DEPARTMENT of ELECTRICAL and ELECTRONIC ENGINEERING
EXAMINATIONS 2002

M.Sc and EEE/ISE PART IV: M.Eng. and ACGI

Solutions 2002
ADVANCED COMMUNICATION THEORY

— There are FOUR questions (Q1 to Q4)
— Answer Question ONE plus TWO other questions.

Comments for Question Q1:
— Question Q1 has 20 multiple choice questions numbered 1 to 20.
— Circle the answers you think are correct on the answer sheet provided.
— There is only one correct answer per question.

Distribution of marks
Question-1: 40 marks
Question-2: 30 marks
Question-3: 30 marks
Question-4: 30 marks

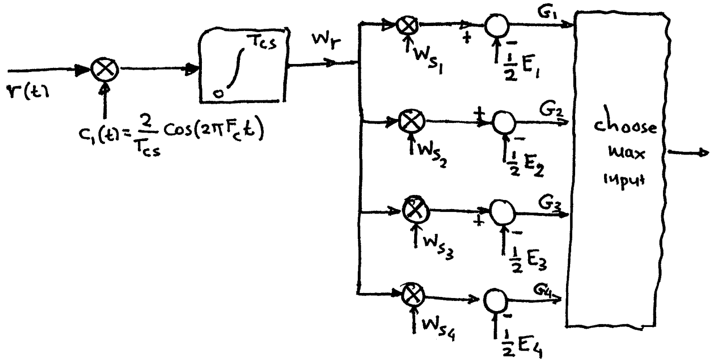
The following are provided:
- ***A table of Fourier Transforms***
- ***A "Gaussian Tail Function" grap***

Examiners responsible: Dr. A. Manikas

ANSWER to Q1

1) A B C D E
2) A B C D E
3) A B C D E
4) A B C D E
5) A B C D E
6) A B C D E
7) A B C D E
8) A B C D E
9) A B C D E
10) A B C D E
11) A B C D E
12) A B C D E
13) A B C D E
14) A B C D E
15) A B C D E
16) A B C D E
17) A B C D E
18) A B C D E
19) A B C D E
20) A B C D E

ANSWER to Q2



i.e. dimensions: $\underset{\sim}{D=1}$

where

$N_0 = 2\times10^{-6}$ W/Hz. & $T_{cs} = 2$ sec

Furthermore

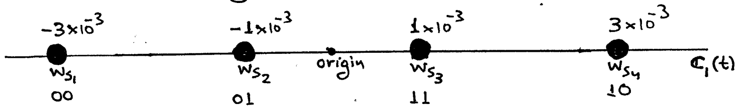
$A_1 = (2-1-4)\times10^{-3} = -3\text{mV} \Rightarrow W_{s_1} = -\sqrt{E_1} = \sqrt{-\frac{A_1^2}{2}T_{cs}} = -3\times10^{-3}$

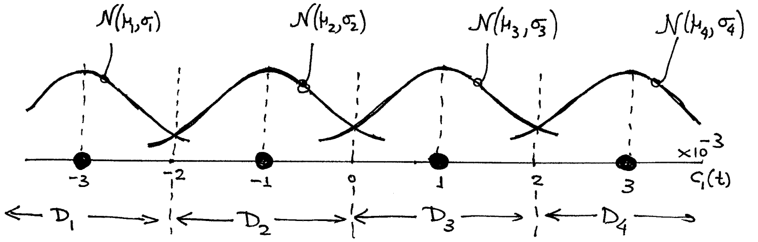
$A_2 = \cdots\cdots \quad = -1\text{mV} \Rightarrow W_{s_2} = -\sqrt{E_2} = \cdots\cdots = -10^{-3}$

$A_3 = \cdots\cdots \quad = 1\text{mV} \Rightarrow W_{s_3} = \sqrt{E_3} = \cdots\cdots = 10^{-3}$

$A_4 = \cdots \quad = 3\text{mV} \Rightarrow W_{s_4} = \sqrt{E_4} = \cdots\cdots = 3\times10^{-3}$

* constellation diagram:





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* $\mu_1 = -3\times10^{-3}$; $\mu_2 = -1\times10^{-3}$; $\mu_3 = 1\times10^{-3}$; $\mu_4 = 3\times10^{-3}$

$\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma$ with $\sigma_1^2 = \frac{N_0}{2}\times 2\underset{\overset{\uparrow}{1/2T_{cs}}}{B}\times T_{cs} = \frac{N_0}{2} = 10^{-6}$

i.e. $\sigma_1^2 = 10^{-6}$ = noise energy over T_{cs}

i.e. $\boxed{\sigma = 10^{-3}}$

* $P_{21} = \Pr(D_2|H_1) = \mathbb{T}\left\{\frac{|-3-2|\times10^{-3}}{10^{-3}}\right\} - \mathbb{T}\left\{\frac{|-3-0|\times10^{-3}}{10^{-3}}\right\}$

$= \mathbb{T}\{1\} - \mathbb{T}\{3\} = 0.16$

$= P_{12} = P_{32} = P_{23} = P_{43} = P_{34}$

where $P_{ij} = \Pr(D_i|H_j)$

* $P_{31} = \mathbb{T}\{3\} - \mathbb{T}\{5\} = 1.4\times10^{-3}$

$= P_{24} = P_{42}$

* $P_{41} = \mathbb{T}\{5\} = P_{14} = 3\times10^{-7}$

* $P_{11} = P_{44} = 1 - \mathbb{T}\{1\} = 0.84$

* $P_{22} = P_{33} = 1 - 2\,\mathbb{T}\{1\} = 0.68$

* overall $\rightarrow$ $$\underline{\underline{F}} = \begin{bmatrix} 0.84 & 0.16 & 1.4\times10^{-3} & 3\times10^{-7} \\ 0.16 & 0.68 & 0.16 & 1.4\times10^{-3} \\ 1.4\times10^{-3} & 0.16 & 0.68 & 0.16 \\ 3\times10^{-7} & 1.4\times10^{-3} & 0.16 & 0.84 \end{bmatrix}$$

* $P_{e,cs} = 1 - \Pr(\text{no-errors}) = 1 - \frac{1}{4}\underbrace{\text{Trace}(\underline{F})}_{\sum_i F_{ii} = 3.04}$

$= 1 - \frac{1}{4}3.04 = 1 - 0.76$

$= 0.24$

* $\frac{P_{e,cs}}{\gamma_{cs}} \leq P_e \leq P_{e,cs} \Rightarrow 0.12 < P_e < 0.24$

ANSWER to Q3

$r_{cs} = 8\,\text{k bits/s}$

$\frac{N_0}{2} = 10^{-12} \Rightarrow N_0 = 2\times 10^{-12}$

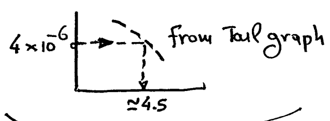
$P_J = 1\text{W}$; $PG = 10^5$; $P_e = 4\times 10^{-6}$; $P_{e,PR} = 4\times 10^{-2}$

$$\frac{B_{ss}}{\underset{r_{cs}}{B}} = PG \Rightarrow B_{ss} = PG \times r_{cs} = 800\,\text{MHz}$$

Baseline Performance

$$B_J = B_{ss} \Rightarrow P_e = \mathrm{T}\left\{\sqrt{(1-\rho)EUE_{equ}}\right\}$$

$$4\times 10^{-6} = \mathrm{T}\left\{\sqrt{(1-\rho)EUE_{equ}}\right\} \leftarrow \text{also } \boxed{\rho=-1}$$



$\Downarrow$

$$\sqrt{2\,EUE_{equ}} = 4.5 \Rightarrow EUE_{equ} = 10.1250$$

$$\Rightarrow \frac{E_b}{N_0 + \frac{P_J}{N_J}} = 10.1250 \Rightarrow E_b = 12\times 10^{-9}$$

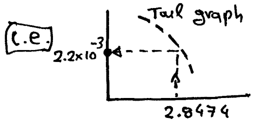
$$\text{i.e.}\quad P_s = E_b/T_{cs} = E_b \cdot r_{cs} = 101\times 10^{-6}$$

$$\therefore \boxed{A = \sqrt{2P_s} = 14.2\,\text{mVolts}}$$

Partial Noise Jammer:

$B_J = 0.4\,B_{ss} = 320\,\text{MHz}$

$$P_e = \mathrm{T}\left\{\sqrt{2\,EUE_{equ}}\right\} = \mathrm{T}\left\{\sqrt{2\underbrace{\frac{E_b}{N_0 + \frac{P_J}{B_J}}}_{=4.05}}\right\} = \mathrm{T}\{2.8474\}$$



i.e. $\underline{P_e = 2.2\times 10^{-3}}$

Pulse Jammer:

$$\left.\begin{array}{l} q = 0.4 \text{ "on"} \\ B_J = B_{ss} \end{array}\right\} \Rightarrow P_e = \underbrace{(1-q)\,\mathrm{T}\left\{\sqrt{2\frac{E_b}{N_0}}\right\}}_{\text{Jammer = "off"}} + q\,\mathrm{T}\left\{\sqrt{2\underbrace{\frac{E_b}{N_0 + \frac{P_J}{q B_{ss}}}}_{=4.05}}\right\}$$

$$\Rightarrow P_e = 0 + 0.4\,\mathrm{T}\{2.8474\}$$

$$\Rightarrow P_e = 0.4 \times 2.2\times 10^{-3}$$

$$\Rightarrow \underline{P_e = 8.8\times 10^{-4}}$$

$$\left.\begin{array}{l} \text{Baseline performance} \\ \text{partial noise jammer} \end{array}\right\} \rightsquigarrow P_{e,PR} = \mathrm{T}\left\{\sqrt{2.EUE_{equ,PR}}\right\}$$

$$\text{pulse Jammer} \longrightarrow P_{e,PR} = q\,\mathrm{T}\left\{\sqrt{2.EUE_{equ,PR}}\right\}$$

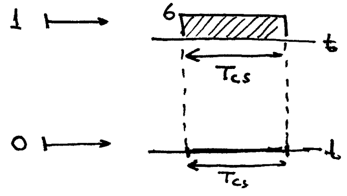
$$\underline{\text{N.B.:}}\quad EUE_{equ,PR} \simeq \begin{cases} \simeq EUE_{J,PR} & \text{(Partial/Baseline)} \\ \simeq q\,EUE_{J,PR} & \text{(pulse Jammer)} \end{cases}$$

$$\therefore \left\{\begin{array}{l} \left.\begin{array}{l}\text{Baseline perf} \\ \& \\ \text{Partial noise Jammer}\end{array}\right\} \rightsquigarrow EUE_{equ,PR} = 1.5325 \\ \text{Pulse Jammer} \longrightarrow EUE_{equ,PR} \simeq 0.8212. \end{array}\right.$$

However, $AJM = 10\log_{10} EUE_{equ} - 10\log_{10} EUE_{equ,PR}$

$$\Rightarrow \begin{array}{l} AJM_{Baseline} = 8.2\,\text{dB} \\ AJM_{partial\ noise} = 4.2\,\text{dB} \\ AJM_{pulse\ Jammer} = 6.9\,\text{dB} \end{array}$$

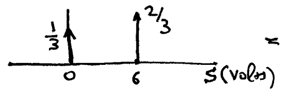
ANSWER to Q4



H_1: a <u>one</u> is being sent

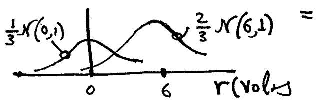
H_0: a <u>zero</u> is being sent

$pdf_s(s) =$



$= \frac{1}{3}\delta(s) + \frac{2}{3}\delta(s-6)$

$pdf_r(r) =$



$= \frac{1}{3}\mathcal{N}(0,1) + \frac{2}{3}\mathcal{N}(6,1)$

likelihood functions: $p_0(r) = pdf_{r|H_0} = \mathcal{N}(0,1)$

$$p_1(r) = pdf_{r|H_1} = \mathcal{N}(6,1)$$

$$\Lambda(r) > \Lambda_0 \text{ where } \Lambda_0 = \frac{\overbrace{Pr(H_0)}^{1/3}\cdot(C_{10}-C_{00})}{\underbrace{Pr(H_1)}_{2/3}\cdot(C_{01}-C_{11})} = \frac{9}{2}$$

observation space: $r \in (-\infty, +\infty)$

likelihood function: $\Lambda(r) = \frac{p_1(r)}{p_0(r)} = \frac{\mathcal{N}(6,1)}{\mathcal{N}(0,1)} =$

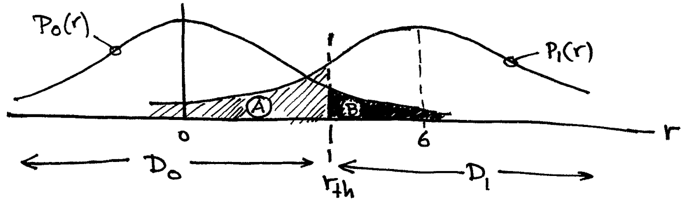
$$= \exp\left(\frac{12r-36}{2}\right) = \exp(6r-18)$$

choose H_1 if $\Lambda(r) > \Lambda_0 \Rightarrow \exp(6r-18) > \frac{9}{2}$

$\Rightarrow$ if $r > \underbrace{3+\frac{1}{6}\ln 4.5}_{\simeq 3.25}$ then choose H_1

solution $\left.\begin{array}{l}\mathcal{R}_0 = (-\infty, 3.25) \\ \mathcal{R}_1 = (3.25, +\infty)\end{array}\right\}$ with $r_{threshold} = 3.25$

ie choose H_0 if $r \in R_0$ and H_1 if $r \in R_1$



area Ⓐ $= Pr(D_0|H_1) = P_{MISS} = T(|3.25-6|) = T(2.75) = 2.8\times10^{-3}$

area Ⓑ $= Pr(D_1|H_0) = P_{FA} = T(3.25) = 5.8\times10^{-4}$

$$Pe = Pr(D_1|H_0)\cdot Pr(H_0) + Pr(D_0|H_1)\cdot Pr(H_1)$$
$$= T(3.25)\cdot\frac{1}{3} + T(2.75)\cdot\frac{2}{3}$$
$$= 5.8\times10^{-4}\times\frac{1}{3} + 2.8\times10^{-3}\times\frac{2}{3} = 2.06\times10^{-3}$$

$$10\log 2^{2\gamma} - 10\log\frac{2^{-0}}{1+4P_e2^{2\gamma}} \underset{\uparrow}{\geq} 1$$

equality $\Longleftrightarrow$ threshold

$$\Rightarrow 10\log\frac{2^{2\gamma}}{\frac{2^{2\gamma}}{1+4P_e2^{2\gamma}}} \geq 1 \Rightarrow 10\log(1+4P_e2^{2\gamma}) \geq 1$$

$$\Rightarrow 1+4P_e2^{2\gamma} \geq 10^{0.1} \Rightarrow 2^{2\gamma} \geq \frac{10^{0.1}-1}{4P_e}$$

$$\Rightarrow 2^{2\gamma} \geq \frac{1}{16P_e} \Rightarrow \gamma \geq \underbrace{\frac{1}{2}\log_2\frac{1}{16P_e}}_{2.46 \text{ bits}}$$