E3.14
AO7

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2004

MSc and EEE PART III/IV: MEng, BEng.and ACGI

POWER ELECTRONICS AND MACHINES

Thursday, 13 May 10:00 am

Time allowed: 3:00 hours

Corrected Copy

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	T.C. Green
	Second Marker(s) :	B.C. Pal

[3.14]

1.

(a) Sketch the circuit diagram of the zero-current switched quasi-resonant version of a buck converter (ZCS QRC). [3]

(b) Explain why the switching loss of ZCS QRC is much lower than that in a standard buck converter. [4]

(c) Explain why it is necessary to arrange that $\frac{V_I}{\omega_R L_R} > I_O$,

where V_I is the input voltage, ω_R is the resonant frequency, L_R is the resonant inductor and I_O is the output current. [3]

(d) Explain how the output voltage of the circuit is controlled. [3]

(e) A ZCS QRC is built with L_R = 5μH and C_R = 10 nF. It is operated from an input voltage of 13 V and gives an output voltage of 5 V. Calculate the following:

(i) the maximum permissible output current [2]

(ii) the switching frequency of the transistor [3]

(iii) the worst case pre-charge time of the inductor (time between first turn-on of the transistor and the beginning of the resonant action) [2]

2.

(a) Describe the mechanisms by which a *pn* junction can breakdown under reverse bias and state what measures are taken in the design of a diode to achieve a high breakdown voltage. [3]

(b) Describe the characteristics of the thyristor and the GTO thyristor. Under what circumstances might a thyristor be preferred to a GTO thyristor? [3]

(c) Compare the properties of an IGBT and a MOSFET [4]

(d) Figure 2 shows snubber circuit applied to a transistor switching an inductive load. The load has been approximated by a constant current. Other details of the circuit are: supply voltage, $V_S = 500$ V; snubber capacitor $C_S = 22$ nF; transistor current fall time, $t_{fi} = 500$ ns and transistor voltage rise time, $t_{rv} = 100$ns.

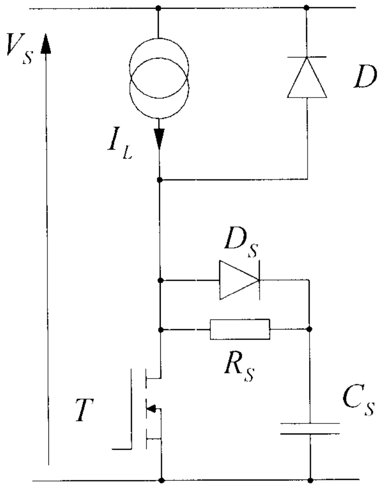


Figure 2

(i) Explain why the transistor has lower switching loss with the snubber than without. [2]

(ii) Find the load current at which the snubber capacitor fully charges in exactly the time taken for the transistor current to fall. [3]

(iii) Calculate the energy dissipated in the transistor at turn-off with the snubber in place and compare this to the energy that would be dissipated without the snubber. [5]

3.

(a) Explain the operation of the Ćuk switch-mode power supply, SMPS [5]

(b) Describe why a Ćuk SMPS might be a better design than the Buck-Boost SMPS [3]

(c) A Ćuk SMPS has been built using two 200 μH inductors and two 470 μF capacitors with effective series resistance of 50 mΩ. It is used to provide a -5.0 V output from an input voltage that varies between 12.0 and 17.0 V.

(i) Choose a switching frequency that will keep the output voltage ripple below 10 mV. You may assume that the ripple is dominated by the voltage across the effective series resistance. [7]

(ii) Calculate the average voltage and voltage ripple across the central coupling capacitor when the output current is -10 A. You may neglect the effective series resistance in this calculation [5]

[3.14]

4.

(a) A DC machine has separate field and armature connections and has the following properties:

Maximum field current I_F = 1.0A
Armature resistance R_A = 0.3 Ω
Maximum armature current, I_A^{Max} = 5 A
The armature has a back EMF of 32 V when the field current is at its maximum and shaft speed is 1,000 rpm.

The machine is supplied via the circuit of figure Q4.1 with an input voltage of V_{DC} = 48 V.

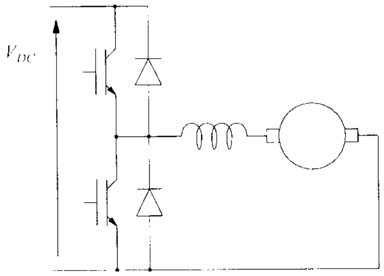


Figure 4.1

Figure 4.2 shows 9 operating points on the torque-speed plane which are further described in Table 4 (overleaf).

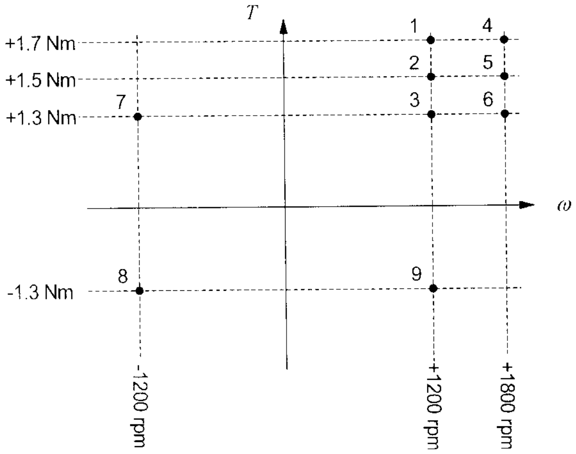


Figure 4.2

[3.14]

Operating Point	Speed (rpm)	Torque (Nm)
1	+1200	1.7
2	+1200	1.5
3	+1200	1.3
4	+1800	1.7
5	+1800	1.5
6	+1800	1.3
7	-1200	1.3
8	-1200	-1.3
9	+1200	-1.3

Table 4

(i) Determine which of the operating points can be achieved with the machine and chopper without the use of field weakening. [8]

(ii) Determine whether any of the remaining points can be achieved by using field weakening. [4]

(b) With the aid of diagrams, describe the construction and operation of a brushless DC machine drive. [8]

5.

(a) Draw a block diagram of the control system for an induction machine that uses controlled slip and describe the operation of the system. [6]

(b) Figure Q5 shows an approximate equivalent circuit of an induction machine. The machine has the following properties:

Number of pole-pairs, $P = 1$
Stator resistance, $R_S = 1.8\ \Omega$
Stator Leakage Inductance, $L_S = 3$ mH
Magnetising Inductance, $L_M = 100$ mH
Referred rotor resistance, $R'_R = 1.2\ \Omega$
Maximum referred rotor current, $I'^{Max}_R = 12$ A RMS
Maximum magnetising current, $I_M^{Max} = 6$ A RMS

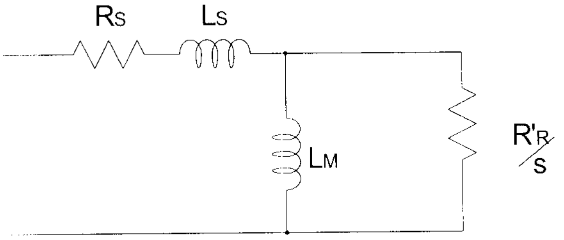


Figure 5

(i) Show that if the magnetising current is maintained at its maximum value then the rotor speed that gives the maximum allowed rotor current is a fixed interval below the synchronous speed and is given by:

$$\omega_R = \omega_S - \frac{I'_R R'_R}{P L_M I_M^{Max}}$$ [4]

(ii) Find the torque that is developed under the condition in (i) [3]

(iii) Find the required stator voltage for operation under the condition in (i) at 4,000 rpm and the minimum DC supply voltage to the inverter need to achieve this. [4]

(iv) With the voltage magnitude limited at the value found in (iii), estimate the maximum torque that can be achieved at a speed of 6,000 rpm. [3]

[3.14]

6.

(a) Figure 6 shows the waveform of AC side current of a rectifier and its relationship to the AC voltage.

(i) Calculate the RMS value of the current [4]

(ii) Calculate the fundamental component of the current [4]

(iii) Calculate the power factor of the current [2]

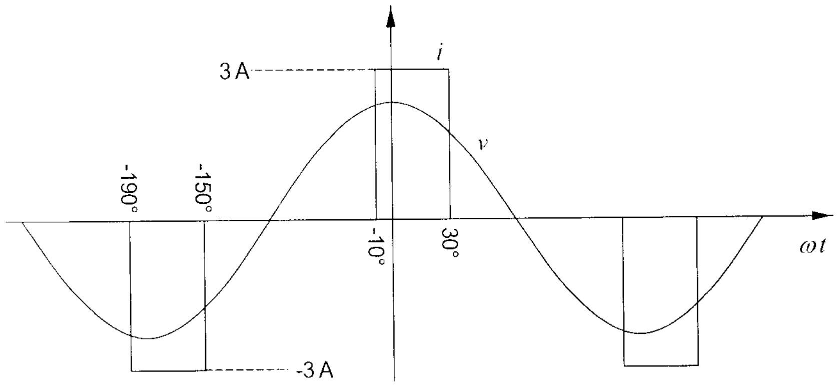


Figure 6

(b) Describe why circuits that draw non-sinusoidal currents from the AC main should not be used. [3]

(c) Describe a circuit and a control scheme that converters AC to DC and maintains a sinusoidal AC current [7]

Power Electronics + Machines

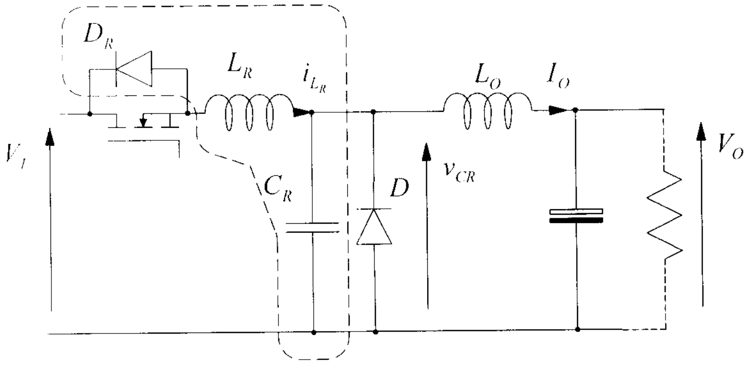
Solutions 2004

[3.14]

1.

(a) Sketch the circuit diagram of the zero-current switched quasi-resonant version of a buck converter (ZCS QRC). [3]

A ZCS QRC



(b) Explain why the switching loss of ZCS QRC is much lower than that in a standard buck converter. [4]

At turn-on of the MOSFET, the current is zero and constrained to rise slowly by the presence of LR in series. Thus the losses during turn-on are very small

The current through the resonant circuit (LR/CR) will reverse during the second half of the resonant cycle and that current is carried through DR rather than the MOSFET. The MOSFET current reduces to zero while it is still switched on and the MOSFET can be turned off while it is not carrying current. Turn off is thus lossless

(c) Explain why it is necessary to arrange that $\frac{V_I}{\omega_R L_R} > I_O$,

where V_I is the input voltage, ω_R is the resonant frequency, L_R is the resonant inductor and I_O is the output current. [3]

The current in LR is a resonant oscillation of amplitude $\frac{V_I}{\omega_R L_R}$ *about the "steady-state" current of Io that flows in Lo. It is important that this total current becomes negative at some point in order to achieve lossless turn off. Thus the peak of the oscillatory term must be greater than the DC offset.*

(d) Explain how the output voltage of the circuit is controlled. [3]

Each turn-on of the MOSFET initiates a single resonant cycle that is of fixed duration (neglecting some variation in the pre-charge time of the inductor). A fixed-duration fixed-amplitude voltage pulse is applied to the LC output filter. The only freedom to vary the average value of this pulse train is to vary the off-time of the transistor. The circuit is operated with a fixed on-time, variable off-time.

(e) A ZCS QRC is built with L_R = 5μH and C_R = 10 nF. It is operated from an input voltage of 13 V and gives an output voltage of 5 V. Calculate the following:

(i) the maximum permissible output current [2]

Amplitude of oscillatory current is:

[3.14]

$$I_R = \frac{V_I}{\omega_R L_R} = V_I\sqrt{\frac{C_R}{L_R}} = 13\sqrt{\frac{10\times10^{-9}}{5\times10^{-6}}} = 0.581A$$

The output current should be kept below this value.

(ii) the switching frequency of the transistor [3]

Approximately:

$$\frac{V_O}{V_I} = \frac{T_R}{T_{SW}} = \frac{f_{SW}}{f_R}$$

$$f_{SW} = \frac{1}{2\pi\sqrt{L_R C_R}}\frac{V_O}{V_I} = \frac{1}{2\pi\sqrt{5\mu\times10\text{n}}}\frac{5}{13} = \frac{5}{13}\times711.8\,\text{kHz} = 273\,\text{kHz}$$

(iii) the worst case pre-charge time of the inductor (time between first turn-on of the transistor and the beginning of the resonant action) [2]

The maximum output current was established as 0.58A

The inductor charges up under the input voltage

$$t_{PC} = \frac{L}{V_I}I_L^{Max} = \frac{5\times10^{-6}\times0.58}{13} = 223ns$$

2.

(a) Describe the mechanisms by which a *pn* junction can breakdown under reverse bias and state what measures are taken in the design of a diode to achieve a high breakdown voltage. [3]

There are two mechanisms for high voltage diodes, avalanche breakdown and punch-through. If the peak electric field strength (potential gradient) becomes sufficiently high then atoms will be ionised and the free electrons will accelerate under the field gaining enough energy to ionise further atoms on collision. With sufficient energy the ionisation becomes a chain reaction releasing many carriers and allowing sustained current flow.

Under high reverse voltage the depletion layer grows far from the junction and will eventually reach the metal contact at the edge of the die. At this point punch-through occurs and continuous conduction is possible.

One side of the junction is lightly doped so that the depletion layer charge density and field strength are low so as to avoid avalanche. Because this leads to a deep depletion layer, the device must be deep to avoid punch through.

(b) Describe the characteristics of the thyristor and the GTO thyristor. Under what circumstances might a thyristor be preferred to a GTO thyristor? [3]

A thyristor acts like a diode in which conduction can be held off. Under forward bias the device will not conduct until a pulse of gate current is applied. Once conduction is initiated the device latches on and only turns off, rather like a diode, when the external circuit causes the current flow to cease and reverse bias is applied. The GTO is a gate turn-off version of the thyristor in which conduction can be stopped on demand by applying negative gate current at some substantial fraction of anode-cathode current

The thyristor is more restricted in use because it can't be turned off at will but has the advantage of being available to higher ratings than any other semiconductor. It is used in very high power rectifiers where the AC voltages periodically turn the device off.

(c) Compare the properties of an IGBT and a MOSFET [4]

Both devices are controlled by a gate voltage across a MOS capacitance. The mosfet is a majority carrier device with rapid turn-on and turn-off. The IGBT has both majority and minority carrier conduction paths. The turn-off has a relatively slow tail because the stored minority charge can not be removed through the external contacts. The on-state voltage drop of the mosfet is resistive while that of the IGBT is a fixed drop of 2-3 V plus a smaller resistive element. In low voltage designs the mosfet has the lower drop but in high voltage designs the IGBT is better. IGBTs are available to much higher voltage ratings than mosfets

(d) Figure Q2 shows snubber circuit applied to a transistor switching an inductive load. The load has been approximated by a constant current. Other details of the circuit are: supply voltage, V_S = 500 V; snubber capacitor C_S = 22 nF; transistor current fall time, t_{fi} = 500 ns and transistor voltage rise time, t_{rv} = 100ns.

[3.14]

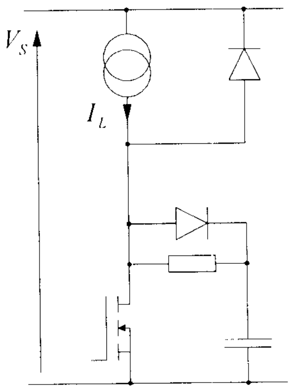


Figure 2

(i) Explain why the transistor has lower switching loss with the snubber than without. [2]

At turn-off, the portion of load current not flowing in the transistor can divert into the capacitor rather than into the diode. Forcing the diode into conduction raises the voltage across the transistor to the full supply voltage during the turn-off. In contrast, the capacitor holds the voltage relatively low during the early stages of turn-off as it slowly charges. Thus the voltage across the transistor is low while the current is high and the instantaneous power dissipation is low.

(ii) Find the load current at which the snubber capacitor fully charges in exactly the time taken for the transistor current to fall. [3]

Assume the transistor current fall is linear and therefore the rise of capacitor current is also linear. The charge delivered to the capacitor during turn off is: $q = \frac{1}{2} I t_{fi}$. *The current to achieve the necessary voltage rise is therefore:* $I = \frac{2CV_S}{t_{fi}} = \frac{2 \times 22 \times 10^{-9} \times 500}{500 \times 10^{-9}} = 44\,A$

(iii) Calculate the energy dissipated in the transistor at turn-off with the snubber in place and compare this to the energy that would be dissipated without the snubber. [5]

With the snubber there is dissipation in the transistor only during the current fall (and not during the voltage rise)

$$
\begin{aligned}
E_{TS} &= \int_0^{t_{fi}} I\left[1 - \frac{t}{t_{fi}}\right] V_S \left[\frac{t}{t_{fi}}\right]^2 dt \\
&= IV_S \left[\frac{t^3}{3t_{fi}^2} - \frac{t^4}{4t_{fi}^3}\right]_0^{t_{fi}} \\
&= \tfrac{1}{12} IV_S t_{fi} \\
&= \tfrac{1}{12} \times 44 \times 500 \times 500 \times 10^{-9} \\
&= 0.92\,mJ
\end{aligned}
$$

Without the snubber there is dissipation in both periods

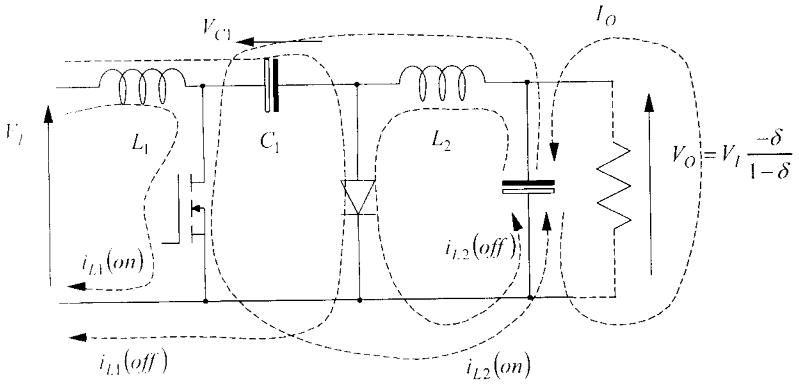
$$
\begin{aligned}
E_T &= \tfrac{1}{2} I V_S \left[t_{rv} + t_{fi}\right] \\
&= \tfrac{1}{2} \times 44 \times 500 \times \left[600 \times 10^{-9}\right] \\
&= 6.6\,mJ
\end{aligned}
$$

[3.14]

3.

(a) Explain the operation of the Ćuk SMPS [5]

Currents during the on-state and off-state are given in the figure



With the mosfet on, current (and energy) are built up in the input inductor. Current in the output inductor also increase by drawing energy from the coupling capacitor. The output capacitor (and load) also receive energy via the output inductor.

With the mosfet off, the current in the input inductor flows through the coupling capacitor replenishing the energy stored there. The output inductor transfers energy to the output.

The direction of current flow in the output inductor is such that the output capacitor is charged to a negative voltage.

The coupling capacitor is charge and discharge during the cycle and holds a steady voltage that is the difference between input and output voltages.

(b) Describe why a Ćuk SMPS might be a better design than the Buck-Boost SMPS [3]

In the Cuk SMPS, both the input and output currents flow continuously and energy is transferred continuously in both connections giving a relatively smooth output voltage and low interference input.

In the Buck-Boost both input and output currents are pulse waveforms. The output capacitor has a large ripple current and consequently a large ripple voltage. The input current is rich in high frequency terms and a source of interference

(c) A Ćuk SMPS has been built using two 200 μH inductors and two 470 μF capacitors with effective series resistance of 50 mΩ. It is used to provide a -5.0 V output from an input voltage that varies between 12.0 and 17.0 V.

(i) Choose a switching frequency that will keep the output voltage ripple below 10 mV. You may assume that the ripple is dominated by the voltage across the effective series resistance. [7]

[3.14]

$$\Delta V = R_{ESR}\,\Delta i_C$$

$$\Delta i_C = \Delta i_2$$

$$\Delta i_2 = \frac{1-\delta}{f}\frac{V_O}{L_2}$$

$$\frac{V_O}{V_I} = \frac{-\delta}{1-\delta} \qquad \delta = \frac{-V_O}{V_I - V_O} \qquad 1-\delta = \frac{V_I}{V_I - V_O}$$

$$\Delta i_2 = \frac{1}{fL_2}\frac{V_I V_O}{V_I - V_O}$$

With a fixed VO and range of VI, the current (and voltage) ripple will be highest with the highest input voltage. The current ripple calculated here is negative.

$$f = \frac{V_I V_O}{V_I - V_O}\frac{R_{ESR}}{L\Delta V} = \frac{17.0\times -5.0}{17.0+5.0}\frac{0.05}{200\times 10^{-6}\times^{-} 0.01} = 96.6\,kHz$$

(ii) Calculate the average voltage and voltage ripple across the central coupling capacitor when the output current is -10 A. You may neglect the effective series resistance in this calculation [5]

The capacitor current is +i1 for δT and –i2 for (1-δ)T.

Charge delivered in on state is

$$q = \frac{\delta}{f} I_2$$

I2 is known and worst case q will occur for largest δ and highest input voltage

$$\Delta V_C = \frac{1-\delta}{fC} I_2 = \frac{\frac{5.0}{12.0+5.0}}{96.6\times 10^{3}\times 470\times 10^{-6}}\times 10 = 64.8\,mV$$

[3.14]

4.

(a) A DC machine has separate field and armature connections and has the following properties:

Maximum field current I_F = 1.0A
Armature resistance R_A = 0.3 Ω
Maximum armature current, I_A^{Max} = 5 A
The armature has a back EMF of 32 V when the field current is at its maximum and shaft speed is 1,000 rpm.

The machine is supplied via the circuit of figure Q4.1 with an input voltage of V_{DC} = 48 V.

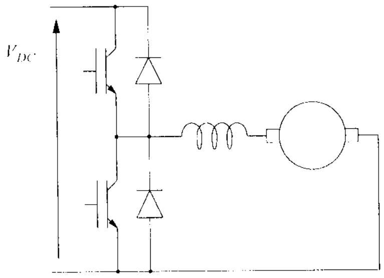


Figure Q4.1

Figure Q4.2 shows 9 operating points on the torque-speed plane.

Operating Point	Speed (rpm)	Torque (Nm)
1	+1200	1.7
2	+1200	1.5
3	+1200	1.3
4	+1800	1.7
5	+1800	1.5
6	+1800	1.3
7	-1200	1.3
8	-1200	-1.3
9	+1200	-1.3

[3.14]

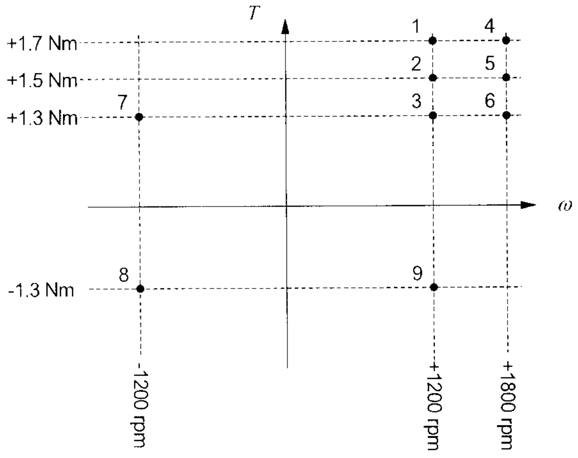


Figure Q4.2

(i) Determine which of the operating points can be achieved with the machine and chopper without the use of field weakening. [8]

Operating points 7 and 8 require negative armature voltage which is not possible with the circuit given. (2 marks)

First find armature constant

$$k'_A\phi = \frac{E}{n} = \frac{32}{1{,}000} = 0.032$$

$$k_A\phi = \frac{E}{\omega} = \frac{32}{2\pi \times 1{,}000 / 60} = 0.3056$$

Maximum torque set by maximum armature current

$$T^{Max} = k_A\phi I_A^{Max} = 0.3056 \times 5 = 1.53\,Nm$$

This result shows that operating points 1 and 4 are not possible (3 marks)

When the chopper duty cycle is 1.0 the armature will be 48.0 V. Subtracting the maximum resistive voltage drop will yield the maximum back EMF possible under full torque.

$$E = \delta V_{DC} - IR_A = 48.0 - 5.0 \times 0.3 = 46.5V$$

This corresponds to a speed of:

$$n = \frac{E}{k'_A\phi} = \frac{46.5}{0.032} = 1453\,rpm$$

Thus operating points 2 and 3 are achievable without field weakening but 5 and 6 are not.(2 marks)

Operating point 9 is also achievable because the given circuit can supply negative current and positive voltage. (1 mark)

(ii) Determine whether any of the remaining points can be achieved by using field weakening. [4]

Operating points 5 and 6 are candidates for field weakening. The reduction in field need to obtain 1,800 prm is:

$$\delta V = k'_A \phi' n + I_A^{Max} R_A$$

$$k'_A \phi' = \frac{\delta V_{DC} - I_A^{Max} R_A}{n} = \frac{46.5}{1{,}800} = 0.02583$$

$$I'_F = \frac{k'_A \phi'}{k'_A \phi} I_F = \frac{0.02583}{0.032} \times 1.0 = 0.807\,A$$

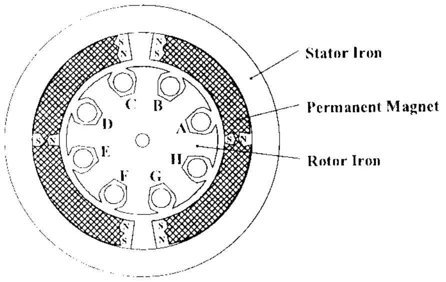
The approx 20% reduction in field will cause a proportional fall in maximum torque

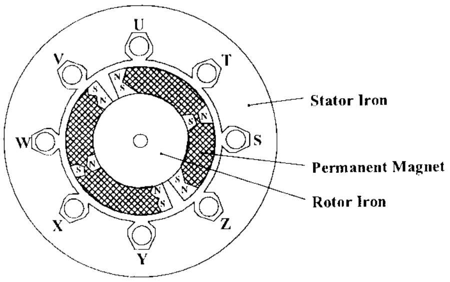
$$T'^{Max} = k'_A \phi I_A^{Max} = 0.807 \times 0.3056 \times 5.0 = 1.23\,Nm$$

So, although 1,800 rpm can be reached the maximum torque is less than required for either operating point 5 or 6.

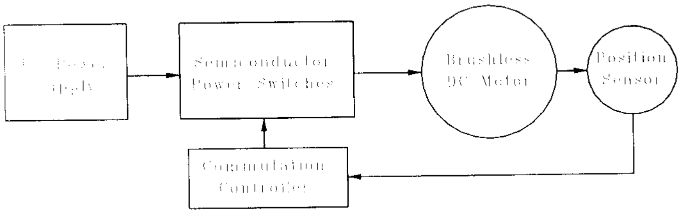
(b) With the aid of diagrams, describe the construction and operation of a brushless DC machine drive. [8]

A brushless DC machine is arranged with a permanent magnet field on the rotor so that no electrical contacts are required to be made to the moving part of the machine. The armature winding is therefore on the stator. Commutation is still required so that the current carried by each armature conductor matches the polarity of the rotor field as that field rotates. The rotor position must be sensed and this information used to set a current reference for each phase of the stator. The magnitude of the current reference is set in proportion to the torque required. Local feedback of current is used to ensure the current reference is followed. The machine layout of a brushed and a brushless machine are compared below.



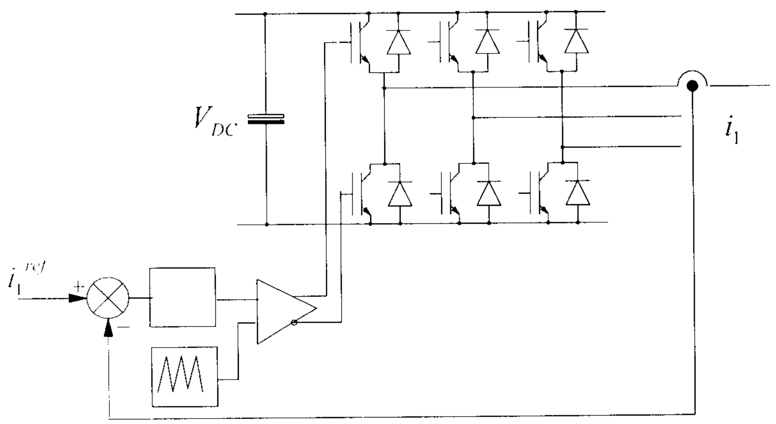


The block diagram of the system is:



The current control system for a burshless machine is as follows:

[3.14]

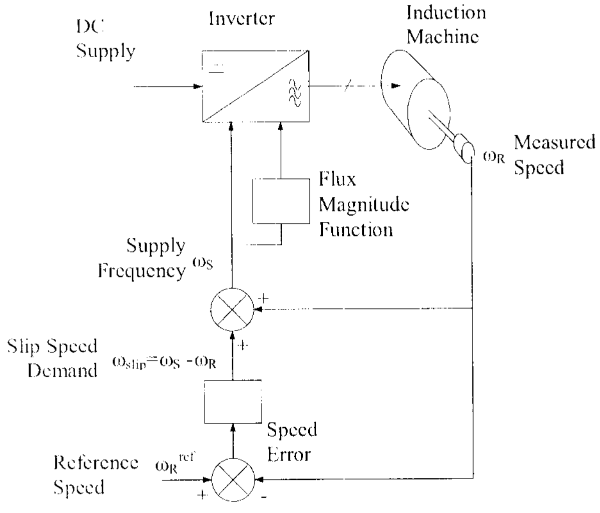


[3.14]

5.

(a) Draw a block diagram of the control system for an induction machine that uses slip compensation and describe the operation of the system. [6]

Slip Compensated
Speed Control



Slip compensated speed control relies on the fact that the torque produced by an induction machine is approximately linearly proportional to the slip speed provide the slip speed is not greater than about 5% of base speed. The slip speed is defined as the difference between the rotational speed of the flux wave (synchronous speed) and the rotational speed of the rotor. The speed error is processed by a standard linear controller (PI etc.) and the output interpreted as a slip speed demand. This is imposed by adding the measured rotational speed to the slip speed to form the synchronous speed to be synthesised by the inverter. The voltage magnitude to be synthesised by the inverter is set so as to maintain the flux-linkage magnitude constant. Most commonly this is approximated as varying the applied voltage in proportion to the frequency and at low frequency, where the stator resistance voltage drop dominates, adding a voltage boost.

(b) Figure 5 shows an approximate equivalent circuit of an induction machine. The machine has the following properties:

Number of pole-pairs, $P = 1$
Stator resistance, $R_S = 1.8\ \Omega$
Stator Leakage Inductance, $L_S = 3$ mH
Magnetising Inductance, $L_M = 100$ mH
Referred rotor resistance, $R'_R = 1.2\ \Omega$
Maximum referred rotor current, $I'^{Max}_R = 12$ A RMS
Maximum magnetising current, $I^{Max}_M = 6$ A RMS

[3.14]

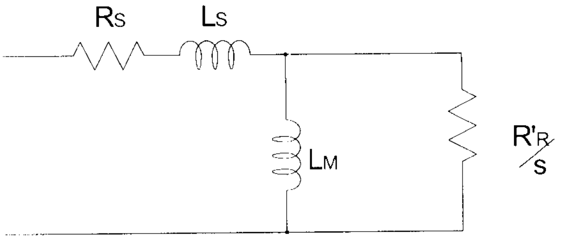


Figure 5

(i) Show that if the magnetising current is maintained at its maximum value then rotor speed that gives the maximum rotor current is a fixed interval below the synchronous speed and is given by:

$$\omega_R = \omega_S - \frac{I'_R R'_R}{P L_M I_M^{Max}}$$ [4]

The voltage across the magnetising inductance at full magnetising current is:

$$V_{AG} = j\omega L_M I_M^{Max}$$

The rotor current driven by this same voltage is:

$$I'_R = s\frac{V_{AG}}{R'_R} = \frac{\omega_S - \omega_R}{\omega_S} \cdot \frac{j\omega L_M I_M^{Max}}{R'_R}$$

Re-arranging and using ωS=ω/P

$$\omega_R = \omega_S - \frac{I'_R R'_R}{P L_M I_M^{Max}}$$

(ii) Find the torque that is developed under the condition in (i) [3]

$$T = \frac{3I'^2_R R'_R}{\omega_R}\left(\frac{1}{s} - 1\right) = \frac{3I'^2_R R'_R}{\omega_S - \omega_R} = 3I'^2_R R'_R \frac{P L_M I_M^{Max}}{I'_R R'_R} = 3PL_M I_M^{Max} I'_R$$

$$= 3 \times 1 \times 0.1 \times 6 \times 12 = 21.6\,Nm$$

or use $T = 3\frac{P^2\psi_M^2}{R'_R}s\omega_S = 3P\psi_M I'_R \sin(\delta)$

(iii) Find the required stator voltage for operation under the condition in (i) at 4,000 rpm and the minimum DC supply voltage to the inverter need to achieve this. [4]

4,000 rpm requires a supply frequency of:

$$\omega_S = \omega_R + (\omega_S - \omega_R) = \frac{2\pi\, n_R}{60} + \frac{I'_R R'_R}{P L_M I_M^{Max}} = 418.9 + 24 = 442.9\,rad/s$$

Stator current is:

$$I_S = I_M + I'_R = 6 + j12\,A$$

Stator voltage, taking mag current as reference phasor, is:

[3.14]

$$\begin{aligned} V_S &= I_S\left(R_S + j\omega L_S\right) + j\omega L_M I_M \\ &= I'_R R_S + I_M R_S + j\omega L_S I'_R + j\omega\left(L_S + L_M\right) I_M \\ &= j12 \times 1.8 + 6 \times 1.8 - 418.9 \times 0.003 \times 12 + j442.9 \times \left(0.1 + 0.003\right) \times 6 \\ &= \left(10.8 - 15.1\right) + j\left(21.6 + 265.7\right) \\ &= 4.2 + j287.3 \\ &= 287.3\, V_{RMS} \end{aligned}$$

The peak phase voltage is 406V

With simple sinusoidal modulation, the DC link voltage must exceed the peak-to-peak phase voltage. A DC link voltage of 812V or greater is required. (With third harmonic addition, this could be reduced by $\sqrt{3}/2$

(iv) With the voltage magnitude limited at the value found in (iii), estimate the maximum torque that can be achieved at a speed of 6,000 rpm. [3]

Approximate by neglecting the voltage drop across the stator components and using $\omega_S \approx \omega_R$.

Magnetising current is then:

$$I_M = \frac{V}{\omega L_M} = \frac{287.3}{2\pi \frac{6000}{60} \times 0.1} = 4.75\, A$$

$$T = 3PL_M I_M I'_R = 3 \times 0.1 \times 4.64 \times 12 = 17.1\, Nm$$

[3.14]

6.

(a) Figure 6 shows the waveform of AC side current of a rectifier and its relationship to the AC voltage.

(i) Calculate the RMS value of the current [4]

(ii) Calculate the fundamental component of the current [4]

(iii) Calculate the power factor of the current [2]

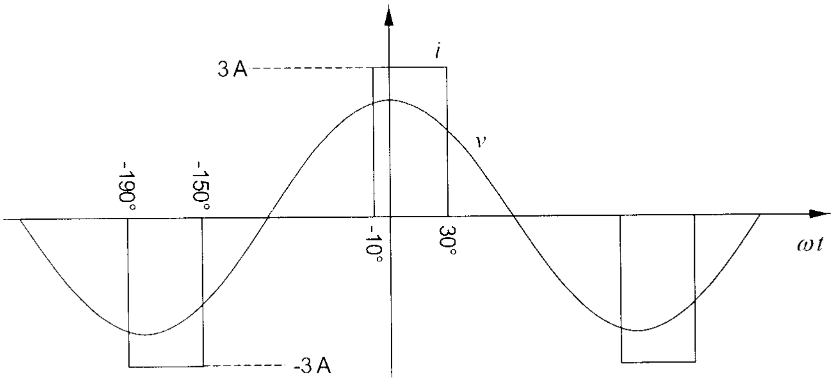


Figure 6

RMS Current

$$I_{RMS} = \sqrt{\frac{1}{2\pi}\int_{\pi}^{\pi} i^2(\omega t).d\omega t} = \sqrt{\frac{I^2}{\pi}[\omega t]_{-10^\circ}^{30^\circ}}$$

$$= 3\sqrt{\frac{30+10}{180}} = 1.41$$

Fundamental Current

$$I_{A1} = \frac{1}{\pi}\int_{\pi}^{\pi} i(\omega t)\cos(\omega t).d\omega t = \frac{2}{\pi}\int_{-10^\circ}^{30^\circ} I\cos(\omega t).d\omega t$$

$$= \frac{2}{\pi} I[\sin(\omega t)]_{-10}^{30} = \frac{2\times 3}{\pi}[\sin 30^\circ - \sin -10^\circ] = 1.286\,A$$

$$I_{B1} = \frac{1}{\pi}\int_{\pi}^{\pi} i(\omega t)\sin(\omega t).d\omega t = \frac{2}{\pi}\int_{-10^\circ}^{30^\circ} I\sin(\omega t).d\omega t$$

$$= \frac{2}{\pi} I[-\cos(\omega t)]_{-10}^{30} = \frac{2\times 3}{\pi}[-\cos 30^\circ + \cos -10^\circ] = 0.227\,A$$

$$I_1 = 1.307\angle -10^\circ$$

$$I_{1RMS} = \frac{1.307}{\sqrt{2}} = 0.924 \quad A$$

Displacement factor is $\cos(10^\circ) = 0.985$

Distortion factor is $\frac{I_1}{I_{RMS}} = \frac{0.924}{1.41} = 0.654$

Power factor is 0.643

[3.14]

(b) Describe why circuits that draw non-sinusoidal currents from the AC main should not be used. [3]

The harmonic components of current do not transfer real power but do contribute to losses in the supply lines and transformers of the distribution system leading to inefficient operation. Utility operators may penalise low power factor (and therefore low distortion factor).

The harmonic current flows cause harmonic voltage drops in the distribution system and cause other consumers to be supplied with a distorted voltage waveform. This may cause mal-operation of equipment. EMC regulations, such as EN 61000, prohibit equipment from drawing harmonic current.

(c) Describe a circuit and a control scheme that converts AC to DC and maintains a sinusoidal AC current [7]

Example of single-phase system is given. Three-phase example could be used.

A boost type SMPS can be arranged to draw a controlled current from a varying voltage. A diode bridge is used to rectify the AC voltage and a control loop set to force the current to be a full wave rectified sinewave also (a simple sinewave will be drawn from the AC supply). The magnitude of the sinewave is set by a second controller according to the error between the DC output voltage and its reference value.

Because the circuit is under closed loop control the output voltage can reject output current disturbance and changes in input voltage magnitude. The current control can easily be amended to provide current limiting to prevent large fault currents.

The circuit suffers a twice line-frequency output voltage ripple because of the inherent pulsation of a single phase power source. Although the circuit can be controlled to avoid low frequency current harmonics there are emission of high frequency current multiples of the switching frequency.