations : · Session	Confidential
MODEL ANSWER and MARKING SCHEME	

First Examiner — Paper Code

Second Examiner — Question — Page 11 out of 13

Question labels in left margin — Marks allocations in right margin

Q5 a) i) The fluid flow model assumes that the number of cells generated during a talk spurt is so large that it appears like a continuous flow of fluid. The buffer occupancy thus becomes a continuous random variable x 2

ii) - The units of x are defined to be the number of cells arriving during a talk spurt.

- One voice source, generating cells at the rate of V cells/sec during a talk spurt of average (cell) length $1/\alpha$ sec will, on the average, increment x by V/α cells during a talk spurt.
- The buffer is emptying at rate αC. Therefore the buffer is filling if $i > C$ and emptying if $i < C$ 2

iii) $F_i(t,x)$ $0 \le i \le N$: cumulative probability distribution at time t with the system in state i.

Assume queue infinite

$F_i(t+\Delta t, x)$: at an incremental time Δt later in terms of the probabilities at time t.

Using the buffer filling / emptying rates:

- if there are i sources in talkspurt they are "pumping" αi "units of information" per second into the buffer 1

2004

Examinations : · Session *Confidential*

MODEL ANSWER and MARKING SCHEME

First Examiner | Paper Code
Second Examiner | Question | Page 12 out of 13

Question labels in left margin | Marks allocations in right margin

Q5 a) Cont

$$F_i(t+\Delta t, x) = [N-(i-1)]\lambda \Delta t F_{i-1}(t,x) + (i+1)\alpha \Delta t F_{i+1}(t,x) + \{1-[(N-i)\lambda + i\alpha]\Delta t\} F_i[t, x-(i-c)\alpha\Delta t] + o(\Delta t)$$

It is assumed that $F_{-1}()$ and $F_{N+1}()$ are equal to zero.

Now expand $F_i(t+\Delta t, x)$ and $F_i(t, x-\Delta x)$, ($\Delta x \equiv (i-c)\alpha\Delta t$) in their respective Taylor series, assuming approximate continuity conditions are met, and let $\Delta t \to 0$. The previous equation simplifies to

$$\frac{\partial F_i(t,x)}{\partial t} = [N-(i-1)]\lambda F_{i-1}(t,x) + (i+1)\alpha F_{i+1}(t,x) - [(N-i)\lambda + i\alpha] F_i(t,x) - (i-c)\alpha \frac{\partial F_i(t,x)}{\partial x}$$

5

2004.

Examinations : · Session	Confidential
MODEL ANSWER and MARKING SCHEME	
First Examiner	Paper Code
Second Examiner	Question Page 13 out of 13
Question labels in left margin	Marks allocations in right margin

Q5 b)

Grade of service = 0.005 $B_c = 0.005$

15 Erlangs ⇒ 25 channels 2

3 links / 5 Erlangs ⇒ 3×12 = 36 channels 2

5 links / 3 Erlangs ⇒ 5×9 = 45 channels 2

discussion

mean channel occupancy

$$\frac{(1-B_r)\rho}{N}$$

ρ = offered traffic 4

N = channels