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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2006

MSc and EEE PART III/IV: MEng, BEng.and ACGI

POWER ELECTRONICS AND MACHINES

Friday, 5 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks.

Figure 3.1 is on a separate sheet. If you answer Q3, please ensure that your candidate number is on this sheet and tie it securely into your answer book.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : T.C. Green
Second Marker(s) : B.C. Pal

Special instructions for invigilators

The answer form for question 3(a) must be collected with scripts.

Special instructions for students

The answer form for question 3(a) must be tied inside the back cover of your main script.

The Questions

1. The circuit in Figure Q1.1 is a type of Flyback converter.

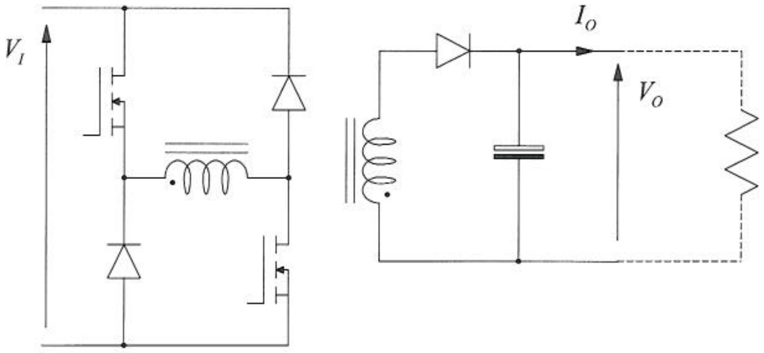


Figure Q1.1 A double-switched flyback converter

(a) Explain the advantages of using two switching devices rather than one. Comment on the voltage ratings of the switches. [4]

(b) The circuit of figure 1.1 is to be used to convert a 12 V input to a 100 V output. The mutual inductances have been chosen to have a turns-ratio of 1:10. The circuit is to be used in discontinuous flux mode with a switching frequency of 100 kHz.

(i) Calculate the energy that must be transferred from input to output in each switching cycle to deliver 10 W to the output. [2]

(ii) Show that the energy stored in the primary inductance at the end of the on-time is given by Eqn Q1.1 [3]

$$E_1 = \frac{\frac{1}{2} V_I^{\ 2}\, t_{On}^{\ \ 2}}{L_1} \qquad \text{(Eqn. Q1.1)}$$

(iii) Find an equation for the minimum off-time to ensure discontinuous operation. [5]

(iv) Find the value of primary winding inductance that causes the circuit to operate on the border of continuous and discontinuous operation at an output power of 10 W. [6]

2. Figure Q2.1 shows a turn-off snubber used with an IGBT that switches a constant current load, I_L = 10 A. The supply voltage of the circuit is V_{DC} = 250 V. The switching frequency is 10 kHz.

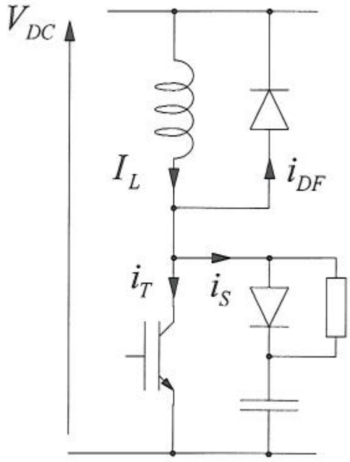


Figure Q2.1

(a) Calculate the switching power loss at turn-on and turn-off for the circuit without a snubber given the following data:

Rise time of current, t_{ri} =	380 ns
Fall time of voltage, t_{fv} =	120 ns
Rise time of voltage, t_{rv} =	100 ns
Fall time of current, t_{fi} =	670 ns

[3]

(b) Two values of capacitor are proposed for the snubber: 4.7 nF and 22 nF. For each of the two capacitor values calculate the items below and comment on how the results would influence the choice of capacitor.

(i) The time taken for the snubber voltage to rise to V_{DC}. [4]

(ii) The power loss in the device at turn-off. [5]

(iii) The power dissipation in the snubber resistor. [2]

(iv) The value of resistor required given that snubber reset must complete within 3 µs [2]

(v) The (approximate) additional turn-on loss caused by the flow of snubber reset current. Assume that the rise of IGBT current at turn-on happens at the same rate-of-change as without the snubber. [4]

3 (a) Figure Q3.1, available on a separate answer form, shows a section through a MOSFET. Mark on that diagram your answers to the following.

(i) Indicate the drain, gate and source connections. [2]

(ii) Indicate which parts of the semiconductor are *n*- and *p*-type and indicate any regions which are heavily or lightly doped. [2]

(iii) Indicate where the channels form in the on-state and where the depletion layer forms in the off-state. [2]

(b) Considering the design of a power MOSFET, explain:

(i) what dimensions and doping would be changed to increase the voltage [3]

(ii) what changes would be made to increase the current rating [3]

(c) Explain why a MOSFET designed for high blocking voltage would also have a high on-state voltage. [3]

(d) Explain how the fabrication of an IGBT differs from that of a MOSFET and why this leads to a lower on-state voltage for a given blocking voltage. [5]

4. (a) Explain how a quasi-resonant SMPS achieves low switching loss. [3]

(b) Explain why the output voltage of, for instance, a buck SMPS is less in practice than that indicated by the product of duty-cycle and input voltage (δV_I). [3]

(c) Explain what is meant by reverse recovery of a diode and what consequence it has for power loss in switching devices. [3]

(d) Describe the form of the spectrum of the output voltage of a single-phase DC to AC converter. [3]

(e) Describe the measures that might be taken to produce a servo motor with a low moment-of-inertia. [4]

(f) Explain why an on-line UPS is able to offer a greater range of features than an off-line UPS. [4]

5. (a) The rectifier circuit shown in Figure Q5.1 draws the phase current shown from phase voltage shown. Calculate the real power and power factor. [6]

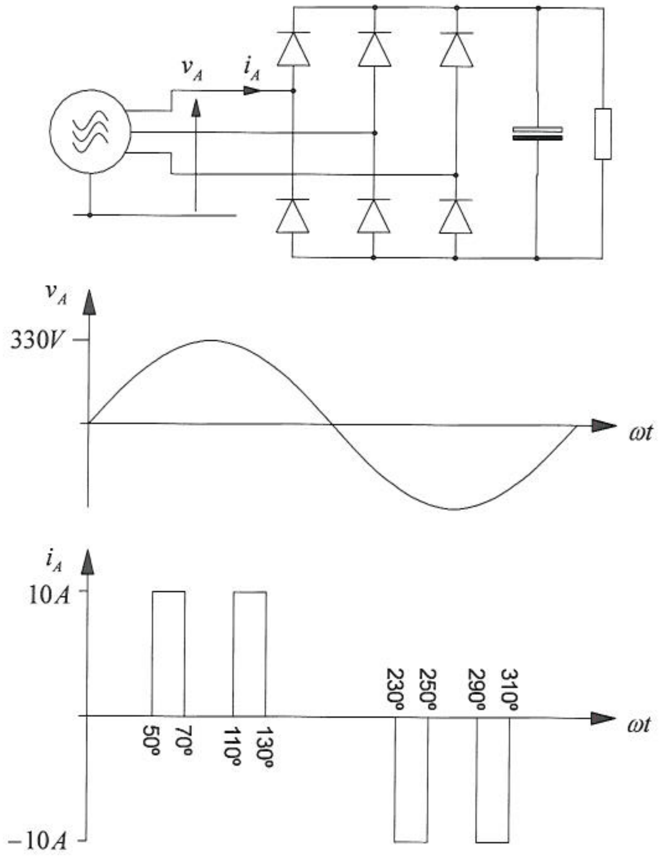


Figure Q5.1 A three-phase diode rectifier and voltage and current waveforms

(b) Explain why diode rectifiers such as that in Figure Q5.1 cause problems in electrical networks. [3]

(c) Draw a circuit diagram of a single-phase AC to DC converter capable of being controlled to draw sinusoidal current and describe its operation and control. [7]

(d) A three-phase AC to DC converter is connected as shown in figure Q5.2 to a 415 V(line), 50 Hz three-phase system via a set of inductors of 4 mH each. Calculate the inverter voltage (magnitude and phase) necessary to draw 35 kW from the inverter to the three-phase system at unity displacement factor. Calculate the voltage required to return 35 kW to the three-phase system from the DC side. [4]

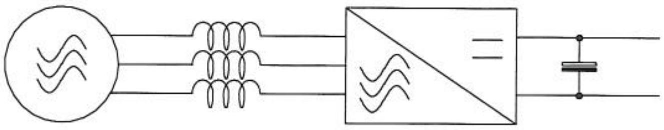


Figure Q5.2 An AC to DC converter and AC supply

6. (a) Describe (using sketches) the main elements of a drive system based on an induction machine. Include a control loop that will control the slip in order to regulate the speed. [7]

(b) For the induction machine model in Figure Q6.1, show that the torque equation $T = 3P\Psi I_R \sin(\delta)$, where P is the number of pole-pairs and Ψ is the magnitude of the air-gap flux-linkage, becomes Eqn 6.1. [5]

$$T = \frac{3P\Psi^2}{R_R}\left(\omega_S - \omega_R\right) \qquad \text{(Eqn 6.1)}$$

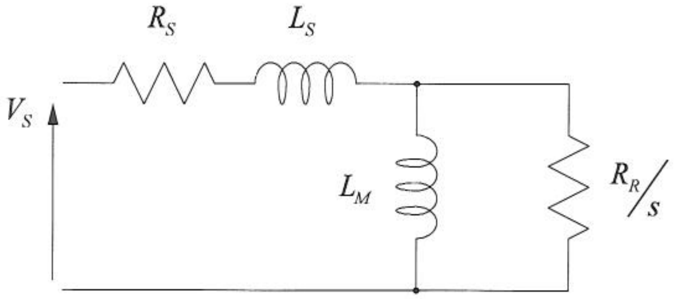


Figure Q6.1 Per-phase equivalent circuit of an induction machine (with negligible rotor leakage inductance)

(c) The induction machine shown in Figure Q6.1 has the following data:

$L_M = 0.035$ H;
$L_S = 0.002$ H;
$R_S = 0.1\ \Omega$;
$R_R = 0.1\ \Omega$;
$P = 1$;
I_R (max) = 40 A;
ω_E (base) = 100π rad/s;
Ψ(max) = 0.65 Wb.

(i) For operation at the base speed (frequency) and maximum flux-linkage, calculate: the torque, the difference between synchronous and rotor speeds, the rotor speed, the output power and the necessary stator voltage. [4]

(ii) Recalculate the same quantities for operation at twice the base speed and half the maximum flux linkage and justify the changes found. [4]

Answer Form for Q3(a)

Candidate Number ____________________

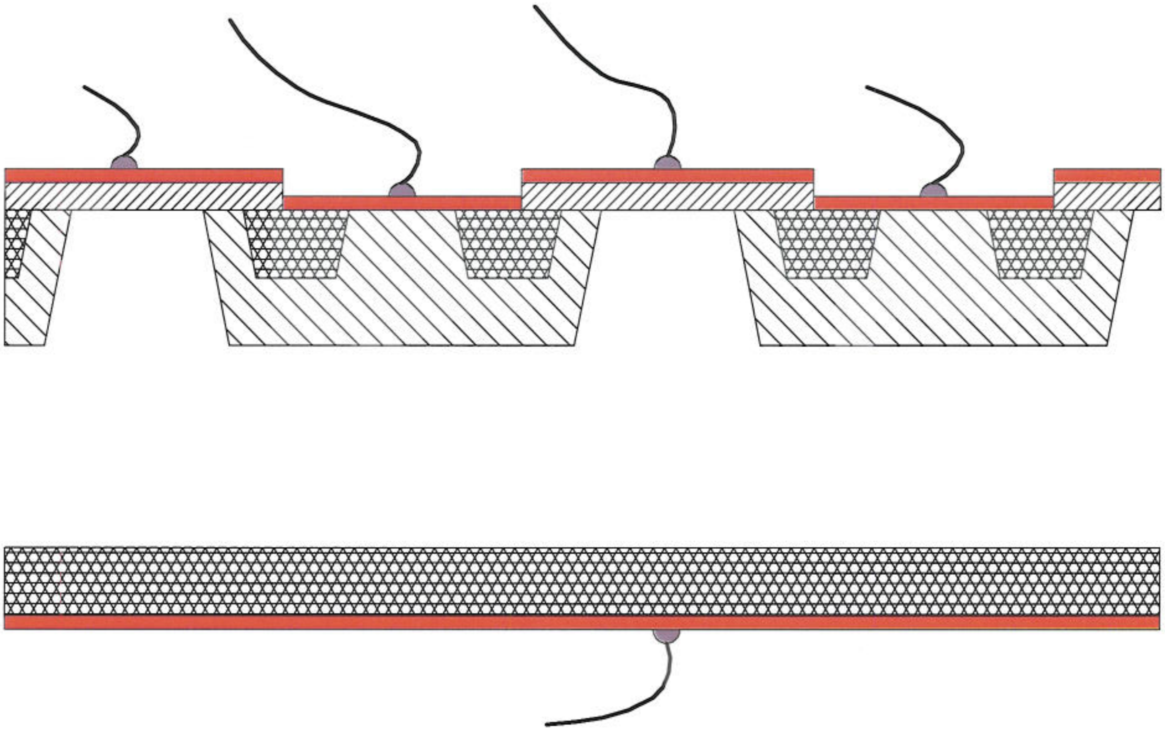


Figure Q3.1 Section through a double diffused MOSFET

The Answers

1. The circuit in Figure Q1.1 is a type of Flyback converter.

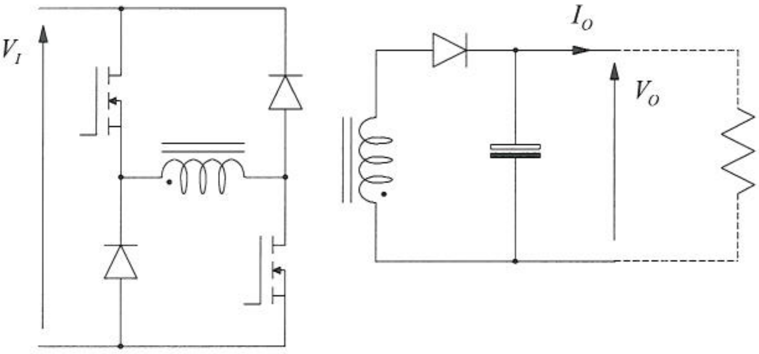


Figure Q1.1 A double-switched flyback converter

(a) Explain the advantages of using two switching devices rather than one. Comment on the voltage ratings of the switches. [4]

Mention to be made of primary leakage inductance preventing instantaneous commutation of current from primary to secondary and the trapped energy in the leakage inductance causing overshoot of the switch voltage at turn-off. Doubled-switched circuit provides diode path for recovery of this energy. Circuit uses two switches rated that the input voltage. Single-switch circuit has switch rated at input plus reflected output voltage.

(b) The circuit of figure 1.1 is to be used to convert a 12 V input to a 100 V output. The mutual inductances have been chosen to have a turns-ratio of 1:10. The circuit is to be used in discontinuous flux mode with a switching frequency of 100 kHz.

(i) Calculate the energy that must be transferred from input to output in each switching cycle to deliver 10 W to the output. [2]

$$E_{SW} = \frac{P}{f_{SW}} = \frac{10}{10^5} = 0.1\,mJ$$

(ii) Show that the energy stored in the primary inductance at the end of the on-time is given by Eqn Q1.1 [3]

$$E_1 = \frac{\frac{1}{2} V_I^{\ 2}\ t_{On}^{\ \ 2}}{L_1} \qquad \text{(Eqn. Q1.1)}$$

Current will increase linearly during the on-time

$$i_{L1}\big|_{t=t_{On}} = \frac{V_I}{L_1} t_{On}$$

$$E_1 = \tfrac{1}{2} L_1 i_{L1}^{\ 2} = \frac{\frac{1}{2} V_I^{\ 2}\ t_{On}^{\ \ 2}}{L_1}$$

(iii) Find an equation for the minimum off-time to ensure discontinuous operation. [5]

Current in the secondary at the start of the off-time can be found from an ampere-turns balance or by matching the stored energy

$$E_1 = \tfrac{1}{2} L_1 {i_{L1}}^2$$
$$E_2 = \tfrac{1}{2} L_2 {i_{L2}}^2$$
$$i_{L2} = i_{L1}\sqrt{\frac{L_1}{L_2}} \qquad \left\{= i_{L1}\frac{N_1}{N_2}\right\}$$

The current will decay linearly while the diode conducts and the diode conduction time can be found

$$\frac{di_{L2}}{dt} = \frac{V_O}{L_2}$$
$$\begin{aligned} t_D &= i_{L2}\frac{L_2}{V_O} \\ &= \frac{V_I\, t_{On}}{L_1}\sqrt{\frac{L_1}{L_2}}\frac{L_2}{V_O} \\ &= t_{On}\frac{V_I}{V_O}\sqrt{\frac{L_2}{L_1}} \\ &= t_{On}\frac{V_I}{V_O}\frac{N_2}{N_1} \end{aligned}$$

The minimum off-time is the diode conduction time
(answer can be obtained via several other routes including considering the increase and decrease of flux rather than current)

(iv) Find the value of primary winding inductance that causes the circuit to operate on the border of continuous and discontinuous operation at an output power of 10 W. [6]

At the boundary of discontinuous operation the on-time and diode-time sum to the switching period.

$$\begin{aligned} T_{SW} &= t_{On} + t_D \\ &= t_{On}\left(1 + \frac{V_I}{V_O}\frac{N_2}{N_1}\right) \end{aligned}$$

And the energy transfer must be satisfied

$$L_1 = \frac{\tfrac{1}{2}{V_I}^2\, {t_{On}}^2}{E_1} = \frac{\tfrac{1}{2}{V_I}^2 {T_{SW}}^2}{E_1\left(1 + \frac{V_I}{V_O}\frac{N_2}{N_1}\right)^2}$$

$$\begin{aligned} L_1 &= \frac{\tfrac{1}{2}\times 12^2 \times \left(10^{-5}\right)^2}{0.1\times 10^{-3}\times\left(1 + \frac{12}{100}\times\frac{10}{1}\right)^2} \\ &= 10.22\,\mu H \end{aligned}$$

Should be 14.88μH

2. Figure Q2.1 shows a turn-off snubber used with an IGBT that switches a constant current load, I_L = 10 A. The supply voltage of the circuit is V_{DC} = 250 V. The switching frequency is 10 kHz.

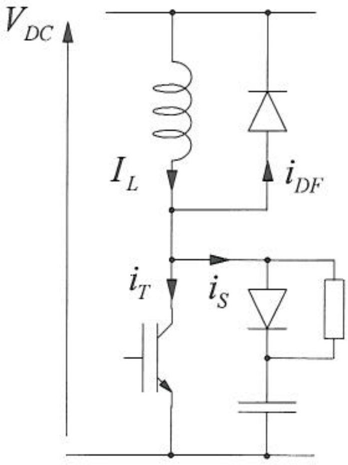


Figure Q2.1

(a) Calculate the switching power loss at turn-on and turn-off for the circuit without a snubber given the following data:
Rise time of current, t_{ri} = 380 ns
Fall time of voltage, t_{fv} = 120 ns
Rise time of voltage, t_{rv} = 100 ns
Fall time of current, t_{fi} = 670 ns
[3]

$$\begin{aligned}P_{On} &= f\,V_{DC}I_L\tfrac{1}{2}\left(t_{ri}+t_{fv}\right)\\ &= 10^4\times250\times10\times\tfrac{1}{2}\times(380+120)\times10^{-9}\\ &= 6.25\,W\\ P_{Off} &= f\,V_{DC}I_L\tfrac{1}{2}\left(t_{rv}+t_{fi}\right)\\ &= 10^4\times250\times10\times\tfrac{1}{2}\times(100+670)\times10^{-9}\\ &= 9.625\,W\end{aligned}$$

(b) Two values of capacitor are proposed for the snubber: 4.7 nF and 22 nF. For each of the two capacitor values calculate the items below and comment on how the results would influence the choice of capacitor.

(i) The time taken for the snubber voltage to rise to V_{DC}. [4]

Initially, the snubber voltage rises quadratically

$$i_T = I_L\left(1 - \frac{t}{t_{fi}}\right)$$

$$i_S = I_L \frac{t}{t_{fi}}$$

$$\frac{dv_S}{dt} = \frac{i_S}{C}$$

$$v_S = \frac{I_L}{C}\frac{\frac{1}{2}t^2}{t_{fi}}$$

Establish if either capacitor completes its rise within the current fall time.

C = 4.7 nF

$$v_S = \frac{I_L}{C}\tfrac{1}{2}t_{fi} = \frac{10}{4.7\times10^{-9}}\times\tfrac{1}{2}\times670\times10^{-9} = 712V$$

So, time is less than this:

$$t^{C=4.7} = \sqrt{\frac{2V_{DC}\,C\,t_{fi}}{I_L}} = \sqrt{\frac{2\times250\times4.7\times10^{-9}\times670\times10^{-9}}{10}}$$
$$= 397\ ns$$

C=22 nF

$$v_S = \frac{I_L}{C}\tfrac{1}{2}t_{fi} = \frac{10}{22\times10^{-9}}\times\tfrac{1}{2}\times670\times10^{-9} = 152V$$

So, voltage rise must continue (linearly) after current fall

$$t^{C=22} = t_{fi} + \left(V_{DC} - v_S^{t=t_{fi}}\right)\frac{I_L}{C} = 670\times10^{-9} + (250-152)\times\frac{22\times10^{-9}}{10}$$
$$= 885\ ns$$

(ii) The power loss in the device at turn-off. [5]

C=4.7 nF

$$E = \int i_T v_S\ dt$$
$$= \int_0^{t^{C=4.7}} I_L\left(1-\frac{t}{t_{fi}}\right)\left(\frac{I_L}{C}\frac{\frac{1}{2}t^2}{t_{fi}}\right)dt + \int_{t^{C=4.7}}^{t_{fi}} I_L\left(1-\frac{t}{t_{fi}}\right)V_{DC}\ dt$$
$$= \frac{{I_L}^2}{C}\left[\frac{\frac{1}{6}t^3}{t_{fi}} - \frac{\frac{1}{8}t^4}{{t_{fi}}^2}\right]_0^{t^{C=4.7}} + I_L V_{DC}\left[t - \frac{\frac{1}{2}t^2}{t_{fi}}\right]_{t^{C=4.7}}^{t_{fi}}$$
$$= \frac{10^2}{4.7\times10^{-9}}\left[\frac{\frac{1}{6}397ns^3}{670ns} - \frac{\frac{1}{8}397ns^4}{670ns^2}\right] + 10\times250\left[670ns - 397ns - \frac{\frac{1}{2}670ns^2}{670ns} + \frac{\frac{1}{2}397ns^2}{670ns}\right]$$
$$= 0.184\ mJ + 0.139\ mJ$$

$$P_{On}{}^{C=4.7} = f\,E = 3.23\,W$$

C=22 nF

$$
\begin{aligned}
E &= \int i_T v_S \, dt \\
&= \int_0^{t_{fi}} I_L \left(1 - \frac{t}{t_{fi}}\right)\left(\frac{I_L}{C}\frac{\frac{1}{2}t^2}{t_{fi}}\right) dt \\
&= \frac{{I_L}^2}{C}\left[\frac{\frac{1}{6}t^3}{t_{fi}} - \frac{\frac{1}{8}t^4}{{t_{fi}}^2}\right]_0^{t_{fi}} \\
&= \frac{10^2}{22\times10^{-9}}\left[\tfrac{1}{6}670ns^2 - \tfrac{1}{8}670ns^2\right] = 0.085\,mJ
\end{aligned}
$$

$P_{On}^{\ C=22} = f\,E = 0.85\,W$

4.7 nF snubber reduces the turn off loss by a factor of about 50 and the 22nF achieves a further factor of 2.

(iii) The power dissipation in the snubber resistor. [2]

C = 4.7 nF

$P_R^{\ C=4.7} = f\,\tfrac{1}{2}C\,{V_{DC}}^2 = 10^4 \times \tfrac{1}{2}\times 4.7\times10^{-9}\times 250^2 = 1.47\,W$

C = 22 nF

$P_R^{\ C=22} = f\,\tfrac{1}{2}C\,{V_{DC}}^2 = 10^4 \times \tfrac{1}{2}\times 22\times10^{-9}\times 250^2 = 6.87\,W$

The 22nF snubber causes resistor power loss of about 2/3rd of the original device loss. The 4.7nF snubber is about a ¼ of that.

(iv) The value of resistor required given that snubber reset must complete within 3 μs [2]

Assume 5 time-constants required for discharge of snubber resistor

C = 4.7 nF

$$R = \frac{t_{\mathrm{Re}set}}{5C} = \frac{3\times10^{-6}}{5\times4.7\times10^{-9}} = 127.6\,\Omega$$

C = 22 nF

$$R = \frac{t_{\mathrm{Re}set}}{5C} = \frac{3\times10^{-6}}{5\times22\times10^{-9}} = 27.3\,\Omega$$

(v) The (approximate) additional turn-on loss caused by the flow of snubber reset current. Assume that the rise of IGBT current at turn-on happens at the same rate-of-change as without the snubber. [4]

Assume fall of snubber voltage during turn-on of the switch is slow compared to the turn-on process itself. Assume turn-on can be treated as being conducted at a higher current due to snubber discharge current and at a linearly extended current rise time.

C = 4.7 nF

$$I = I_L + \frac{V_{DC}}{R} = \left(10 + \frac{250}{127.6}\right) = 11.95$$

$$t'_{ri} = t_{ri}\frac{I}{I_L} = 380\times10^{-9}\times\frac{11.95}{10} = 454\ ns$$

$$\begin{aligned} P_{On}^{C=4.7} &= f\,\tfrac{1}{2}V_{DC}I\left(t'_{ri}+t_{fv}\right) \\ &= 10^4\times\tfrac{1}{2}\times250\times11.95\times\left(454+120\right)\times10^{-9} \\ &= 8.58\,W \\ P_{On}^{C=0} &= 6.25\,W \\ P_{On}^{C=4.7} - P_{On}^{C=0} &= 2.33\,W \end{aligned}$$

C = 22 nF

$$I = I_L + \frac{V_{DC}}{R} = \left(10 + \frac{250}{27.3}\right) = 19.16\ A$$

$$t'_{ri} = t_{ri}\frac{I}{I_L} = 380\times10^{-9}\times\frac{19.16}{10} = 728\ ns$$

$$\begin{aligned} P_{On}^{C=4.7} &= f\,\tfrac{1}{2}V_{DC}I\left(t'_{ri}+t_{fv}\right) \\ &= 10^4\times\tfrac{1}{2}\times250\times19.16\times\left(728+120\right)\times10^{-9} \\ &= 20.31\,W \\ P_{On}^{C=0} &= 6.25\,W \\ P_{On}^{C=4.7} - P_{On}^{C=0} &= 14.06\,W \end{aligned}$$

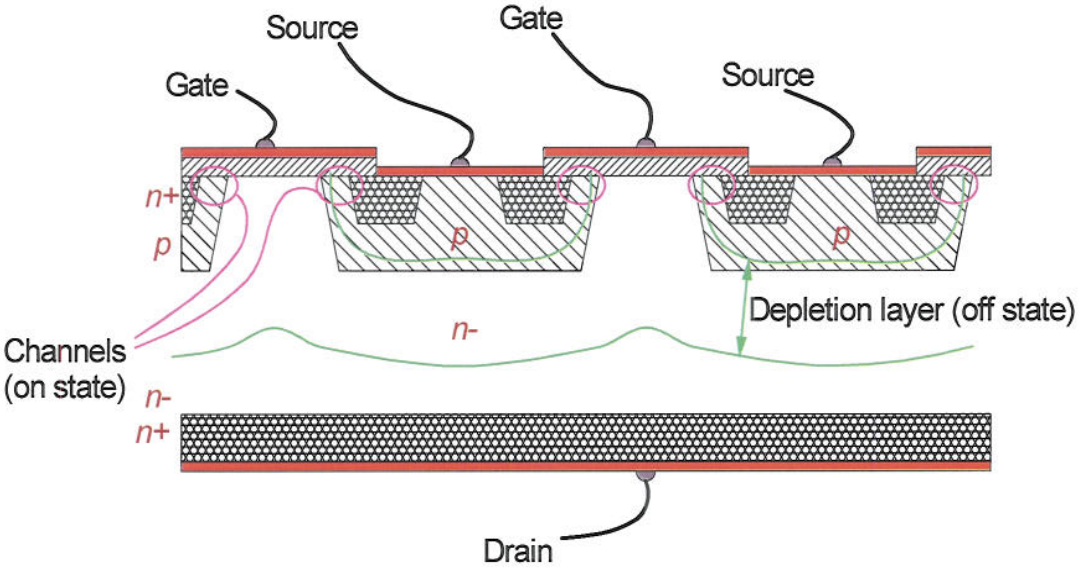
Extra turn-on loss with 4.7nF is substantial but less than the saved turn-off loss. For the 22nF snubber the overall loss is increased (and a turn-on snubber would also be needed).

3 (a) Figure Q3.1, available on a separate page, shows a section through a MOSFET. Mark on that diagram your answers to the following.

(i) Indicate the drain, gate and source connections. [2]

(ii) Indicate which parts of the semiconductor are *n*- and *p*-type and indicate any regions which are heavily or lightly doped. [2]

(iii) Indicate where the channels form in the on-state and where the depletion layer forms in the off-state. [2]



(b) Considering the design of a power MOSFET, explain:

(i) what dimensions and doping would be changed to increase the voltage rating [3]

The depth of the *n*- drain region would be increased to allow a deeper depletion layer and the doping of this region would be decreased to keep the peak electric field strength below the breakdown value (which in turn requires the depletion layer to grow deeper still).

(ii) what changes would be made to increase the current rating [3]

The requirement would be for a greater channel width. This would be achieved by fabricating more parallel connected cells on a larger die.

(c) Explain why a MOSFET designed for high blocking voltage would also have a high on-state voltage. [3]

The deep lightly-doped drain region of a high voltage MOSFET has a poor conductivity (both majority and minority carrier concentrations at almost intrinsic levels) and so a large ohmic voltage drop develops in series with the voltage dropped across the channel itself.

(d) Explain how the fabrication of an IGBT differs from that of a MOSFET and why this leads to a lower on-state voltage for a given blocking voltage. [5]

The IGBT has a p^+ layer where the MOSFET has an n^+ layer at the drain contact. This forms a p^+n^- junction that is forward biased for forward conduction. Electrons, (majority carriers in the mosfet channel and the drain) are injected across this junction from n- to p+. The injection across the junction is in proportion to the doping and there is a much larger injection of holes into the n- drain and the majority carrier concentration rises. Charge neutrality in the drain will require the minority carrier concentration to rise also. The carrier concentrations rise far above the near intrinsic values of the drain (high level injection). This increases the conductivity of the lightly doped drain and reduces the voltage drop.

4 (a) Explain how a quasi-resonant SMPS achieves low switching loss. [3]

A zero-current switched circuit, for instance, places an inductor in series with the switch. At turn-on, the current is initially zero and constrained to rise slowly. The inductor forms part of a series resonant circuit which is excited by the switch turning on. At some point later the current in the inductor will reverse and flow in a parallel diode rather than the switch. The switch can then be turned off at zero current.

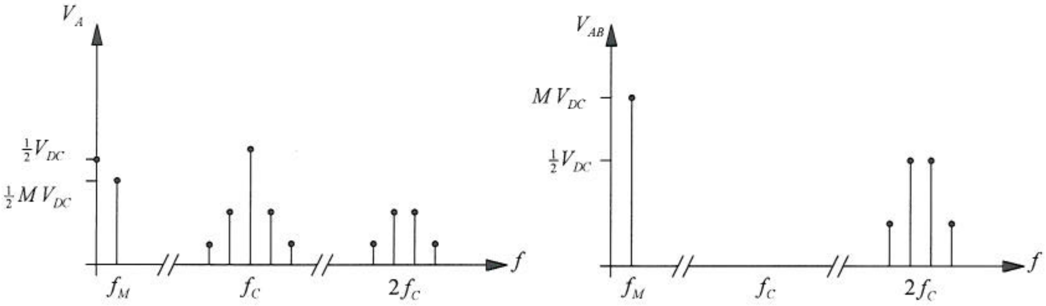
(b) Explain why the output voltage of, for instance, a buck SMPS is less in practice than the indicated by δV_I. [3]

The standard equation is derived assuming that no voltage is dropped across the semiconductors during conduction and that inductors have no series resistance. Both assumptions are no true in practice and both tend to decrease the output voltage.

(c) Explain what is meant by reverse recovery of a diode and what consequence it has for power loss in switching devices. [3]

A *pn* junction diode in forward conduction has raised levels of minority carriers (and majority carriers) at the depletion layer edge that move away by diffusion along a concentration gradient. This stored charge is available for reverse conduction if the circuit conditions dictate. However, in reverse conduction the charge that leaves the region is not replenished by injection and so reverse conduction last only until the stored charge is evacuated and the diode "recovers" its reverse blocking capability. Reverse recovery tends to happen as a switch turns on and the recovery current flows through the switch as it is completing its turn on operation. Thus, turn-on losses of that switch are increased.

(d) Describe the form of the spectrum of the output voltage of a single-phase DC to AC converter. [3]



Spectrum consists of the desired term at LF plus a harmonic series of carrier terms each with sidebands. The carriers are at multiples of the switching frequency or twice the switching frequency depending on the modulation scheme.

(e) Describe the measures that might be taken to produce a servo motor with a low moment-of-inertia. [4]

Reducing the radius and increasing the axial length (to maintain the rotor volume) reduces the inertia. For high pole-number, machines it is possible to hollow out the rotor to form an annular shape because the flux density in the centre of the rotor is very low anyway. For relatively low torque machines where the forces on the rotor conductors are small it is possible to form an iron-less rotor in which the rotor conductors are supported in a thin epoxy carrier that fits within the airgap. All of the magnetic material (steel and PM) is stationary and does not contribute to the inertia. This method can be applied to axial or radial flux machines.

(f) Explain why an on-line UPS is able to offer a greater range of features than an off-line UPS. [4]

Off-line UPS is only active during a grid supply failure. The on-line UPS is always active and processes all of the power flowing from input to output and thus completely decouples the two. The output can be made immune to input problems such as sag/swell, harmonic distortion, frequency drift. The input can be made immune to output disturbances such as harmonically distorted load current.

5. (a) The rectifier circuit shown in Figure Q5.1 draws the phase current shown from phase voltage shown. Calculate the real power and power factor. [6]

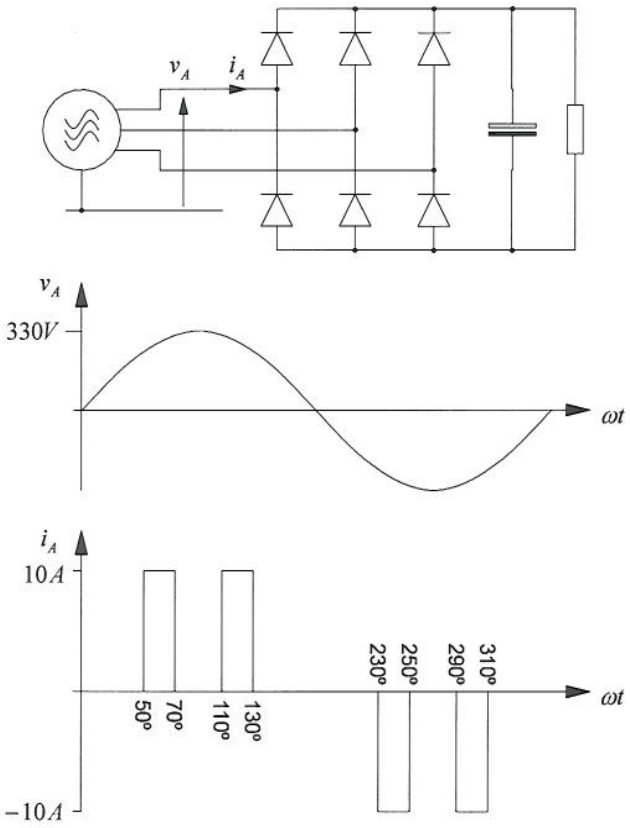


Figure Q5.1 A three-phase diode rectifier and voltage and current waveforms

Real power found by averaging instantaneous power (over three phases)

$$
\begin{aligned}
P &= 3\frac{1}{2\pi}\int_{-\pi}^{\pi} v_A i_A \, d(\omega t) \\
&= \frac{3}{\pi} V_A I_A \left(\int_{50}^{70} \sin(\theta) d\theta + \int_{110}^{130} \sin(\theta) d\theta \right) \\
&= \frac{3}{\pi} \times 330 \times 10 \times \left[-\cos 70^\circ + \cos 50^\circ - \cos 130^\circ + \cos 110^\circ \right] \\
&= 1.895 \, kW
\end{aligned}
$$

Find RMS current and apparent power to find power factor

$$
\begin{aligned}
I_{RMS} &= \sqrt{\frac{1}{2\pi}\int_{0}^{2\pi} i(\omega t)^2 \, d(\omega t)} \\
&= \sqrt{\frac{1}{360^\circ} 10^2 \left[4 \times 20^\circ \right]} \\
&= 4.71 \, A
\end{aligned}
$$

$$V_{RMS} = \frac{\hat{V}}{\sqrt{2}} = 233.3\,V$$

$$PF = \frac{P}{3V_{RMS}I_{RMS}} = \frac{1895}{3\times 233.3\times 4.71} = 0.575$$

(b) Explain why diode rectifiers such as that in Figure Q5.1 cause problems in electrical networks. [3]

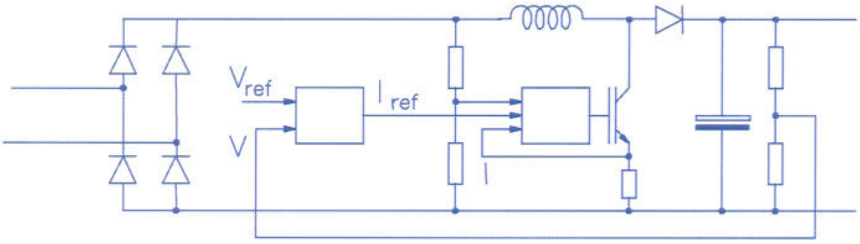
Two issues: harmonic currents causing heating of cables and transformers and harmonic voltage drops distorting consumer supplies and causing interference.

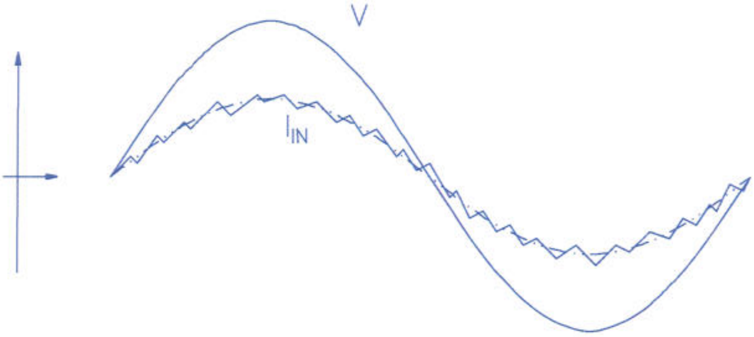
(c) Draw a circuit diagram of a single-phase AC to DC converter capable of being controlled to draw sinusoidal current and describe its operation and control. [7]

A boost type SMPS can be arranged to draw a controlled current from a varying voltage. A diode bridge is used to rectify the AC voltage and a control loop set to force the current to be a full wave rectified sinewave also (a simple sinewave will be drawn from the AC supply). The magnitude of the sinewave is set by a second controller according to the error between the DC output voltage and its reference value.

Because the circuit is under closed loop control the output voltage can reject output current disturbance and changes in input voltage magnitude. The current control can easily be amended to provide current limiting to prevent large fault currents.

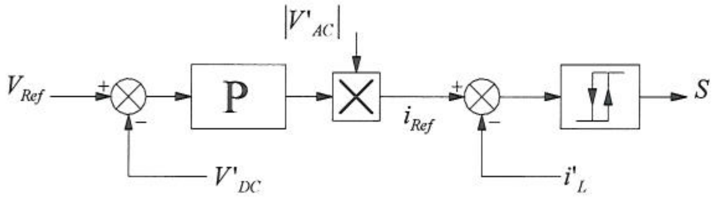
The circuit suffers a twice line-frequency output voltage ripple because of the inherent pulsation of a single phase power source. Although the circuit can be controlled to avoid low frequency current harmonics there are emission of high frequency current multiples of the switching frequency.

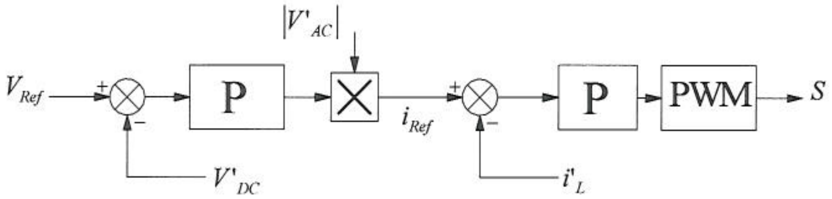




This has become a popular topology and several manufactures produce ICs which include these control functions. Two variants exist. The first uses a hysteresis controller for the current loop and results in fast response but a varying switching frequency. The second uses fixed-frequency PWM. The spread spectrum resulting from a varying switching frequency can make filter design difficult but it does have the advantage of spreading emissions thinly over a range rather than producing high level emissions concentrated at particular frequencies.

The control loop for the output voltage must be designed with a limited bandwidth typically around 5 Hz. The energy drawn from a single-phase supply will vary at twice line frequency even if sinusoidal current wave-shape is achieved. Thus, there will be an unavoidable output voltage ripple at twice line frequency which the control loop should not attempt to reject.





(d) A three-phase AC to DC converter is connected as shown in figure Q5.2 to a 415 V(line), 50 Hz three-phase system via a set of inductors of 4 mH each. Calculate the inverter voltage (magnitude and phase) necessary to draw 35 kW from the inverter to the three-phase system at unity displacement factor. Calculate the voltage required to return 35 kW to the three-phase system from the DC side. [4]

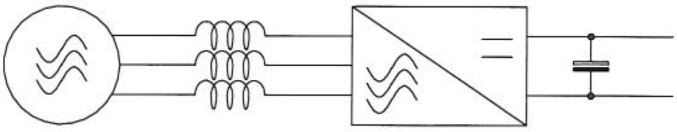


Figure Q5.2 An AC to DC converter and AC supply

Phase voltage of source is 240V

Phase current required and inductive voltage drop are

$$I_P = \frac{P}{3V_P \cos\phi} = \frac{35{,}000}{3 \times 240} = 48.6\ A$$

$$V_{XL} = j\omega L I_P = j2\pi \times 50 \times 0.04 \times 48.6 = j61.1\,V$$

To import power the current is in phase with the supply voltage and the inductive voltage leads the supply. The converter voltage is

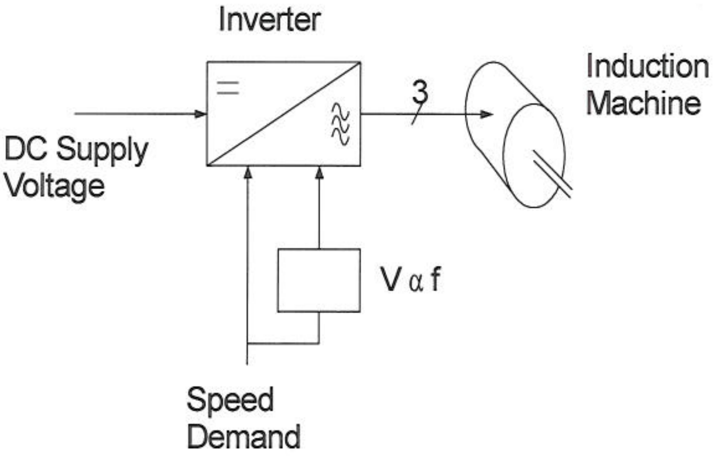
$$V_C = V_S - V_{XL} = 240 - j61.1 = 247.6\angle -14.2^\circ \quad V$$

To export power the current must be in anti-phase and the voltage drop reverses.

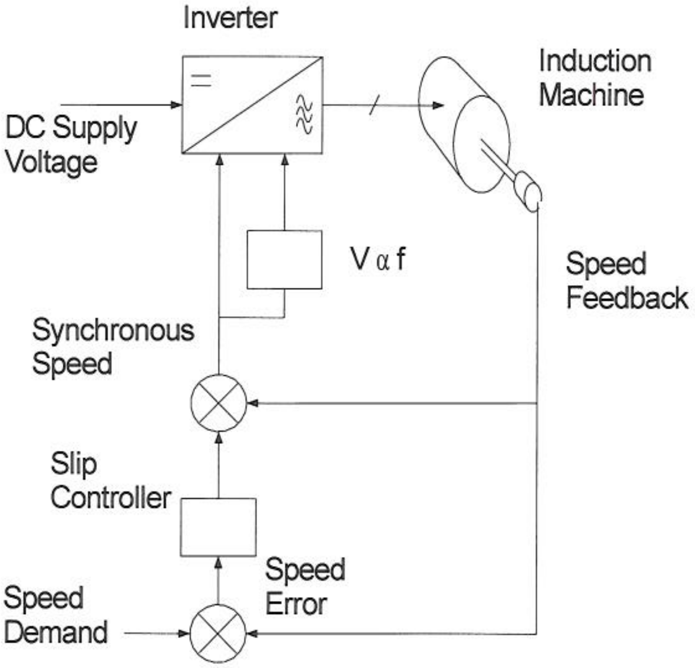
$$V_C = V_S - V_{XL} = 240 + j61.1 = 247.6\angle +14.2^\circ \quad V$$

6. (a) Describe (using sketches) the main elements of a drive system based on an induction machine. Include a control loop that will control the slip in order to regulate the speed. [7]

In the simplest case the speed demand is translated directly to a supply frequency reference for the inverter (taking account of the number of poles of the machine). It is assumed that the slip of the machine is small and that this small speed error is unimportant. In order to keep the air-gap flux magnitude at approximately its design value, the voltage magnitude is varied in proportion to the frequency. Neglecting the effect of the stator winding impedance, this will keep the magnetising current constant.



If the error introduced by the slip is unacceptable then speed feedback is used. The speed error is calculated and from this a slip demand established to increase or decrease the torque. The desired synchronous speed is calculated by adding the slip to the measured speed. The inverter frequency is then set. The applied voltage is again calculated in proportion to the frequency.



(b) For the induction machine model in Figure Q6.1, show that the torque equation $T = 3P\Psi I_R \sin(\delta)$, where P is the number of pole-pairs and Ψ is the magnitude of the air-gap flux-linkage, becomes Eqn 6.1. [5]

$$T = \frac{3P^2\Psi^2}{R_R}\left(\omega_S - \omega_R\right) \text{ (Eqn 6.1)}$$

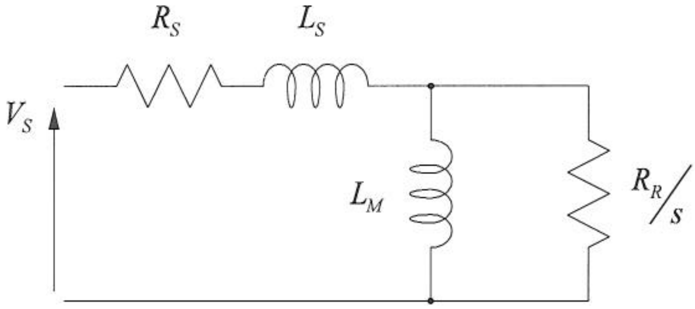


Figure Q6.1 Per-phase equivalent circuit of an induction machine (with negligible rotor leakage inductance)

Defining E as the voltage across L_M.

$$\overline{\Psi} = \overline{I_M} \cdot L_M$$

$$\overline{E} = j\omega_E \overline{\Psi}$$

$$\overline{I_R} = \frac{\overline{E}}{R_R/s} = \frac{s\overline{E}}{R_R}$$

$$T = 3P\left|\overline{\Psi}\right| \cdot \left|\overline{I_R}\right| \sin(\delta)$$

$$= 3P\left|\overline{\Psi}\right| \cdot \frac{s\omega_E\left|\overline{\Psi}\right|}{R_R} \sin\left(\angle j\overline{\Psi} - \angle\overline{\Psi}\right)$$

$$= \frac{3P\Psi^2}{R_R} s\omega_E$$

$$= \frac{3P^2\Psi^2}{R_R}\left(\omega_S - \omega_R\right)$$

(c) The induction machine shown in Figure Q6.1 has the following data:

$L_M = 0.035$ H;
$L_S = 0.002$ H;
$R_S = 0.1\ \Omega$;
$R_R = 0.1\ \Omega$;
$P = 1$;
I_R (max) = 40 A;
ω (base) = 100π rad/s;
Ψ(max) = 0.65 Wb.

(i) For operation at the base speed and maximum flux-linkage, calculate: the torque, the difference between synchronous and rotor speeds, the rotor speed, the output power and the necessary stator voltage. [4]

Because the rotor branch is resistive the load angle is zero.

$$\begin{aligned} T &= 3P\Psi I_R \sin(\delta) \quad \sin(\delta)=1 \\ &= 3\times1\times0.65\times40 \\ &= 78\ Nm \end{aligned}$$

$$\begin{aligned} \omega_R - \omega_S &= \frac{I_R R_R}{\omega_E \Psi}\omega_S \\ &= \frac{40\times0.1}{100\pi\times0.65}\times100\pi \\ &= 6.15\ rad/s \end{aligned}$$

$$\begin{aligned} \omega_R &= 100\pi - 6.15 \\ &= 308\ rad/s \end{aligned}$$

$$\begin{aligned} P &= T\omega_R \\ &= 78\times308 \\ &= 24\ kW \end{aligned}$$

$$\begin{aligned} \overline{V} &= \overline{E} + (R_S + j\omega_E L_S)(\overline{I_R} - j\overline{I_M}) \\ &= 100\pi\times0.65 + (0.1 + j100\pi\times0.002)(40 - j18.6) \\ &= 220 + j23 \end{aligned}$$

$$V = 221\ V$$

(ii) Recalculate the same quantities for operation at twice the base speed and half the maximum flux linkage and justify the changes found. [4]

At $\omega = 2\omega_B$ and $\Psi = {}^{\Psi_B}\!/_{2}$

$$
\begin{aligned}
T &= 3P\Psi I_R \sin(\delta) \quad \sin(\delta) = 1 \\
&= 3 \times 1 \times {}^{0.65}\!/_{2} \times 40 \\
&= 39\ Nm
\end{aligned}
$$

$$
\begin{aligned}
\omega_R - \omega_S &= \frac{I_R R_R}{\omega_E \Psi} \omega_S \\
&= \frac{40 \times 0.1}{100\pi \times {}^{0.65}\!/_{2}} \times 100\pi \\
&= 12.30\ rad/s
\end{aligned}
$$

$$
\begin{aligned}
\omega_R &= 2 \times 100\pi - 12.30 \\
&= 616\ rad/s
\end{aligned}
$$

$$
\begin{aligned}
P &= T\omega_R \\
&= 39 \times 616 \\
&= 24\ kW
\end{aligned}
$$

$$
\begin{aligned}
\overline{V} &= \overline{E} + (R_S + j\omega_E L_S)(\overline{I_R} - j\overline{I_M}) \\
&= 2 \times 100\pi \times {}^{0.65}\!/_{2} + (0.1 + j2 \times 100\pi \times 0.002)(40 - j9.3) \\
&= 220 + j48 \\
V &= 225\ V
\end{aligned}
$$