Paper Number(s): **E4.05**
SO7

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2002

MSc and EEE PART IV: M.Eng. and ACGI

TRAFFIC THEORY & QUEUEING SYSTEMS

Wednesday, 24 April 10:00 am

There are FIVE questions on this paper.

Answer FOUR questions.

Time allowed: 3:00 hours

Corrected Copy

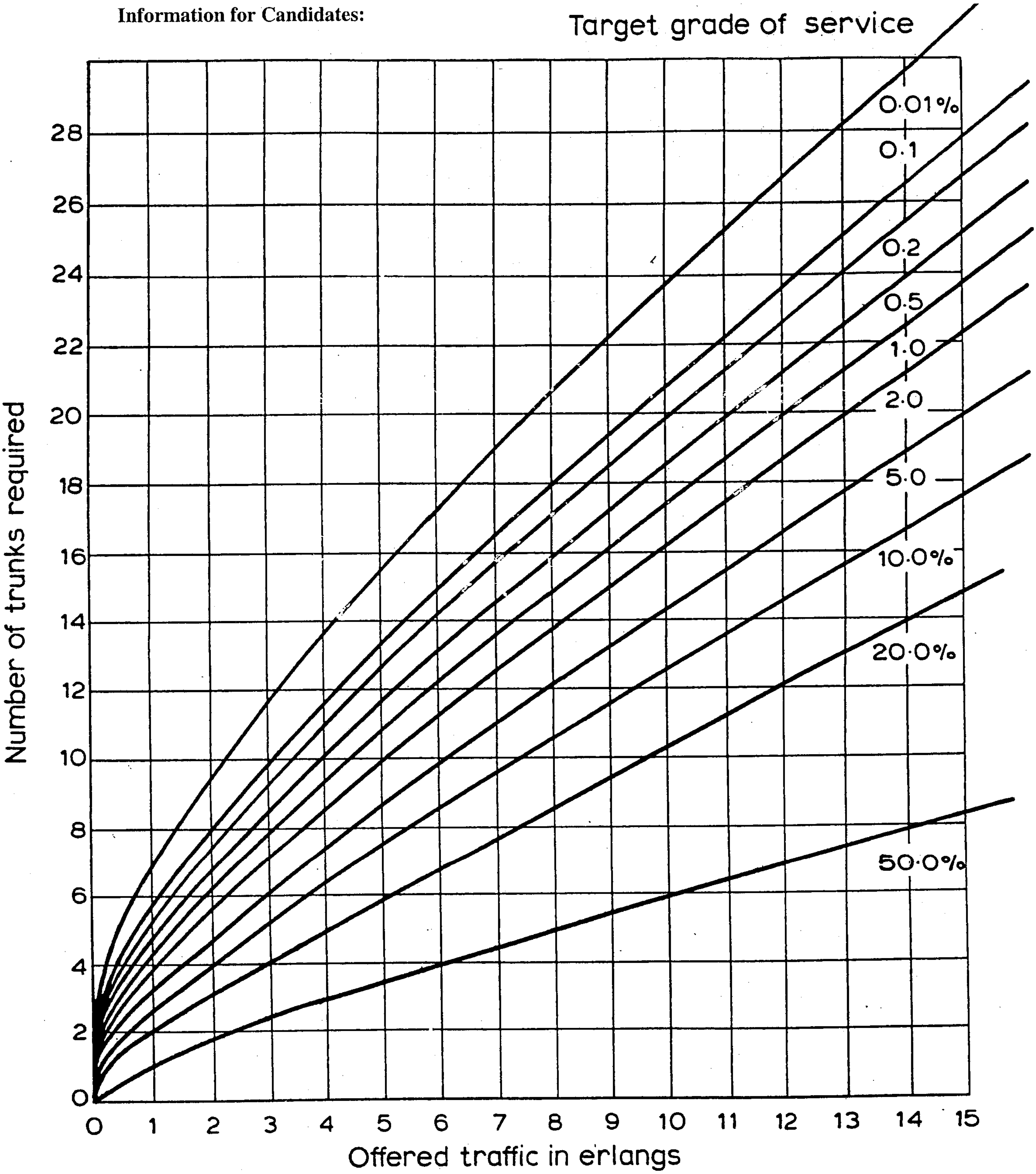
Examiners responsible:

First Marker(s): Barria,J.A.

Second Marker(s): De Wilde,P.

Special Information for Invigilators: **NIL**

Information for Candidates:



Traffic capacity on basis of Erlang B. formula.

1. (a) For the Engset model:

   (i) Describe and list the basic assumptions underlying the model. [2]

   (ii) Derive the death and birth coefficients. [2]

   (iii) Derive the equilibrium traffic distribution if $N < M$. Where M is the number of independent active Poisson sources that is offered to an N-channel communications link. [6]

   (b) Two telephone exchanges are connected by a multi-channel link operating with a loss probability of 0.005. If the aggregated calling rate in the two directions is 1200 calls/hour, and the average call holding time is 180 seconds:

   (i) Determine the total offered traffic and the total carried traffic for the link. [5]

   (ii) Estimate the size (i.e. number of channels) of the link. [5]

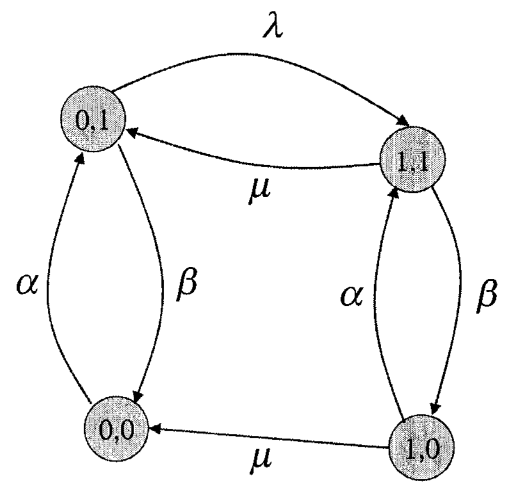
   Hint: you can assume that for large values of the offered traffic, the B-curves are approximately linear.

2. (a) For the *M/M/K/N* system, derive the Queue-length distribution. [10]

(b) The two dimensional Birth and Death process represented in *Figure 2.1* represents the traffic being offered to an overflow link. If we define the state of the system by (N_t, Y_t) where N_t is the number of busy channels on the overflow link, and Y_t is an ON/OFF switching process:

(i) define and explain the meaning of all parameters, [5]

(ii) calculate the blocking probability for the overflow link. [5]



$\alpha = 0.145$
$\beta = 7.08$
$\lambda = 3$
$\mu = 0.33$
$\pi_0 = P(N_t = 0, Y_t = 0)$
$\pi_1 = P(N_t = 1, Y_t = 0)$
$\sigma_0 = P(N_t = 0, Y_t = 1)$
$\sigma_1 = P(N_t = 1, Y_t = 1)$

Figure 2.1

3. (a) (i) For queuing systems define and explain two types of priority schemes known to you. [4]

(ii) Derive the expression for the mean waiting time of class k traffic in a non-pre-emptive priority scheme. Explain all variables involved in the derivation and clearly state and discuss any assumptions made. [6]

(b) A Poisson stream of messages with rate λ is fed into a single-channel communication link via a large input buffer. The message stream consists of a random mixture of 1-packet messages and 2-packet messages, each packet being of length B bits. If $\lambda = 1800$ messages/minute, B = 60 bits, and 60 % of the messages are single-packet messages:

determine the overall mean message transit time across the link (i.e. waiting time plus transmission time) when the queue discipline is FIFO and the channel transmission rate is 64 Kbits/seconds. [10]

4. (a) (i) Define the stochastic Knapsack problem. [4]

(ii) *Figure 4.1* represents a Knapsack problem. The resources required by n_1 type traffic is $B1 = 1$ [unit] and by n_2 type traffic is $B2 = 2$ [units].

What is the capacity of the system ? [2]

Suppose the probabilities associated with the state space (n_1, n_2) are given by:

$P(i,0) = 0.05$ for $i = 0, ..,8$

$P(i,1) = 0.05$ for $i = 0, ..,6$

$P(i,2) = 0.03$ for $i = 0, ..,4$

$P(i,3) = 0.01$ for $i = 0, ..,2$

$P(i,4) = 0.02$ for $i = 0.$

Derive the blocking probability of n_2 type traffic. [4]

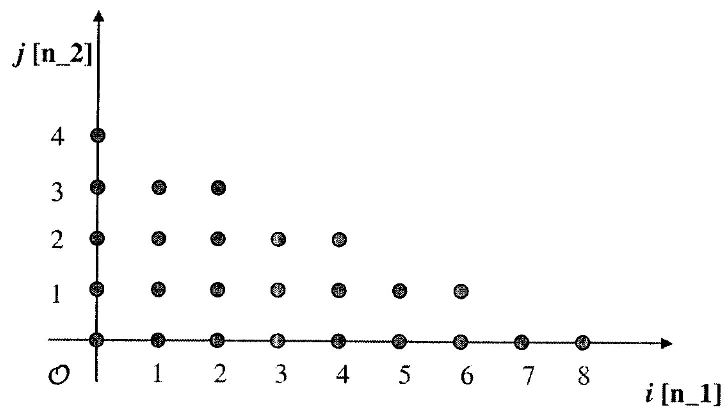


Figure 4.1

(b) In broadband networks, admission and access control schemes play important roles. Discuss briefly the importance of admission and access control in broadband networks. Give examples as necessary. [10]

5. (a) For the N- sources stochastic fluid model represented in *Figure 5.1*:

(i) explain the meaning of the random variable x, [3]

(ii) explain the relationship between the parameter αC and the system capacity VC cells/seconds, [3]

(iii) derive the stationary probability that the buffer occupancy is less than or equal to x given that i sources are in talkspurt. [6]

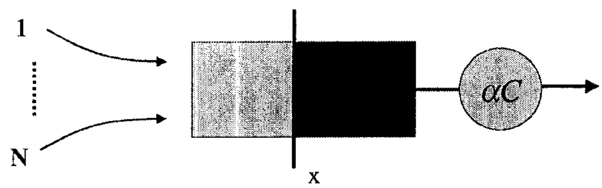


Figure 5.1

(b) A well known measurement of equivalent capacity is given by the following expression:

$$C_l = \min[C_{ls}, C_{lf}]$$

where

$$C_{ls} = mR_p + R_p\sigma\sqrt{-\ln(2\pi) - 2\ln\varepsilon}$$

$$C_{lf} = R_p N\left(\frac{1-k}{2}\right) + R_p N\sqrt{\left(\frac{1-k}{2}\right)^2 + kp}\ .$$

(i) Explain the meaning of all parameters of C_{ls}. [4]

(ii) Explain the meaning of all parameters of C_{lf}. [4]

Department of Electrical and Electronic Engineering Examinations 2002 **Confidential**

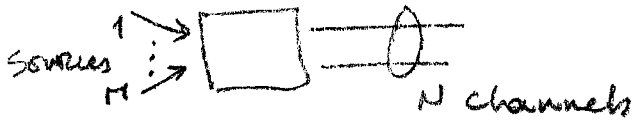
Model Answers and Mark Schemes First Examiner:

Paper Code: 4.05 Second Examiner:

Traffic Theory & Queueing Systems

Question Number etc. in left margin | Mark allocation in right margin

1a (i) Engset model.



since there is no buffering j channels busy $\Leftrightarrow$ j sources busy

$\Rightarrow$ total arrival falls as N_t increases

Assumptions

(1) Each idle source is a Poisson source:

P[source generates new demand in $(t, t+\Delta t)$ | source is idle] $= \lambda \Delta t$

(2) channel holding times are exponential with mean $(1/\mu)$

(3) full-availability access

(ii) Birth coefficients

no. of idle sources in state i is $(M-i)$:

$$\lambda_i = (M-i)\lambda \qquad i < i_{max}$$
$$= (1 - i/M) M\lambda$$
$$= (1 - i/M)\lambda_0$$

Death coefficients (as in Erlang model)

$$\mu_i = i\mu \quad ; \; i > 0$$

(iii) $N < M$

$$\pi_i = \left(\frac{\lambda_{i-1}}{\mu_i}\right)\pi_{i-1} = \left[\frac{M-(i-1)\lambda}{i\mu}\right]\pi_{i-1}$$

$$\pi_i = \binom{M}{i}\alpha^i \pi_0 \quad , \; i = 1, 2, \ldots, N \qquad \text{and } \alpha = \left(\frac{\lambda}{\mu}\right)$$

$$\pi_0 = \left[\sum_{j=0}^{N}\binom{M}{j}\alpha^j\right]^{-1} \qquad \alpha = \frac{p}{1-p}$$

$$\pi_i = \left[\frac{\binom{M}{i} p^i (1-p)^{M-i}}{\sum_{j=0}^{N}\binom{M}{j} p^j (1-p)^{M-j}}\right] \qquad i = 0, 1, \ldots, N$$

Department of Electrical and Electronic Engineering Examinations 2002 **Confidential**

Model Answers and Mark Schemes First Examiner:

Paper Code: 4.05 Second Examiner:

Question Number etc. in left margin | Mark allocation in right margin

1b (i) Total calling rate = 1200 calls/hour = 20 calls/min

mean call duration = 180 sec = 3 min

$\Rightarrow$ offered traffic = 20×3 = 60 Erlangs

loss probability = $B_c = 0.005$

$\Rightarrow$ carried traffic = $60(1 - 0.005) = 59.7$ Erlangs

(ii) From Erlang chart, N is approximately linear in ρ for large ρ.

For $B_c = 0.005$ $N \approx 1.33\rho + 5$

and so for ρ = 60 Erlangs:

$N \approx 85$ channels

3

Department of Electrical and Electronic Engineering Examinations 2002 | Confidential

Model Answers and Mark Schemes | First Examiner:

Paper Code: 4.05 | Second Examiner:

Question Number etc. in left margin | Mark allocation in right margin

2(a) Queue length distribution for M/M/K/N system local balance equation give

$$\pi_i = \left(\frac{A^i}{i!}\right)\pi_0 \qquad 0 \le i \le K$$

$$= \left(\frac{A^k}{k!}\right)\rho^{i-k}\pi_0 \qquad K \le i \le K+B$$

$$\pi_0 = S^{-1} \; ; \; S = \left(\frac{A^K}{K!}\right)\left[E_K^{-1}(A) + \frac{\rho(1-\rho^B)}{1-\rho}\right]$$

$$\pi_0 = \frac{1}{(A^k/k!)}\left[\frac{(1-\rho)E_K(A)}{(1-\rho)+\rho(1-\rho^B)E_K(A)}\right]$$

$$P[\text{Delay}] = P[\text{all } K \text{ server busy} \mid \text{buffer not full}]$$

$$= P[K \le N_t < K+B]$$

$$= \pi_K\left[\frac{1-\rho^B}{1-\rho}\right]$$

$$P[\text{loss}] = P[\text{buffer full}]$$

$$= P[N_t = K+B]$$

$$= \pi_K \rho^B$$

$$\pi_K = \left(\frac{A^k}{K!}\right)\pi_0$$

$$P[Q_t = i \mid \text{Delay}] = P[Q_t = i \mid K \le N_t < K+B]$$

$$= \frac{P[N_t = K+i]}{P[\text{Delay}]} = \frac{\pi_K \rho^i}{\pi_K\left[\frac{1-\rho^B}{1-\rho}\right]} \qquad i = 0,1,\ldots,B-1$$

$$= \rho^i\left[\frac{1-\rho}{1-\rho^B}\right] \qquad i = 0,1,\ldots,B-1$$

- The queue length seen by rejected arrivals is $Q_t = B$ (i.e. buffer full)
- $P[Q_t = i \mid \text{Delay}]$: Queue length seen by arrivals which are accepted but delayed.

2b (i) This B/D process represent an IPP model for a single channel overflow link

N_t = nr of busy channel on the overflow link

$y_t = 0$ arrival stream is off

$y_t = 1$ arrival stream is on

λ = mean overflow arrival rate

μ = holding time of a call.

(ii)

$$P(\text{overflow traffic is } \underline{\text{ON}}) = \frac{\alpha}{\alpha+\beta} = 0.02$$

$$P(\text{overflow traffic is } \underline{\text{OFF}}) = \frac{\beta}{\alpha+\beta} = 0.98$$

$$P(\text{overflow channel busy}) = \pi_1 + \sigma_1 = 0.115$$

$$P(\text{overflow channel idle}) = \pi_0 + \sigma_0 = 0.885$$

<u>global balance equations</u>

$$\alpha\pi_0 = \beta\sigma_0 + \mu\pi_1$$

$$(\beta+\lambda)\sigma_0 = \alpha\pi_0 + \mu\sigma_1$$

$$(\alpha+\mu)\pi_1 = \beta\sigma_1$$

$$(\beta+\mu)\sigma_1 = \alpha\pi_1 + \lambda\sigma_0$$

$$\pi_0+\sigma_0 = \left[\frac{\alpha+\mu(1+\delta)}{\alpha(1+\beta\delta)+\mu(1+\delta)}\right] \qquad \delta = \frac{\lambda}{\mu+\alpha+\beta}$$

Department of Electrical and Electronic Engineering Examinations 2002 **Confidential**

Model Answers and Mark Schemes First Examiner:

Paper Code: 4.05 Second Examiner:

Question Number etc. in left margin | Mark allocation in right margin

3a (i) Two main type of priority in common use:

(1) Non-pre-emptive priority: service of an item may not be interrupted by a high-priority arrival

(2) Pre-emptive priority: service is interrupted by any higher-priority arrival

In this case there are two possibilities

(2a): after the period of interruption, the interrupted service is resumed at the point where it was interrupted – pre-emptive resume

(2b): after the period of interruption, service starts again from the beginning – pre-emptive re-start

(ii) class 1 $\Rightarrow$ $E(W_1) = E(R) + E(Q_1)E(S_1)$

$$E(W_1) = \left[\frac{E(R)}{1-\rho_1}\right]$$

class 2 $\Rightarrow$ $E(W_2) = E(R) + [E(Q_1)E(S_1) + E(Q_2)E(S_2)] + [\lambda_1 E(W_2)]E(S_1)$

$$E(W_2) = E(R) + [\rho_1 E(W_1) + \rho_2 E(W_2)] + \rho_1 E(W_2)$$

$$E(W_2) = \frac{E(R) + \rho_1 E(W_1)}{1-\rho_1-\rho_2} = \frac{E(W_1)}{1-\rho_1-\rho_2}$$

$$E(W_2) = \frac{E(R)}{(1-\rho_1)(1-\rho_2-\rho_1)}$$

class k

(define $\sigma_{K-1} = \sum_{i=1}^{k-1} \rho_i$

$$E(W_k) = \frac{E(R)}{(1-\sigma_{K-1})(1-\sigma_K)}$$

Department of Electrical and Electronic Engineering Examinations 2002 **Confidential**

Model Answers and Mark Schemes First Examiner:

Paper Code: 4.05 Second Examiner:

Question Number etc. in left margin

Mark allocation in right margin

3b This is an M/G/1 system

mean message length $= B\frac{6}{10} + 2\frac{4}{10}B = 1.2B$

mean square message length $= \frac{6}{10}B^2 + \frac{4}{10}(2B)^2 = 2.2B^2$

$$E(S) = \frac{1.2B}{64} = \frac{1.2 \times 60}{64} = 1.125 \text{ ms}$$

$$E(S^2) = \frac{6}{10}\frac{60^2}{64^2} + \frac{4}{10}\,4 \times \frac{60^2}{64^2} = \left(\frac{60}{64}\right)^2$$

$$= 0.8789 \text{ msec}^2$$

$$E(W) = \left[\frac{\lambda E(S^2)}{2(1-\rho)}\right] = \frac{300 \times 0.8789}{2(0.6625)} = 0.19899 \text{ msec}$$

$$E(T) = E(W) + E(S)$$

$$\rho = \lambda E(S) = 1.125 \times 10^{-3} \times 300 = 0.3375$$

Department of Electrical and Electronic Engineering Examinations 2002 | **Confidential**

Model Answers and Mark Schemes | First Examiner:

Paper Code: 4.05 | Second Examiner:

Question Number etc. in left margin | Mark allocation in right margin

4a Stochastic Knapsack

(i) - C resource units

- Arrivals from k classes
  - Poisson arrival λ_k
  - Exponential holding time $1/\mu_k$
  - hold b_k resources units
- pure loss system

$n = (n_1, \dots, n_k)$ state of the system

$b = (b_1, \dots, b_k)$

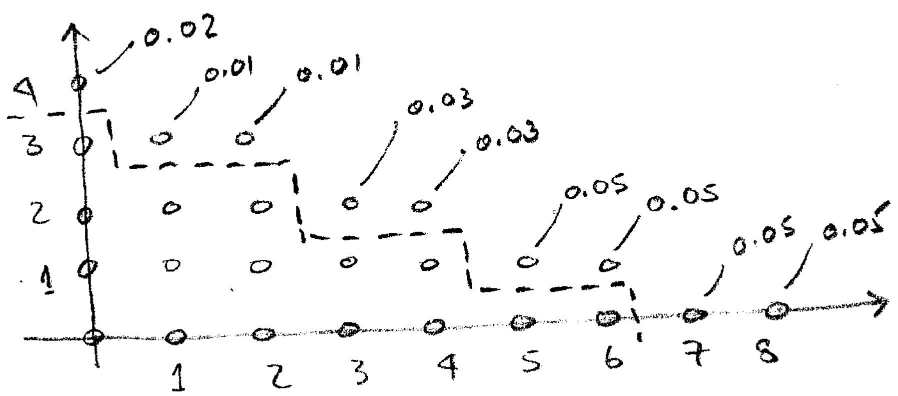
- admit class-k arrival iff $b_k < C - b \cdot n$

Dynamic Knapsack problem

$S \equiv \{ n \in I^k : b \cdot n \leq C \}$

$X(t) = (X_1(t), \dots, X_k(t))$ state at t

(ii) $C = 8$, $b_1 = 1$, $b_2 = 2$ (capacity of system = 8)



S = all o

$S_K = \{ n \in S : b \cdot n \leq C - b_k \}$ $\quad S_2 = \{ n \in S : b \cdot n \leq C - 2 \}$

$B_K = 1 - \sum_{n \in S_K} \pi(n)$ $\quad B_2 = 1 - 0.3 = 0.7$

Department of Electrical and Electronic Engineering Examinations 2002 **Confidential**

Model Answers and Mark Schemes First Examiner:

Paper Code: 4.05 Second Examiner:

Question Number etc. in left margin | Mark allocation in right margin

4b Admission and access control aim at answering the following question: How much traffic can it handle if a prescribed QoS for each traffic class is to be maintain while the network utilisation (i.e. throughput) is to meet some minimum goal?

- Admission control in the context of B-ISDN: given VPs set up in a network, how many virtual connections (i.e. calls with specified QoS) can it handle? given this number, does one admits a new call with specified QoS?

- Discussion on admission policies
Access control: traffic can only be described statistically and therefore congestion may develop despite good Admission policy

To prevent congestion from occuring, control at the access point as well as within the network must be exerted (also called "policing function")
(ATM refers to user parameter control UPC)

UPC is used to ensure that user do not violate their traffic contract, negotiated during admission control.

- Discussion on e.g. leaky bucket scheme.

Department of Electrical and Electronic Engineering Examinations 2002 **Confidential**

Model Answers and Mark Schemes First Examiner:

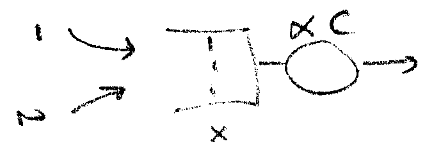
Paper Code: 4.05 Second Examiner:

Question Number etc. in left margin | Mark allocation in right margin

5a (i) - one voice source will generate cell at a rate of $v \frac{\text{cells}}{\text{sec}}$ during a talk spurt of average length $1/\alpha$ sec

- Increment x by $\underbrace{\frac{v}{\alpha} \text{ cells}}_{= \text{unit of information}}$ during a talk spurt

(ii) - System with capacity $vc \frac{\text{cell}}{\text{sec}}$ will have an equivalent capacity of $\frac{vc}{\frac{v}{\alpha}} = \alpha c$ "unit of information" per second



- i sources are "pumping" $i\alpha$ "unit of information" to the system $\therefore$ $i\alpha\left[\frac{v}{\alpha}\right] = iv = iv\left[\frac{\text{cells}}{\text{sec}}\right]$

- The buffer at the same time empty up at the rate αc

(iii) $F_i(t+\Delta t, x) = [N-(i-1)]\lambda\Delta t F_{i-1}(t,x) + (i+1)\alpha\Delta t F_{i+1}(t,x) + \{1-[(N-i)\lambda + i\alpha]\Delta t\} F_i[t, x-(i-c)\alpha\Delta t] + o(\Delta t)$

- Expand $F_i(t+\Delta t, x)$ and $F_i(t, x-\Delta x)$ $(\Delta x \equiv (i-c)\alpha\Delta t)$ in their respective Taylor series and $\Delta t \to 0$

$$\frac{\partial F_i(x,t)}{\partial t} = [N-(i-1)]\lambda F_{i-1}(t,x) + (i+1)\alpha F_{i+1}(t,x) - [(N-i)\lambda + i\alpha] F_i(t,x) - (i-c)\alpha \frac{\partial F_i}{\partial x}(t,x)$$

....

$$(i-c)\alpha \frac{dF_i(x)}{\partial x} = [N-(i-1)]\lambda F_{i-1}(x) - [(N-i)\lambda + i\alpha] F_i(x) + (i+1)\alpha F_{i+1}(x)$$

...

$$\frac{dF(x)}{dx} D = F(x) M$$

Department of Electrical and Electronic Engineering Examinations 2002 **Confidential** 1.0

Model Answers and Mark Schemes First Examiner:

Paper Code: 4.05 Second Examiner:

Question Number etc. in left margin | Mark allocation in right margin

5b ON-OFF traffic sources

(i) R_p = peak rate

$m = Np$ (ON-OFF source model)

$\sigma^2 = Np(1-p) = m(1-p)$

$$\varepsilon = \int_{J_0}^{\infty} \frac{e^{-(x-m)^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\, dx$$

if $C - m > 3\sqrt{2}\,\sigma$

$$\varepsilon = \frac{\sigma e^{-(c-m)^2/2\sigma^2}}{\sqrt{2\pi}\,(c-m)}$$

$$C_{G} = mR_p + \sigma\sqrt{-\ln(2\pi) - 2\ln\varepsilon}\; R_p$$

plus explanations

(ii)

$$P_L = e^{-\beta R x / R_p}$$

$$R = (1-\rho)\left(1+\frac{\alpha}{\beta}\right) \Big/ \left(1 - \frac{C_L}{NR_p}\right)$$

$$k \equiv \frac{\beta x}{R_p}(1-p)\ln\left(\frac{1}{P_L}\right)$$

$$C_{LF} = \left(\frac{1-k}{2}\right)R_p N + \sqrt{\left(\frac{1-k}{2}\right)^2 + kp}\; R_p N$$