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Paper Number(s): **E3.14**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2002

EEE PART III/IV: M.Eng., B.Eng. and ACGI

POWER ELECTRONICS AND MACHINES

Friday, 26 April 10:00 am

There are SIX questions on this paper.

Answer FOUR questions.

Time allowed: 3:00 hours

Examiners responsible:

First Marker(s): Green,T.C.

Second Marker(s): Pal,B.C.

1. (a) Explain why switch-mode power supplies are normally used with closed-loop control rather than in open-loop.

[3]

(b) Figure 1.1 shows a variation on the Ćuk switch-mode power supply that is known as the Zeta switch-mode power supply.

(i) Assume that the capacitor voltages and inductor currents are positive in the senses shown and sketch the current paths during the on and off states of the transistor.

[4]

(ii) By considering the rates-of-change of the currents in the inductors and by imposing the condition of steady-state operation, derive an equation for the output voltage as a function of input voltage and duty cycle. You may assume that both inductors are in continuous conduction.

[8]

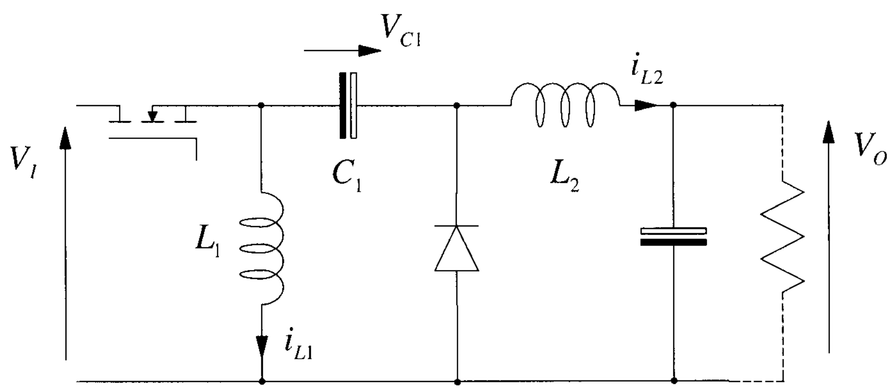


Figure 1.1 The Zeta switch-mode power supply

(c) Compare and contrast the Zeta switch-mode power supply with the Ćuk and Flyback.

[5]

2. (a) Outline the differences in structure between a MOSFET and an IGBT. Briefly describe the advantages and disadvantages of the IGBT structure.

[5]

(b) Describe the property of a *pn* junction diode known as reverse recovery and draw current and voltage waveforms to illustrate your answer. Describe what is meant by snap recovery and soft recovery.

[5]

(c) Figure 2.1 shows an IGBT used to switch an inductive load at a supply voltage of $V_S = 500$ V. A *pn* diode provides a current path for the load when the IGBT is switched off. The current in the inductor, I_L, can be considered constant at 100 A. At turn-on the IGBT achieves a rate of rise of current of 500 A/μs and its voltage fall time is 100 ns. The recovery current of the diode ceases abruptly when the junction recovers. For a forward current of 100 A, the diode recovery charge is 2.5 μC.

(i) Sketch the collector-emitter voltage, v_{CE}, and collector current, i_C, of the IGBT and the voltage, v_D, and current, i_D, of the diode during turn-on of the IGBT.

(ii) Calculate the diode recovery time.

(iii) Calculate the energy lost in the IGBT at turn-on.

[7]

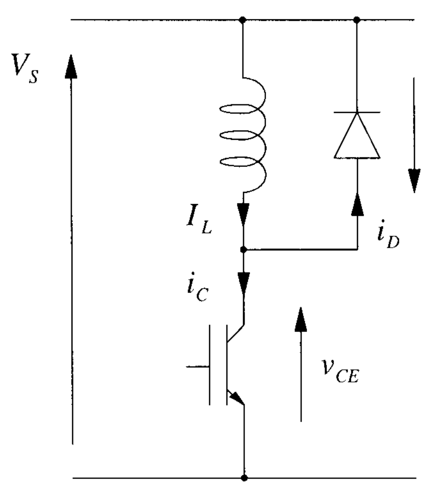


Figure 2.1 IGBT and inductive load

(d) Sketch a turn-on snubber suitable for use with the circuit in Figure 2.1. Explain how this snubber reduces the energy loss in the IGBT.

[3]

3. (a) Define the terms power factor, displacement factor and distortion factor.

[5]

(b) Figure 3.1 shows a three-phase rectifier and a simplified representation of a current and the corresponding phase voltage.

(i) Which of the following components are present in the frequency spectrum of the current: DC, sine-wave fundamental, cosine-wave fundamental, sine-wave harmonic and cosine-wave harmonic?

[3]

(ii) Calculate the displacement factor, distortion factor and power factor of the rectifier.

[6]

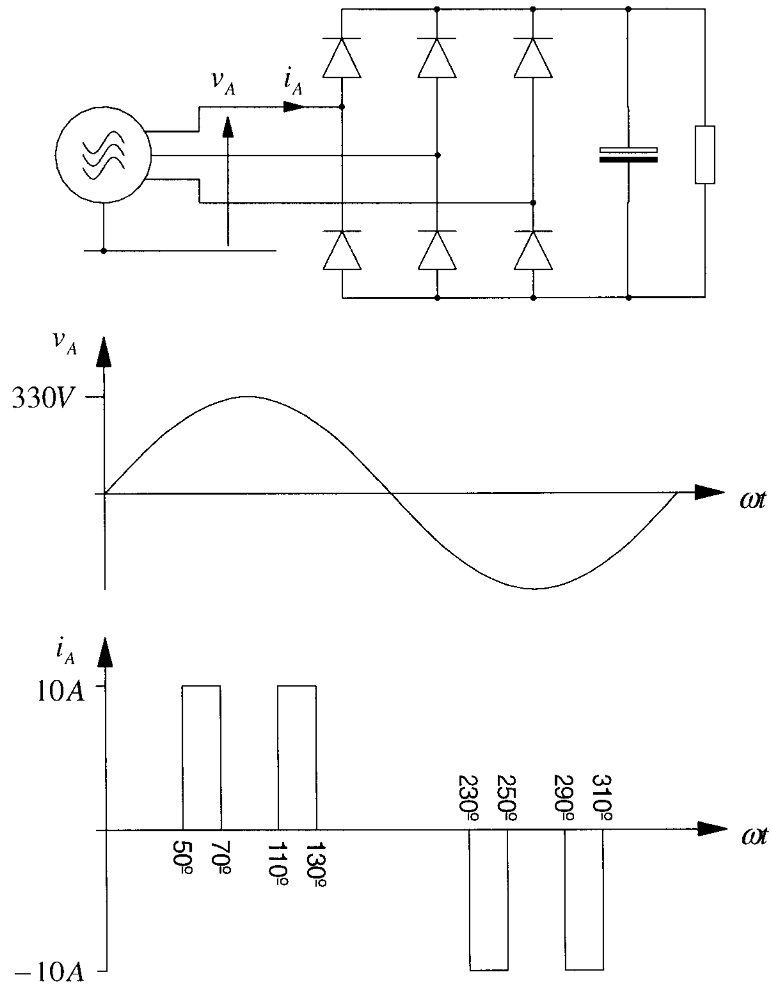


Figure 3.1 A three-phase diode rectifier

(c) Draw a circuit diagram of an alternative to the rectifier in Figure 3.1 that can achieve a low distortion factor. Describe how the circuit should be operated and discuss the displacement factor of the circuit.

[6]

4. (a) Explain why bi-directional use of a transformer core is preferred to uni-directional use.

[3]

(b) Figure 4.1 shows an isolated Flyback switch-mode power supply employing two primary-side switches. Describe the advantages this circuit has over a circuit with one primary-side switch.

[3]

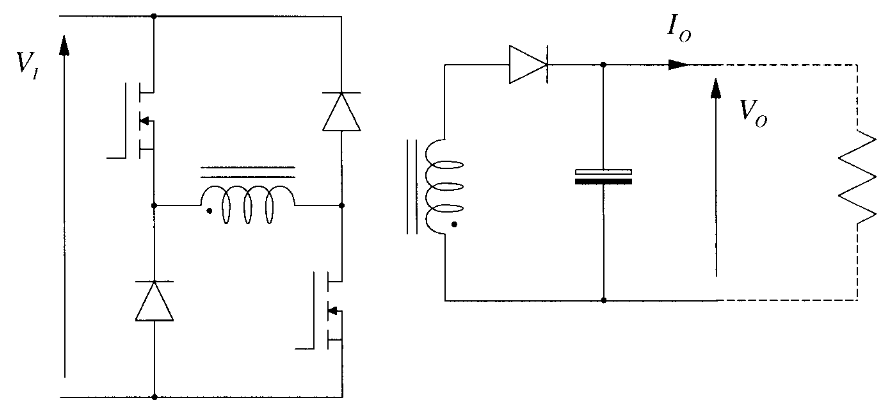


Figure 4.1 An isolated Flyback switch-mode power supply

(c) The circuit of Figure 4.1 has been implemented with a transformer having a 20:1 turns ratio (primary : secondary) and a primary inductance of 100 μH. It is to be used with an input voltage of $V_I = 340$ V and an output voltage of $V_O = 15$ V.

(i) Find expressions for the peak primary current as a function of on-time and diode conduction time (assuming discontinuous conduction) as a function of on-time.

[6]

(ii) Find the minimum frequency at which the circuit can be operated while remaining in discontinuous mode and taking an input power of 100 W.

[6]

(iii) If the operating frequency is doubled, would the circuit be able to process more or less power while remaining in discontinuous mode? Give a physical reason for your answer.

[2]

5. (a) Figure 5.1 shows a single-phase AC/DC converter based on a Boost switch-mode power supply. Sketch the shape of the current waveform drawn from the AC source (assuming that a suitable control circuit has been provided) and describe its frequency spectrum. Sketch the shape of the instantaneous input power.

[4]

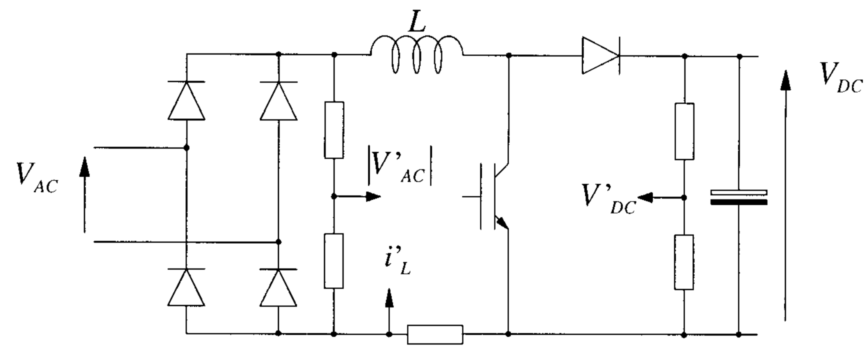


Figure 5.1 A single-phase AC/DC converter

(b) (i) Give expressions for the change in current in the inductor in Figure 5.1 during the on-time and during the off-time of the switch.

[2]

(ii) Assuming that the rise and fall of current are approximately matched over one switching cycle, find an expression for the magnitude of the ripple current.

[4]

(iii) Find the point in the mains cycle at which the maximum ripple current occurs and find the value of the maximum ripple current when the circuit is operated with V_{DC} = 400 V, V_{AC} = 230 V_{rms}, L = 10 mH and a switching frequency of 75 kHz.

[6]

(c) Consider the circuit of Figure 5.1 operated with an output voltage of 400 V and supplying a 200 W load that draws a constant current. Assuming the circuit is ideally (100%) efficient, compare the instantaneous power supplied to the capacitor to the power drawn by the load. For a capacitor of 500 μF, estimate the voltage variation that results from the instantaneous power variation.

[4]

6. (a) Draw a torque against speed curve for a typical induction machine operated at its rated voltage, V_S^R and frequency, f^R. Add to the graph curves for operation at $V_S^R/2$ & $f^R/2$ and for $V_S^R/5$ & $f^R/5$. Justify the shapes of the curves. Describe why a *voltage boost* is normally applied at low frequencies to an induction motor drive operated with voltage varied in proportion to frequency.

[8]

(b) Draw a circuit diagram of a circuit suitable for supplying a three-phase induction machine with a variable frequency three-phase set of voltages and describe its operation.

[4]

(c) Draw a block diagram of a controlled-slip induction machine drive and explain its operation. Describe why it is normal to include a slip limiter.

[8]

master copy - Aug. 02

1/17

Answers

E3.14 Power Electronics 2001/2002

1) (a) Explain why switch-mode power supplies are normally used with closed-loop control rather than in open-loop.

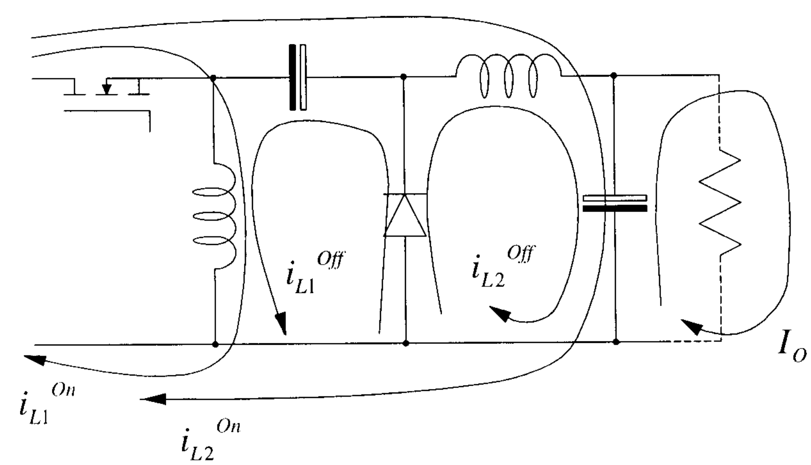
[3]

Only in continuous conduction mode will there be a simple relationship between duty-cycle and output voltage. If the circuit enters discontinuous mode the relationship will change and more importantly will become dependent on either the input current or the output current. It is not possible to operate in open loop if the SMPS is likely to enter discontinuous mode. Even in continuous mode there will be some dependence of output voltage on output current because of voltage drops across switch or inductor resistances and a dependence of output voltage on input voltage. Thus, for good regulation of the output voltage, closed loop control is essential.

(b) Figure 1 shows a variation on the Ćuk switch-mode power supply known as the Zeta switch-mode power supply.

(i) Assume that the capacitor voltages and inductor currents are positive in the senses shown and sketch the current paths during the on and off states of the transistor.

[4]



(ii) By considering the rates-of-change of currents in the inductors and imposing the condition of steady-state operation, derive an equation for the output voltage as a function of input voltage and duty cycle. You may assume that both inductors are in continuous conduction.

[8]

Considering first the current ripple in L_1

2/17

$$\Delta i_{L1}^{\ On} + \Delta i_{L1}^{\ Off} = 0$$

$$\frac{V_I}{L_1} \cdot \delta T + \frac{-V_{C1}}{L_1} \cdot (1-\delta)T = 0$$

$$V_{C1} = \frac{\delta}{1-\delta} V_I$$

And now L_2

$$\Delta i_{L2}^{\ On} + \Delta i_{L2}^{\ Off} = 0$$

$$\frac{V_I + V_{C1} - V_O}{L_2} \cdot \delta T + \frac{-V_O}{L_2} \cdot (1-\delta)T = 0$$

$$V_O = V_I\delta + V_{C1}\delta$$

Substituting for V_{C1} yields:

$$V_O = V_I\delta + \frac{\delta}{1-\delta} V_I \delta$$

$$V_O = \frac{\delta}{1-\delta} V_I$$

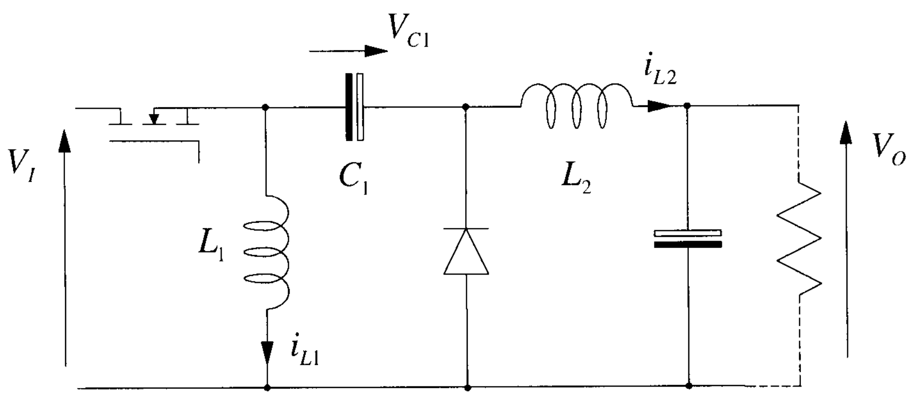


Figure 1, The Zeta switch-mode power supply

(b) Compare and contrast the Zeta switch-mode power supply with the Ćuk and Flyback.

[5]

A description of the advantages and disadvantages is expected that covers most of the points in the table below.

	Flyback	*Ćuk*	*Zeta*
Voltage ratio	$V_O = \frac{-\delta}{1-\delta} V_I$	$V_O = \frac{-\delta}{1-\delta} V_I$	$V_O = \frac{\delta}{1-\delta} V_I$
Polarity of output	*negative*	*negative*	*positive*
Step-Up/Down	*both*	*both*	*both*
Input current	*chopped*	*smooth*	*chopped*
Input interference	*high*	*low*	*high*

Output current	*chopped*	*smooth*	*smooth*
Output voltage ripple	*high*	*low*	*low*
Passives	*C & L*	*2C & 2L*	*2C & 2L*

4/17

2) (a) Outline the differences in structure between a MOSFET and an IGBT. Briefly describe the advantages and disadvantages of the IGBT structure.

[5]

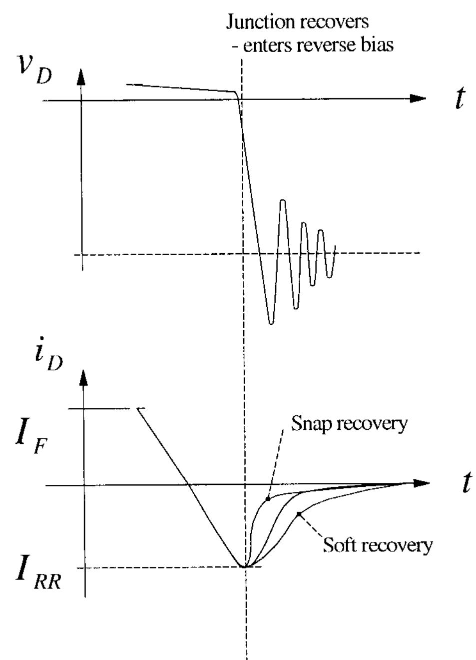
A Mosfet is a three-layer (npn) device and for power applications is a double diffusion vertical structure. The drain-body junction is reverse biased in the off-state with a large depletion layer established in the n-type drain. An n+ contact region is provided to interface with the drain metalization. In the IGBT the n+ contact is modified to p+. This creates a four layer (pnpn) structure with the extra junction forward biased under normal conditions. Off-state conditions are not affected. During the on-state the forward bias junction floods the drain region with excess carriers (high level injection) and reduces the resistance of this region. This enables this region to be lightly doped and deep to support a large off-state voltage without a large penalty in the on-state voltage drop. IGBTs of substantial current rating are feasible up to much higher voltage ratings than Mosfets (4kV v. 500V)

An IGBT is monolithic Mosfet/Bipolar Darlington in which normal base current for the bipolar is provided by the Mosfet. However, at turn-off, there is no path for reverse base current and the bipolar section turns off slowly. Life time control can help but compromises on-state voltage.

(b) Describe the property of a *pn* junction diode known as reverse recovery and draw current and voltage waveforms to illustrate your answer. Describe what is meant by snap recovery and soft recovery.

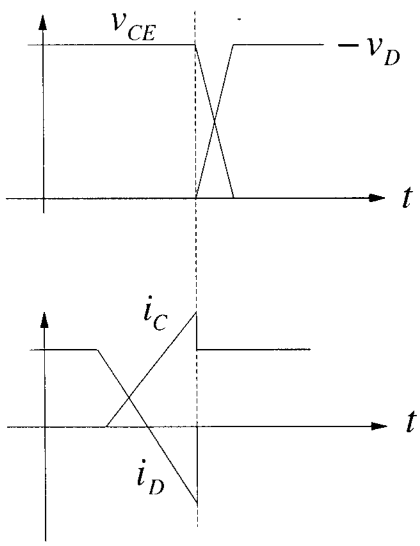
[5]

During forward conduction, minority carriers build up either side of the junction and diffuse away to the metal contacts. When a reverse external voltage is applied the minority carriers are drawn back across the junction and the minority carriers away from the junction diffuse to ward it, i.e., move in the opposite direction. Thus reverse current flow occurs until the stored minority carriers are removed. The junction itself remains in forward bias until the minority carrier concentration at the depletion layer edge is reduced below its zero bias level. A small amount of stored charge remains to be removed from the diode after the junction recovers. The abruptness with which this happens depends on the construction of the diode and is described as either snap or soft recovery.



(c) Figure 2 shows an IGBT used to switch an inductive load at a supply voltage of V_S = 500 V. A *pn* diode provides a current path for the load when the IGBT is switched off. The current in the inductor, I_L can be considered constant at 100 A. At turn-on the IGBT achieves a rate of rise of current of 500 A/μs and its voltage fall time is 100 ns. The recovery current of the diode ceases abruptly when the junction recovers. For a forward current of 100 A, the diode recovery charge is 2.5 μC.

(i) Sketch the collector-emitter voltage, v_{CE} and collector current, i_C of the IGBT and the voltage, v_D and current, i_D of the diode during turn-on of the IGBT.



(ii) Calculate the diode recovery time.

$$Q_{RR} = \tfrac{1}{2} I_{RR} t_{RR} = \tfrac{1}{2} \frac{di_C}{dt} t_{RR}^{\;2}$$

$$t_{RR} = \sqrt{\frac{2 \times 2.5 \times 10^{-6}}{500 \times 10^{6}}} = 100ns$$

(iii) Calculate the energy lost in the IGBT at turn-on

[7]

$$E_{on} = \tfrac{1}{2} V_S \left[(I_L + I_{RR}) \cdot (t_{ri} + t_{RR}) + I_L t_{fv} \right]$$

$$t_{ri} = I_L \Big/ \frac{di_C}{dt} = \frac{100}{500 \times 10^{6}} = 200ns$$

$$I_{RR} = t_{RR} \frac{di_C}{dt} = 100 \times 10^{-9} \times 500 \times 10^{6} = 50A$$

$$E_{on} = \tfrac{1}{2} \times 500 \times \left[150 \times 300 \times 10^{-9} + 100 \times 100 \times 10^{-9} \right] = 13.75mJ$$

6/17

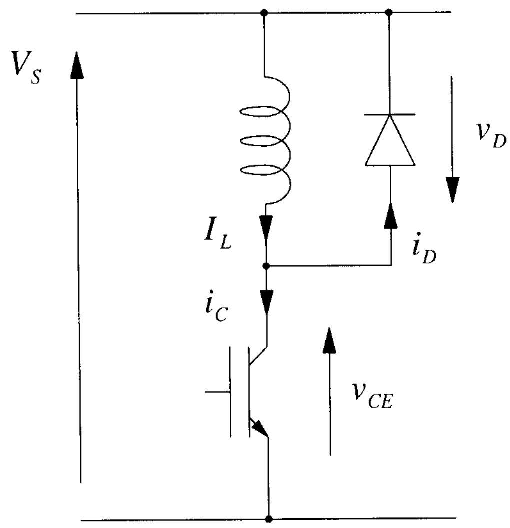
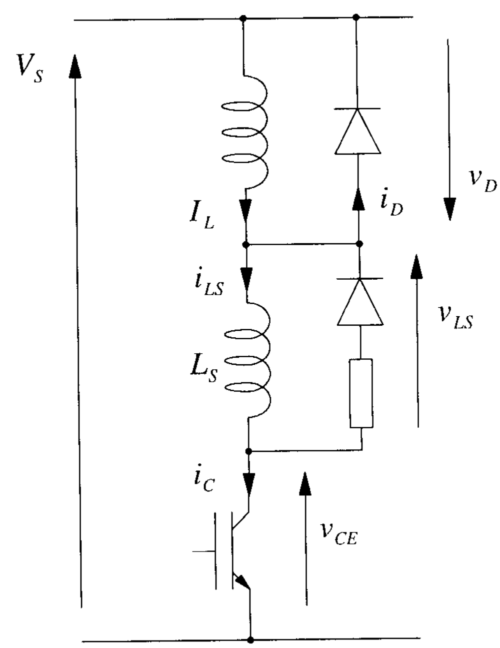


Figure 2 Igbt and inductive load

(d) Sketch a turn-on snubber suitable for use with the circuit in Figure 2. Explain how this snubber reduces the energy loss in the IGBT.

[3]

The addition of a snubber inductor, L_S in series with the switch controls the rate-of-rise of collector current at turn-on. It also allows a voltage to develop, V_{LS} such that the transistor voltage v_{CE} can fall while the diode remains in forward bias (as it must while i_C is less than I_L). Thus, the voltage across the transistor falls while the current through it is still small and rising slowly. The co-incidence of high current and high voltage is avoided and the losses in the transistor are reduced. The extra diode and resistor are provided to discharge the energy stored in the snubber. This occurs at turn-off of the switch..



7/17

3) (a) Define the terms power factor, displacement factor and distortion factor.

[5]

Power factor is the ratio of real power to apparent power. In turn, real power is defined as the average (over a cycle) of the instantaneous power and apparent power is defined as the product of the RMS voltage and RMS current.

Displacement factor is defined as the cosine of the angle between the fundamental component of voltage and the fundamental component of current.

Distortion factor (for sinusoidal voltage conditions) is defined as the ration of the RMS value of the fundamental component of current to the RMS of the total current.

Power factor is the product of the distortion factor and the displacement factor.

(b) Figure 3 shows a three-phase rectifier and a simplified representation of a current and the corresponding phase voltage.

(i) Which of the following components are present in the frequency spectrum of the current: DC, sine-wave fundamental, cosine-wave fundamental, sine-wave harmonic and cosine-wave harmonic?

[3]

The DC component is zero (because the waveform is half-wave symmetric)
The waveform has odd symmetry and so all cosine components are zero.
Both fundamental and harmonic sine components are present.

(ii) Calculate the displacement factor, distortion factor and power factor of the rectifier.

[6]

The displacement factor is unity because the fundamental of the current has no cosine component and is therefore in phase with the voltage.

$$
\begin{aligned}
I_{RMS} &= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)d(\omega t)} \\
&= \sqrt{\frac{4}{2\pi} I^2 [\omega t]_{50^\circ}^{70^\circ}} = \sqrt{4I^2\left[\frac{70-50}{360}\right]} \\
&= I\sqrt{\frac{2}{9}} \\
&= 4.714\ A
\end{aligned}
$$

$$
\begin{aligned}
I_{b1} &= \frac{1}{\pi}\int_0^{2\pi} i(\omega t)\sin(\omega t).d(\omega t) \\
&= \frac{4}{\pi}\int_{50^\circ}^{70^\circ} I\sin(\omega t).d(\omega t) = \frac{4I}{\pi}[-\cos(\omega t)]_{50^\circ}^{70^\circ} = \frac{4I}{\pi}[-0.3420+0.6428] \\
&= 3.829A
\end{aligned}
$$

8/17

$I_{b1\,RMS} = \frac{1}{\sqrt{2}} I_{b1} = 2.708$

Distortion factor:

$$\mu = \frac{I_{b1\,RMS}}{I_{RMS}} = \frac{2.708}{4.714} = 0.574$$

Displacement factor is unity because voltage is sinewave and fundamental current has no cosine component.

Therefore, power factor = 0.574

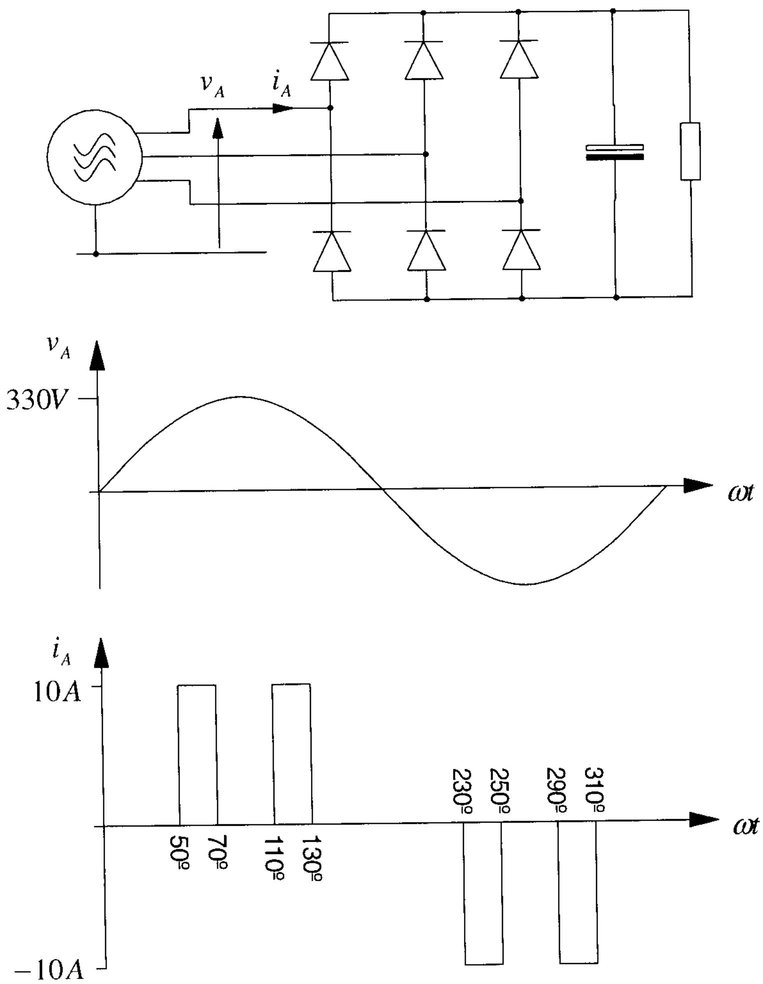
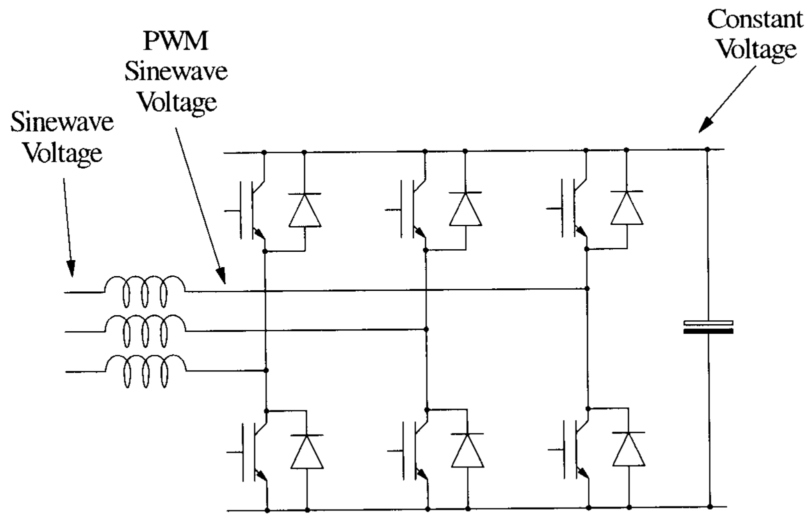


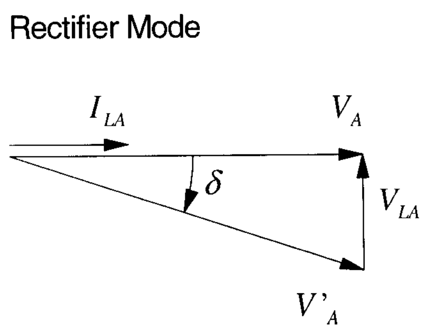
Figure 3 A three-phase diode rectifier

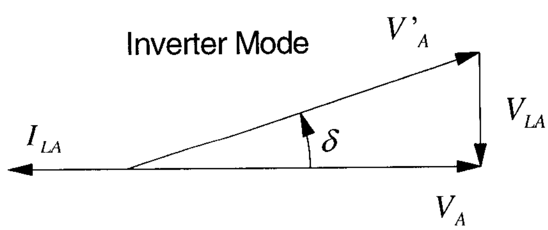
(c) Draw a diagram of an alternative circuit to the rectifier in Figure 3 that can achieve a low distortion factor. Describe how the circuit should be operated and discuss the displacement factor of the circuit.

[6]

The standard 3-phase inverter circuit can be connected to the three-phase mains via inductive impedances. Once charged, the voltage on the DC-side capacitor can be used to synthesis AC voltages and therefore a sinusoidal voltage can be imposed across the inductance of arbitrary magnitude and angle (within the voltage limits of the inverter). Thus, a sinusoidal current can be drawn from the mains at an arbitrary magnitude and angle. The phasor diagram illustrates two cases, unity displacement factor rectification and unity displacement factor re-generation. In fact, any displacement factor (including also zero - reactive power only) can be achieved.







10/17

4) (a) Explain why bi-directional use of a transformer core is preferred to uni-directional use.

[3]

The physical size of a transformer depends on both:

- *the number of turns used and*
- *the cross-sectional area of the core.*

The change in flux in the core of a transformer during a pulse of voltage depends on the applied voltage and the duration. The change of flux could be from zero to the saturation limit (uni-directional use) or from the saturation limit in one direction to the limit in the other. For a given voltage pulse, the saturation flux limit could be halved if the change is made from uni-directional to bi-directional use. This would enable the number of turns or the cross-sectional area to be halved.

(b) **Figure 4** shows an isolated Flyback switch-mode power supply employing two primary-side switches. Describe the advantages this circuit has over a circuit with one primary-side switch.

[3]

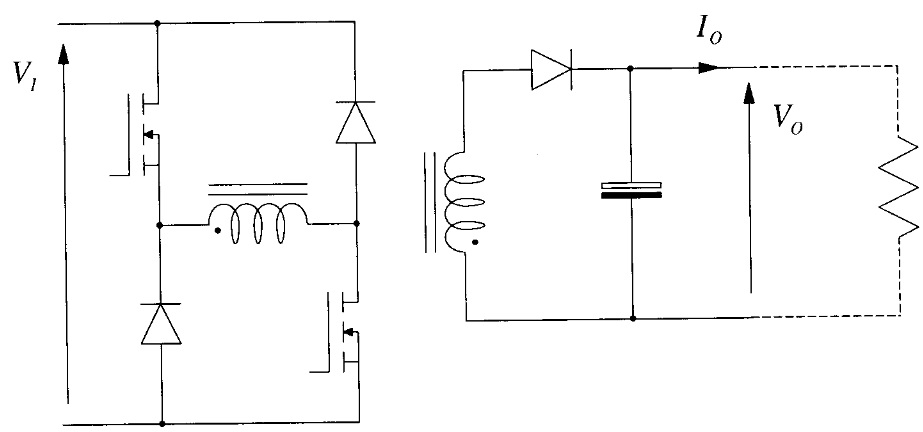


Figure 4, An isolated Flyback switch-mode power supply

It is inevitable that some of the flux produced by the primary does not link the secondary. This leakage flux stores energy that must be released when the primary-side switches turn off but cannot be released through the secondary. In this circuit, the two primary-side diodes provide a path for the primary current to continue when the switches are off. The supply voltage is imposed in reverse across the primary and so the leakage energy is quickly returned to the supply. The single switch alternative can not provide this path and instead the leakage energy transfer to the parasitic capacitances of the circuit, principally C_{DS}, and a poorly damped oscillation occurs with a large voltage overshoot across the transistor. In the two-transistor circuit, the transistors should each be rated for the input voltage. In the single-transistor circuit, the transistor should be rated for the input voltage plus the reflected output voltage plus the voltage overshoot of the trapped leakage energy.

(c) The circuit of **Figure 4** has been implemented with a transformer with a 20:1 turns ratio (primary : secondary) and a primary inductance of 2 mH. It is to be used to with an input voltage of $V_I = 340$ V and an output voltage of $V_O = 15$ V.

11/17

(i) Find expressions for the peak primary current as a function of on-time and diode conduction time (assuming discontinuous conduction) as a function of on-time.

[6]

$$\hat{i}_1 = \frac{V_I}{L_1} t_{on}$$

$$t_{diode} = \hat{i}_2 \frac{L_2}{V_O}$$

$$\hat{i}_1 N_1 = \hat{i}_2 N_2$$

$$\frac{L_1}{L_2} = \frac{N_1^{\,2}}{N_2^{\,2}}$$

$$t_{diode} = \frac{N_1}{N_2} \cdot \frac{V_I}{V_O} \cdot \frac{L_2}{L_1} t_{on}$$

$$= \frac{N_2}{N_1} \cdot \frac{V_I}{V_O} t_{on}$$

(ii) Find the maximum frequency at which the circuit can be operated while remaining in discontinuous mode and taking an input power of 100 W.

[6]

For discontinuous mode $t_{diode} < T - t_{on}$

Input power for discontinuous conditions is:

$$P_I = V_I \, i_1^{Avg}$$

$$= V_I \frac{\frac{1}{2} \frac{V_I}{L_1} t_{on}^{\,2}}{T}$$

The on-time required for a given input power is:

$$t_{on} = \sqrt{\frac{2T\, L_1 P_I}{V_I^{\,2}}}$$

The conduction period that results is therefore:

$$t_{on} + t_{diode} = \sqrt{\frac{2T\, L_1 P_I}{V_I^{\,2}}} \left(1 + \frac{N_2}{N_1} \frac{V_I}{V_O} \right)$$

Setting $T = t_{diode} + t_{on}$ *to find minimum* T

12/17

$$T = \frac{2L_1 P_I}{V_I^2}\left(1 + \frac{N_2}{N_1}\frac{V_I}{V_O}\right)^2$$

$$= \frac{2\times 2\times 10^{-3}\times 100}{340^2}\left(1 + \frac{340}{20\times 15}\right)^2$$

$73.0\mu s$

Maximum frequency is 13.75 kHz

(iii) If the operating frequency is doubled, would the circuit be able to process more or less power while remaining in discontinuous mode. Give a physical reason for your answer.

[2]

If the frequency is doubled, the available power in discontinuous mode will halve. The on-time and diode conduction time will each have to be halved. The energy stored in the core during each on-time is proportional to the current squared and therefore to the period squared. The power delivered is the product of the energy-per-cycle and the frequency. Thus, power is inversely proportional to frequency in this circuit in discontinuous mode.

13/17

5) (a) Figure 5 shows a single-phase AC/DC converter based on a Boost switch-mode power supply. Sketch the shape of the current waveform drawn from the AC source (assuming that a suitable control circuit has been provided) and describe its frequency spectrum. Sketch the shape of the instantaneous input power.

[4]

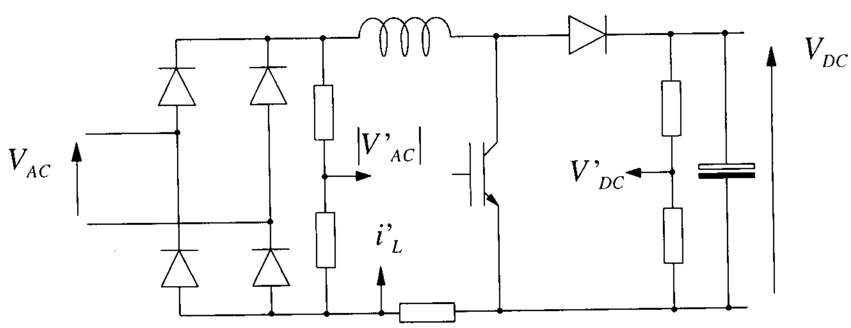
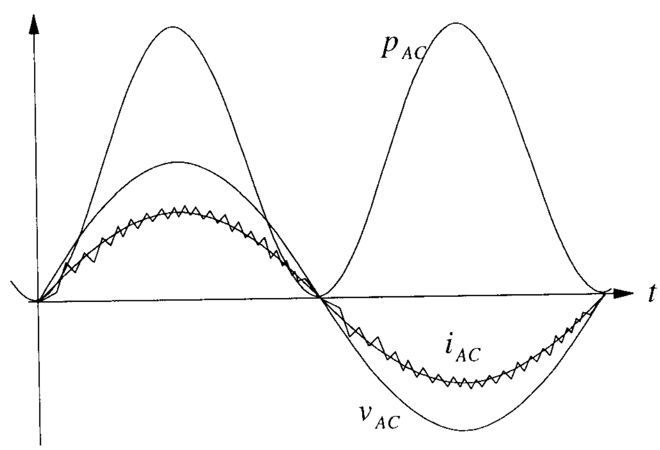


Figure 5 A single-phase AC/DC converter

The input current will be forced to follow a sinusoidal shape subject to a switching frequency ripple. The spectrum will consist of a large 50Hz component, small amplitude low-order harmonics and series of sidebands around multiples of the switching frequency. The input power will be approximately a double frequency sinusoid with zeros the voltage zero-crossings.



(b) (i) Give expressions for the change in current in the inductor in Figure 5 during the on-time and during the off-time of the switch.

[2]

$$\Delta i_L^{On} = \frac{|v_{AC}|}{L} t_{On} \qquad \Delta i_L^{Off} = \frac{|v_{AC}| - V_{DC}}{L} t_{Off}$$

(ii) Assume that the rise and fall of current are approximately matched over one switching cycle and that the inductor is in continuous conduction, find an expression for the magnitude of the ripple current.

[4]

$$\Delta i_L^{On} + \Delta i_L^{Off} = 0$$

$$|v_{AC}|t_{On} + (|v_{AC}| - V_{DC})t_{Off} = 0$$

$$t_{Off} = T - t_{On}$$

$$t_{On} = \frac{(V_{DC} - |v_{AC}|)}{|v_{AC}|}(T - t_{On})$$

$$t_{On} = \frac{\left(\frac{V_{DC}}{|v_{AC}|} - 1\right)T}{\frac{V_{DC}}{|v_{AC}|}} = \left(1 - \frac{|v_{AC}|}{V_{DC}}\right)T$$

$$\Delta i_L^{On} = \frac{|v_{AC}|}{L}\left(1 - \frac{|v_{AC}|}{V_{DC}}\right)T$$

(iii) Find the point in the mains cycle at which the maximum ripple current occurs and find the value of the maximum ripple for a DC-side voltage of 400 V, an AC-side voltage of 230 V_{rms}, a switching frequency of 75 kHz and an inductor of 10 mH.

[6]

For the positive half cycle:

$$\Delta i_L^{On} = \frac{V_{AC}\sin(\omega t)}{L}\left(1 - \frac{V_{AC}\sin(\omega t)}{V_O}\right)T$$

The maximum value occurs when:

$$\frac{\Delta i_L^{On}(\omega t)}{d(\omega t)} = 0$$

$$\frac{\Delta i_L^{On}(\omega t)}{d(\omega t)} = \frac{V_{AC}T}{L}\left[\sin(\omega t)\frac{V_{AC}}{V_O}(-\cos(\omega t)) + \cos(\omega t)\left(1 - \frac{V_{AC}\sin(\omega t)}{V_O}\right)\right]$$

$$= \frac{V_{AC}}{V_O}\frac{T}{L}[-V_{AC}\sin(\omega t)\cos(\omega t) + \cos(\omega t)(V_O - V_{AC}\sin(\omega t))]$$

So, the angle must satisfy:

$$0 = -V_{AC}\sin(\omega t)\cos(\omega t) + \cos(\omega t)(V_O - V_{AC}\sin(\omega t))$$

$$\sin(\omega t) = \frac{V_O}{2V_{AC}} = 0.6149$$

$$\omega t = \sin^{-1}(0.6149) = 37.94°$$

And the maximum ripple will be:

$$\Delta i_L^{On} = \frac{\sqrt{2}\times 230\times 0.6149}{10\times 10^{-3}}\left(1 - \frac{\sqrt{2}\times 230\times 0.6149}{400}\right)\frac{1}{75\times 10^3}$$

$$= 133mA$$

(c) Consider the circuit of Figure 5 operated with an output voltage of 400 V and supplying a 200 W load that draws a constant current. Assuming the circuit is ideally (100%) efficient, compare the instantaneous power supplied to the capacitor to the power drawn by the load. For a capacitor of 500 μF, estimate the voltage variation that results from the instantaneous power variation.

[4]

The input power must have an average value equal to the output power. For the input power to average 200 W, it must have a peak of 400 W because $p_I = P_I\left(1-\cos(2\omega t)\right)$. *For half of the period the input power is less than the output power and the capacitor discharges and for the other half the input power exceeds the output power and the capacitor charges.*

During the half period in which the input power exceeds the average value, the "excess" energy is:

$$\Delta E = \int_{\frac{\pi}{2\omega}}^{\frac{3\pi}{2\omega}} \left(p_I - P_I\right).dt$$

$$= P_I \int_{\frac{\pi}{2\omega}}^{\frac{3\pi}{2\omega}} -\cos(2\omega).dt = P_I\left[\frac{\sin(2\omega t)}{2\omega}\right]_{\frac{\pi}{2\omega}}^{\frac{3\pi}{2\omega}} = \frac{P_I}{2\omega}$$

$$= \frac{200}{4\pi 50} = 0.318J$$

The stored energy in the output capacitor is $\frac{1}{2}CV_O{}^2$

For a small change in energy and a small deviation in voltage

$$E + \Delta E = \tfrac{1}{2}C\left(V_O + \Delta V\right)^2$$

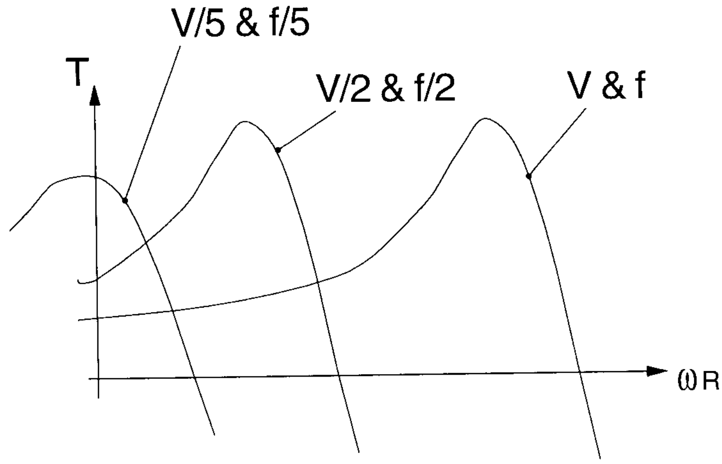
$$\Delta E = \tfrac{1}{2}C\left(2V_O\Delta V + \Delta V^2\right) \approx CV_O\Delta V$$

$$\Delta V \approx \frac{\Delta E}{CV_O} = \frac{0.318}{500\times10^{-6}\times400} = 1.59V$$

16/17

6) (a) Draw a torque against speed curve for a typical induction machine operated at its rated voltage, V_S^R and frequency, f^R. Add to the graph curves for operation at $V_S^R/2$ & $f^R/2$ and for $V_S^R/5$ & $f^R/5$. Justify the shapes of the curves. Describe why a *voltage boost* is normally applied at low frequencies to an induction motor drive operated with voltage varied in proportion to frequency.

[3+3+2]



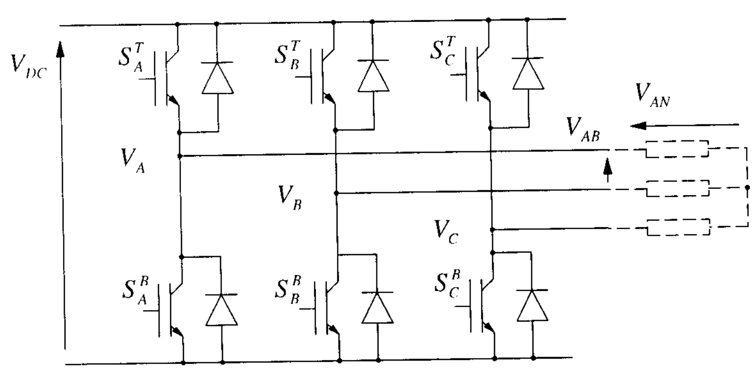
Halving the frequency, halves the synchronous speed and similarly, reducing the frequency by a factor of five reduces the synchronous speed by a factor of five.

The shape of the torque speed curve depends on the flux magnitude and this depends on the applied stator voltage. Because flux and voltage are related by $E = N\frac{d\phi}{dt}$, *if the frequency is halved, the applied voltage should be halved to maintain the flux at its designed value (usually just below the point of saturation). However, at low frequencies, the stator winding resistance is a significant component of the machine's impedance. A significant proportion of the applied voltage is dropped across the resistance and a smaller proportion across the magnetising inductance. Thus, the flux magnitude will be less than the design value and the steepness of the torque/speed curve and the pull-out torque will both be reduced.*

To overcome the reduction in torque capability at low speed under a constant V/f regime, a voltage boost equal to the nominal stator current times the stator resistance is added.

(b) Draw a circuit diagram of circuit suitable for supplying a three-phase induction machine with a variable frequency three-phase set of voltages and describe its operation.

[4]



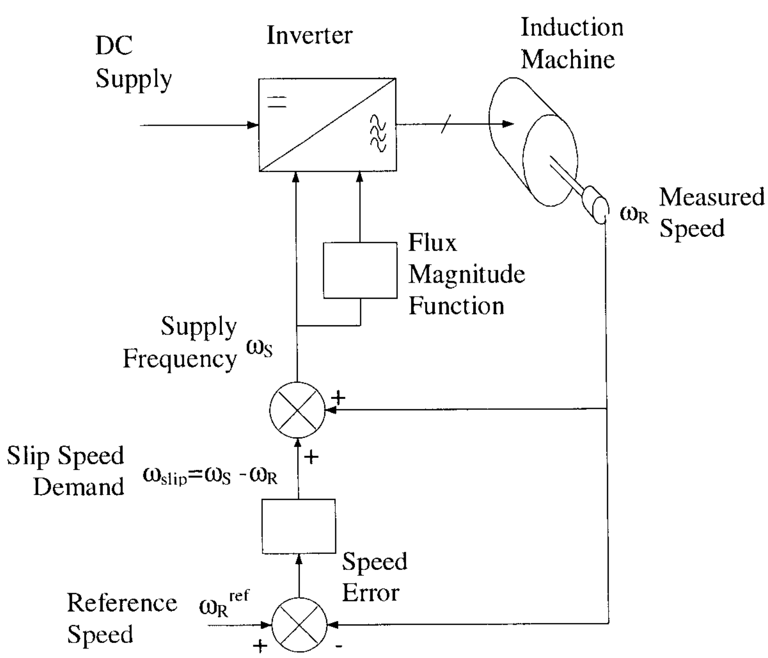
The switches of each phase are switched in a complementary fashion so that each phase voltage is a pulse train of either O or V_{DC}. The pulse widths are modulated through either a *comparison of the modulating wave with a triangular carrier (with all phases using the same carrier) or by programming the timer channels of a microcontroller to effect the same result. The frequency of the modulating wave can be set at will and it's amplitude can be set up to a maximum instantaneous value corresponding to the peak of the carrier and the available DC-link voltage.*

(c) Draw a block diagram of a controlled-slip induction machine drive and explain its operation. Describe why it is normal to include a slip limiter.

[3+3+2]

The torque/speed curve in the region of synchronous speed is steep and approximately linear. The controlled-slip drive aims to always operate the machine in this region and apply a slip proportional to the torque required. The speed of the machine is measured and the stator is supplied with a frequency corresponding to this speed plus the required slip. The error between the measured speed and the speed reference is used, via a controller gain, to set the required torque. The applied voltage magnitude is calculated in proportion to the frequency with a low frequency boost applied.

Slip Compensated
Speed Control



Too high a slip demand will lead to excessive current and so the slip output of the controller is limited. This also means that pull-out can not occur - instead the machine steadyily decellerates with a controlled current in response to an excessive load.