Paper Number(s): **E3.14**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
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EEE PART III: M.Eng., B.Eng. and ACGI

POWER ELECTRONICS AND MACHINES

Friday, 4 May 10:00 am

There are SIX questions on this paper.

Answer FOUR questions.

Time allowed: 3:00 hours

Examiners: Green,T.C. and Vickery,J.C.

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1. (a) The circuit in Figure 1 shows a transistor switching an inductive load which carries a current I_L that can be assumed to be constant. The transistor is switched on and off regularly with a period T and duty-cycle, δ. Sketch the waveforms of the voltage across the transistor and the current through it over one cycle of operation. From this diagram, derive an equation for the average power loss in the transistor.

[6]

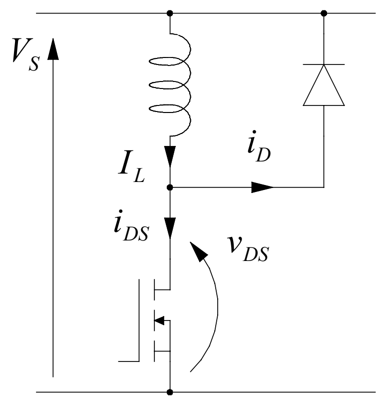


Figure 1 Inductive load switched by a transistor

(b) The circuit of **Figure 2** shows a quasi-resonant switch-mode power supply. It can be assumed that the circuit is in steady-state and that the output voltage V_O and the current I_O are constant over a switching cycle.

(i) Identify whether it is resonant while the switch is on or off and explain the stages of its operation.

[6]

(ii) Write the differential equation that governs the circuit during its resonant period. Identify the initial conditions for the solution to this equation.

[4]

(iii) Calculate a resonant frequency for the circuit if it is to step down from an input voltage of 24 V to an output voltage of 5 V with a transistor switching frequency of 250 kHz.

[4]

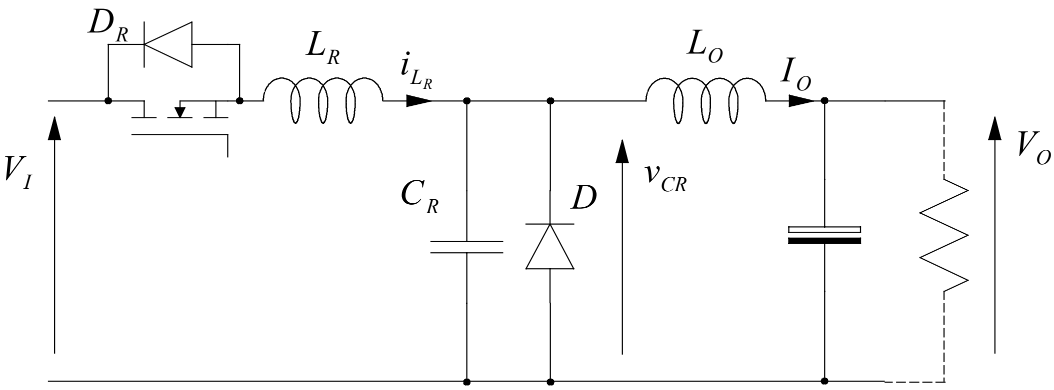


Figure 2 A quasi-resonant switch mode power supply

2. (a) Figure 3 shows a variation on the Ćuk switch-mode power supply known as a SEPIC circuit.

(i) Assume that the capacitor voltages and inductor currents are positive in the senses shown and sketch the current paths during the on and off states of the transistor.

[5]

(ii) By considering the rates-of-change of currents in the inductors and imposing the condition of steady-state operation, derive an equation for the output voltage as a function of input voltage and duty cycle. You may assume that both inductors are in continuous conduction.

[10]

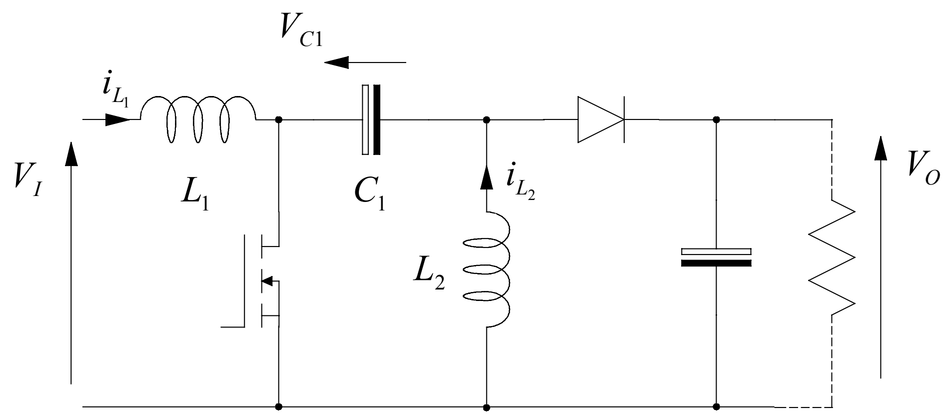


Figure 3 A SEPIC switch-mode power supply

(b) Compare and contrast the Sepic switch-mode power supply with the Ćuk and Flyback power supplies.

[5]

3. (a) Figure 4 shows an isolated Flyback switch-mode power supply. Describe the operation of the circuit using sketches of the principal voltages and currents in discontinuous flux mode.

[6]

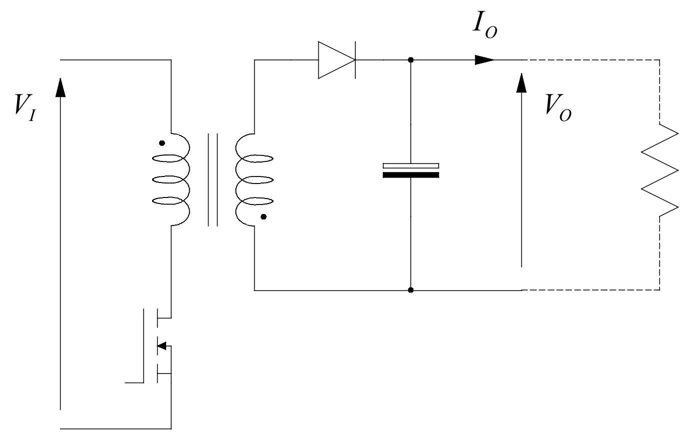


Figure 4 An isolated Flyback switch-mode power supply

(b) Consider an isolated Flyback smps in which the coupling of the two windings is perfect. A design is required to step-up from 48 V to 400 V and deliver a maximum output current of 0.5 A. The circuit is to operate at 100 kHz in discontinuous mode.

(i) Choose a transformer turns ratio to ensure that the transistor voltage does not exceed 90 V.

[3]

(ii) Assuming discontinuous mode, express the average input current as a function of the duty-cycle and primary inductance. Using the assumption of 100% efficiency, relate the input current to the output current.

[3]

(iii) Choose a primary-side inductance to suit a duty-cycle of 0.3 at maximum output current. Find the corresponding secondary-side inductance.

[3]

(iv) Calculate the peak secondary current when delivering the maximum output current.

[3]

(v) Calculate the time for which the diode conducts at maximum output current and check that the circuit is in fact in discontinuous mode.

[2]

4. (a) Sketch the circuit diagram of a DC to 3-phase inverter. Describe how the switching states of the transistors are determined to provide sinusoidal voltages. Sketch the form of the frequency spectrum of the output voltages.

[10]

(b) Describe how a 3-phase inverter can be used as an interface to an AC power distribution grid. Describe the advantages this arrangement has over a conventional diode or thyristor rectifier.

[10]

5. (a) Describe (using sketches) the main elements of a drive system based on an induction machine. Include a control loop that will control the slip in order to regulate the speed.

[7]

(b) For the induction machine model in Figure 5, show that the torque equation $T = 3P\Psi I_R \sin(\delta)$, where P is the number of pole-pairs and Ψ is the magnitude of the air-gap flux-linkage, becomes:

$$T = \frac{3P\Psi^2}{R_R}\left(\omega_S - \omega_R\right)$$

[5]

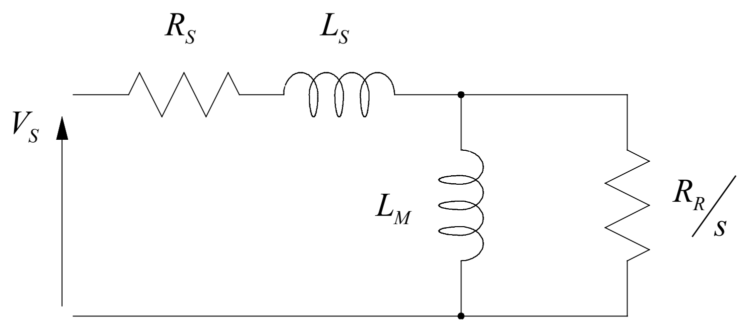


Figure 5 Per-phase equivalent circuit of an induction machine (with negligible rotor leakage inductance)

(c) The induction machine shown in Figure 5 has the following data:

$L_M = 0.035$ H;
$L_S = 0.002$ H;
$R_S = 0.1\ \Omega$;
$R_R = 0.1\ \Omega$;
$P = 1$;
I_R (max) = 40 A;
ω (base) = 100π rad/s;
Ψ(max) = 0.65 Wb.

(i) For operation at the base speed and maximum flux-linkage, calculate: the torque, the difference between synchronous and rotor speeds, the rotor speed, the output power and the necessary stator voltage.

[4]

(ii) Recalculate the same quantities for operation at twice the base speed and half the maximum flux linkage and justify the changes found.

[4]

6. (a) Explain why the physical size of a power semiconductor is in approximate proportion to its power rating.

[4]

(b) A snubber is required to reduce the turn-on power loss of a transistor switching an inductive load of 50 A from a supply voltage of 500 V. Sketch a suitable circuit and make component choices given that the snubber should complete its action over the fall time, t_{fv} of the transistor voltage, which is 200 ns, and that the snubber must reset within 5 μs.

[6]

(c) Calculate the power dissipation in the transistor due to turn-on and the power dissipation in the snubber components for a switching frequency of 5 kHz.

[5]

(d) The transistor and snubber reset resistor are placed on the same heat-sink. Find the temperature of the transistor junction if the transistor has additional losses of 2 W and the various thermal resistances are as follows:

Junction to heat-sink, $R_{th\text{-}JS} = 0.2$ K/W;
Resistor to heat-sink, $R_{th\text{-}RS} = 0.1$ K/W;
Heat-sink to air, $R_{th\text{-}SA} = 0.5$ K/W;
Resistor to air direct, $R_{th\text{-}SA} = 0.7$ K/W.

[5]

Answers to E3.14   2000/01

1) (a) The circuit in Figure 1 shows a transistor switching an inductive load which carries a current I_L that can be assumed to be constant. The transistor is switched on and off regularly with a period T and duty-cylce, δ. Sketch the waveforms of the voltage across the transistor and the current through it over one cycle of operation. From this diagram, derive an equation for the average power loss in the transistor.

[6]

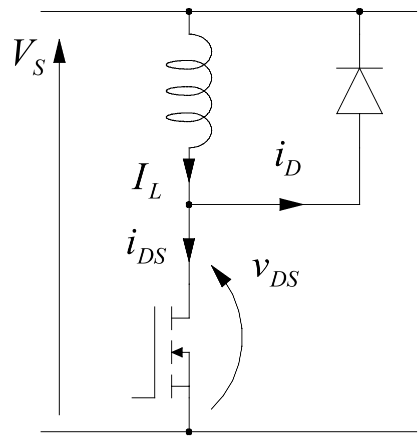
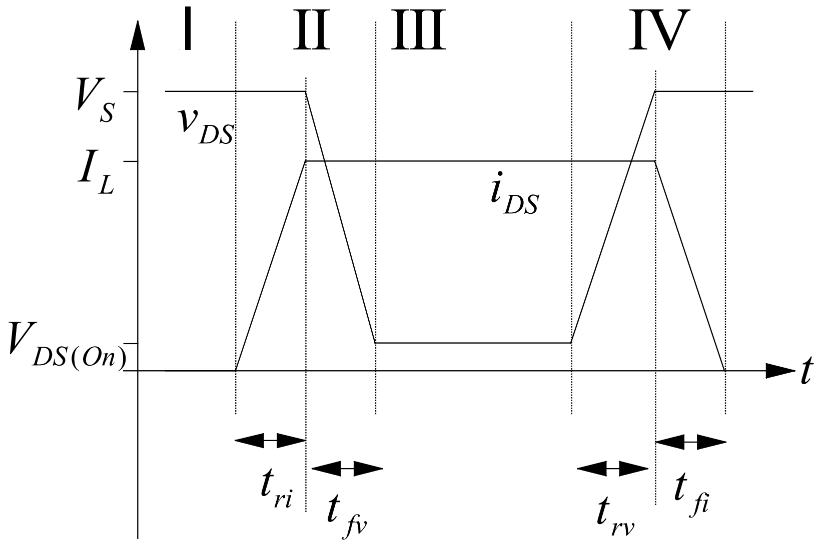


Figure 6, Inductive load switched by a transistor

Treating the rise and fall of voltage and current as linear the waveforms are:



The voltage must remain high, so that the diode is forward biased, while the current is changing. For this reason there is a co-incidence of voltage across the transistor while current flows through it.

$$p(t) = v(t) \cdot i(t) \qquad E = \int_0^T p(t) \cdot dt \qquad P^{Avg} = \frac{1}{T}\int_0^T p(t) \cdot dt$$

$$P_{Loss} = \frac{1}{T}\int^{T} v_T(t)\cdot i_T(t)\cdot dt$$

$$= \frac{1}{T}\left\{\int^{t_I} v_T(t)\cdot i_{Off}(t)\cdot dt + \int^{t_{II}} v_T(t)\cdot i_T(t)\cdot dt + \int^{t_{III}} v_{On}(t)\cdot i_T(t)\cdot dt + \int^{t_{IV}} v_T(t)\cdot i_T(t)\cdot dt\right\}$$

$$= \frac{1}{T}\left\{E_{Cond} + E_{Leak} + E_{Turn\,On} + E_{Turn\,Off}\right\}$$

$$\begin{aligned}
E_{Cond} &= V_{On}\, I_{On(Avg)}\, t_{On} \\
E_{Leak} &= negligable \\
E_{Turn\,On} &= \tfrac{1}{2} V_S\, I_L\left(t_{ri} + t_{fv}\right) \\
E_{Turn\,Off} &= \tfrac{1}{2} V_S\, I_L\left(t_{rv} + t_{fi}\right)
\end{aligned}$$

$$\begin{aligned}
P_{Loss} &= \frac{1}{T}\left(E_{Cond} + E_{Turn\,On} + E_{Turn\,Off}\right) \\
&= \delta\cdot V_{On}\cdot I_{On(Avg)} + f\cdot\left(E_{Turn\,On} + E_{Turn\,Off}\right) \\
&= Conduction\ Loss + Switching\ Loss
\end{aligned}$$

(b) The circuit of **Figure 2** shows a quasi-resonant switch-mode power supply.

(i) Identify whether it is resonant while the switch is on or off and explain the stages of its operation.

[6]

The circuit is paused while the transistor is off and a resonant cycle is initiated by turning the transistor on.

The stages of operation are:

I. *The transistor is off and I_O flows in D. This state is stable.*

II. *The transistor is turned on and the input voltage is imposed across L_R. As a consequence i_{LR} rises linearly. Turn-on occurs while i_{LR} is zero and constrained to rise slowly by L_R. D stays in conduction (and v_{CR} is held close to zero) because i_{LR} is less than I_O. This period lasts until $i_{LR} = I_O$.*

III. *D falls out of conduction and $L_R C_R$ form a resonant circuit governed by a second order differential equation. The current increases, peaks, decreases and reverses. While the current i_{LR} is flowing in reverse (that is through the diode D_R) the transistor can be turned off losslessly.*

Period III ends when the i_{LR} rises again to zero but is blocked from becoming positive because the transistor is off.

IV. *When period III ends there is residual charge on C_R that is discharged (linearly) by the continued flow of I_O. This period ends when v_{CR} reaches zero and D is brought into conduction. The circuit then re-enters the stable state I.*

(ii) Write the differential equation that governs the circuit during its resonant period. Identify the initial conditions for the solution to this equation.

[4]

The resonant action is governed by:

$$V_i = v_{C_r} + L_r \frac{d}{dt}\left(i_{C_R} + I_O\right)$$

$$= v_{C_r} + L_r C_r \frac{d^2 v_{C_r}}{dt^2}$$

The initial conditions are: $i_{LR} = I_O$ and $v_{CR} = 0$.

(iii) Calculate a resonant frequency for the circuit if it is to step down from an input voltage of 24 V to an output voltage of 5 V with a transistor switching frequency of 250 kHz.

[4]

Each pulse of voltage across the resonant capacitor averages to approximately V_I if period IV is considered short. Therefore:

$$\frac{V_O}{V_I} \approx \frac{t_R}{T} \approx \frac{2\pi f}{\omega_R}$$

$$f_R = f\frac{V_I}{V_O} = 250\times10^3 \times \frac{24}{5} = 1.2\,MHz$$

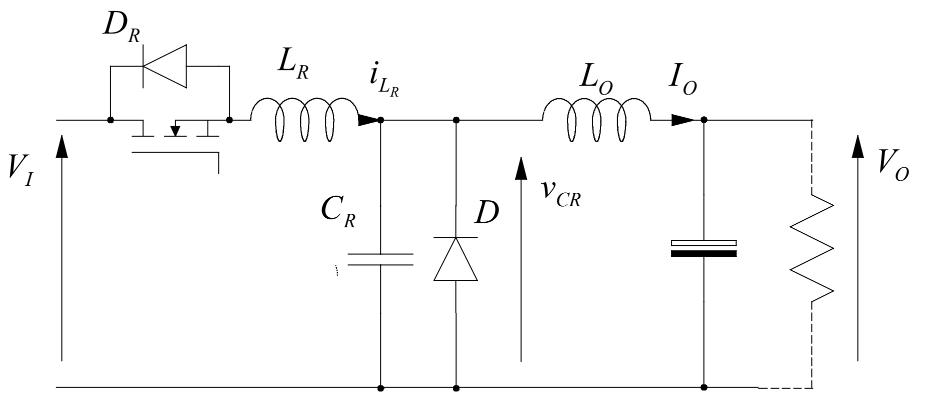


Figure 7, A quasi-resonant switch mode power supply

2) (a) Figure 3 shows a variation on the Ćuk switch-mode power supply known as a SEPIC circuit.

(i) Assume that the capacitor voltages and inductor currents are positive in the senses shown and sketch the current paths during the on and off states of the transistor.

[5]

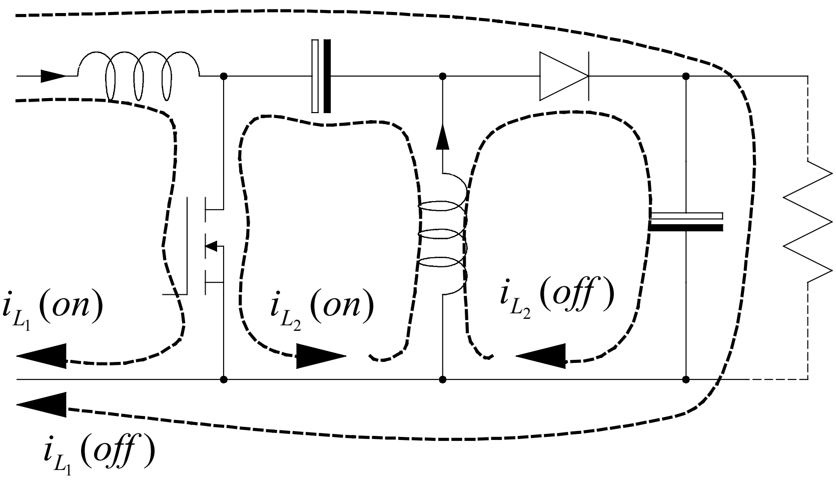


Figure 8 A SEPIC switch-mode power supply

(ii) By considering the rates-of-change of currents in the inductors and imposing the condition of steady-state operation, derive an equation for the output voltage as a function of input voltage and duty cycle. You may assume that both inductors are in continuous conduction.

[10]

Considering i_{L1}
When switch is on:

$$\frac{di_{L1}}{dt} = \frac{V_I}{L_1}$$

When switch is off:

$$\frac{di_{L1}}{dt} = \frac{V_I - V_{C1} - V_O}{L_1}$$

Change of current over one cycle is zero:

$$\Delta i_{L1}(on) + \Delta i_{L1}(off) = 0$$

$$V_I\delta + V_I(1-\delta) - V_{C1}(1-\delta) - V_O(1-\delta) = 0$$

$$(1-\delta)V_O = V_I - V_{C1}(1-\delta)$$

Considering i_{L2}
When switch is on:

$$\frac{di_{L2}}{dt} = \frac{V_{C1}}{L_2}$$

When switch is off:

$$\frac{di_{L2}}{dt} = \frac{-V_O}{L_2}$$

Change of current over one cycle is zero:

$$\Delta i_{L2}(on) + \Delta i_{L2}(off) = 0$$

$$(1-\delta)V_O = V_{C1}\delta$$

Combining the two expressions yields:

$$(1-\delta)V_O = V_I - V_O \frac{(1-\delta)}{\delta}(1-\delta)$$

$$(1-\delta)\left(1+\frac{(1-\delta)}{\delta}\right)V_O = V_I$$

$$\frac{V_O}{V_I} = \frac{\delta}{1-\delta}$$

(b) Compare and contrast the Sepic switch-mode power supply with the Ćuk and Flyback.

[5]

	Flyback	Cuk	Sepic
Voltage Ratio	up/down	up/down	up/down
Polarity	negative	negative	positive
Input Current Shape	pulsed	smooth	smooth
Capacitor Current Shape	pulsed	smooth	pulsed
Voltage Ripple	high	low	high

The Sepic circuit is used where a positive output voltage is required that is to be either above or below the input voltage. The circuit is good in terms of input current distortion but bad in terms of output voltage ripple.

3) (a) Figure 4 shows an isolated Flyback switch-mode power supply. Describe the operation of the circuit using sketches of the principal voltages and currents in discontinuous flux mode.

[6]

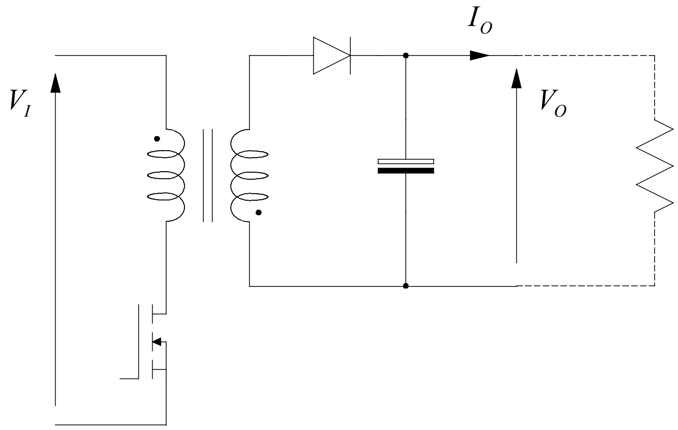


Figure 9, An isolated Flyback switch-mode power supply

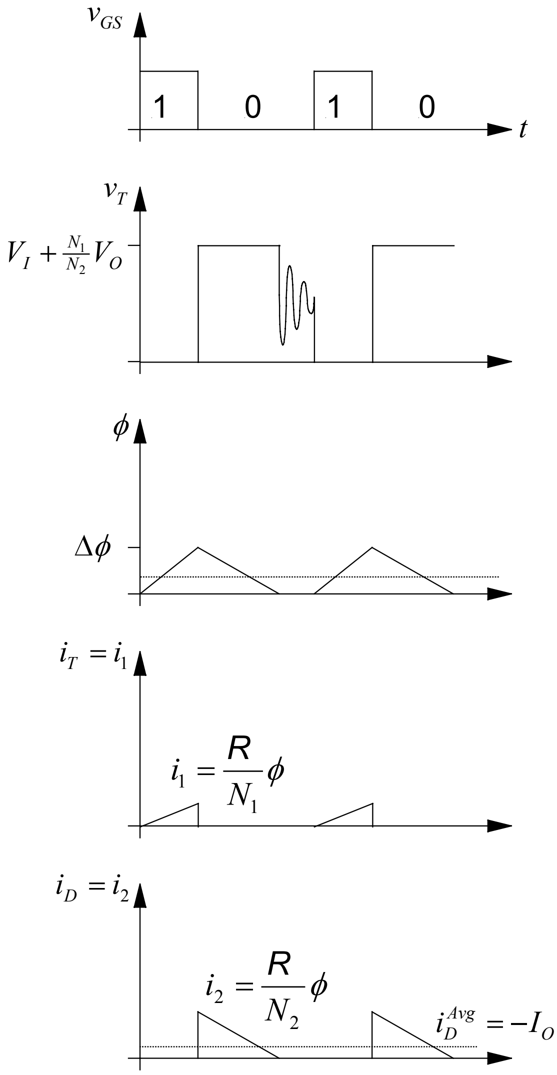
When the transistor is turned on:

- *the input voltage is applied across the primary*
- *flux builds up in the core and energy is stored in the magnetic field*

When the transistor is turned off:

- *the primary current ceases*
- *the flux must be maintained by the flow of current so the diode becomes forward biased and a secondary current flows*
- *the output voltage is applied across the secondary in a sense that decreases the flux*
- *energy is transferred from the core to the output capacitor*

There are two modes of operation: continuous flux mode and discontinuous flux mode (shown in graphs). When one transformer winding has a voltage imposed on it, the other winding sees a reflected copy of this voltage scaled by the turns ratio.



(b) Consider an isolated Flyback smps in which the coupling of the two windings is perfect. A design is required step-up from 48 V to 400 V and deliver a maximum output current of 0.5 A. The circuit is to operate at 100 kHz in discontinuous mode.

(i) Choose a transformer turns ratio to ensure that the transistor voltage does not exceed 90 V.

[3]

$$V_T = V_I + V_O \frac{N_1}{N_2}$$

$$\frac{N_2}{N_1} = \frac{V_O}{V_T - V_I} = \frac{400}{90-48} = 9.523$$

(ii) Assuming discontinuous mode, express the average input current as a function of the duty-cycle and primary inductance. Using the assumption of 100% efficiency, relate the input current to the output current.

[3]

$$I_I = \tfrac{1}{2}\Delta i_L \delta$$
$$= \tfrac{1}{2}\frac{V_I}{L_1}\frac{\delta^2}{f}$$

$$V_I I_I = V_O I_O$$

(iii) Choose a primary-side inductance to suit a duty-cycle of 0.3 at maximum output current. Find the corresponding secondary-side inductance.

[3]

$$L_1 = \tfrac{1}{2}V_I \frac{V_I}{V_O I_O}\frac{\delta^2}{f}$$
$$= \tfrac{1}{2}48\frac{48}{400\times 0.5}\times\frac{0.3^2}{10^5}$$
$$= 5.18\ \mu H$$

$$L_2 = L_1\left(\frac{N_2}{N_1}\right)^2$$
$$= 5.2\times 10^{-6}\times 9.523^2$$
$$= 472\ \mu H$$

(iv) Calculate the peak secondary current when delivering the maximum output current.

[3]

$$\Delta i_1 = \frac{V_I}{L_1}\times\frac{\delta}{f}$$
$$= \frac{48}{5.18\times 10^{-6}}\times\frac{0.3}{10^5} = 27.89A$$

$$\Delta i_2 = \Delta i_1\frac{N_1}{N_2} = 27.80\times\frac{1}{9.523} = 2.91A$$

(v) Calculate the time for which the diode conducts at maximum output current and check that the circuit is in fact in discontinuous mode.

[2]

$$\Delta i_2 = \frac{V_O}{L_2} t_{Diode}$$

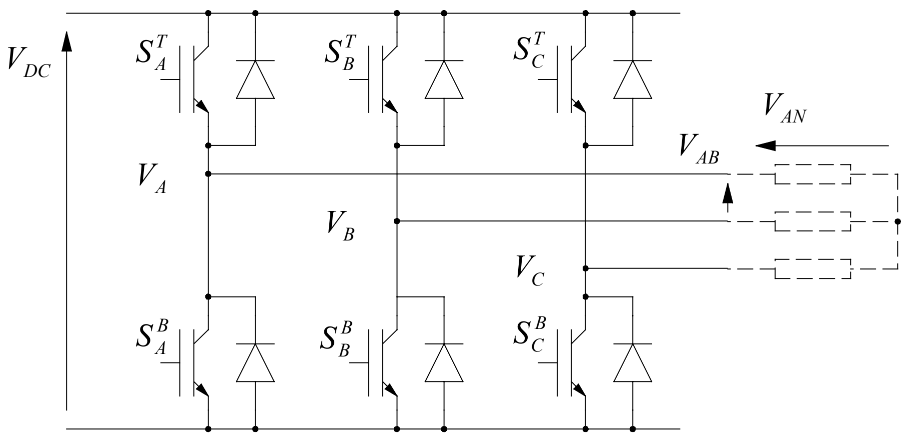
$$t_{Diode} = 2.91 \times \frac{470 \times 10^{-6}}{400}$$

$$t_{Diode} = 3.42\,\mu s$$

$$t_{Off} = \frac{1-\delta}{f} = 0.7 \times 10^{-5} = 7\,\mu s$$

$t_{Diode} < t_{Off}$ so the operation is discontinuous

4) (a) Sketch the circuit diagram of a DC to 3-phase inverter. Describe the how the switching states of the transistors are determined to provide sinusoidal voltages. Sketch the form of the frequency spectrum of the output voltages.



[4]

The switches of each phase are switched in a complementary fashion so that each phase voltage is a pulse train of either O or V_{DC}. The pulse widths are modulated through either a comparison of the modulating wave with a triangular carrier (with all phases using the same carrier) or by programming the timer channels of a microcontroller to effect the same result.

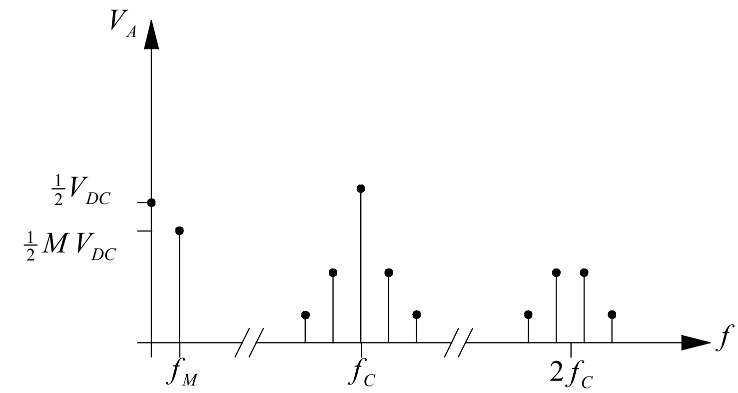
The modulation signal for the three phase voltages V_A, V_B and V_C are a balanced three-phase set with a DC offset:

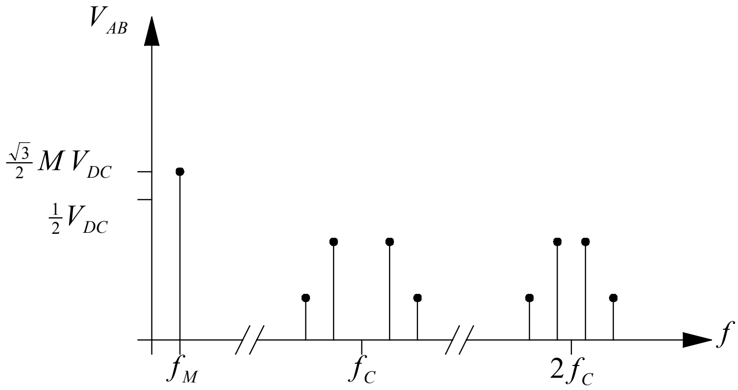
$$v_A = \tfrac{1}{2}V_{DC} + \tfrac{1}{2}V_{DC}M\sin(\omega t)$$
$$v_B = \tfrac{1}{2}V_{DC} + \tfrac{1}{2}V_{DC}M\sin\left(\omega t - \tfrac{2\pi}{3}\right)$$
$$v_C = \tfrac{1}{2}V_{DC} + \tfrac{1}{2}V_{DC}M\sin\left(\omega t + \tfrac{2\pi}{3}\right)$$

$$v_{AB} = \frac{\sqrt{3}}{2}V_{DC}M\sin\left(\omega t + \tfrac{\pi}{6}\right)$$
$$v_{BC} = \frac{\sqrt{3}}{2}V_{DC}M\sin\left(\omega t - \tfrac{\pi}{2}\right)$$
$$v_{CA} = \frac{\sqrt{3}}{2}V_{DC}M\sin\left(\omega t + \tfrac{5\pi}{6}\right)$$

[4]

The spectra of the phase and line voltages are:





[2]

(b) Describe how a 3-phase inverter can be used as an interface to an AC power distribution grid. Describe the advantages this arrangement has over a conventional diode or thyristor rectifier.

- *The inverter is connected to the grid via a set of inductors*
- *The inverter must be phase-locked to the 3-phase grid.*
- *The DC link voltage of the inverter is used to synthesise a 3-phase voltage set.*
- *The current through the interface inductors can be controlled by imposing a voltage across them.*
- *Approximately, the difference in angle between the inverter voltage and the gird voltage sets the real power flow and the magnitude difference sets the reactive power flow.*

[4]

The circuit is superior to a diode rectifier because:

- *the current drawn from the mains is sinusoidal*
- *the voltage on the DC side can be varied and regulated*
- *the circuit can step up to high voltages*

- *the circuit is bi-directional and power can be returned to the grid if the DC load re-generates.*
- *the circuit can also provide reactive power support*

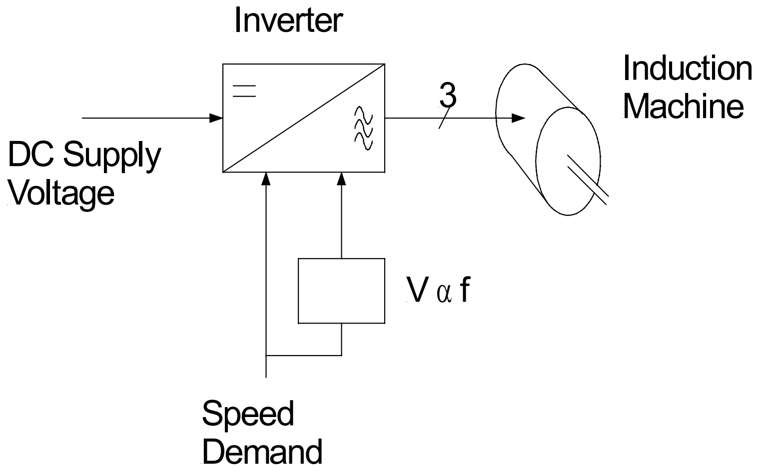
The circuit is superior to a thyristor phase-angle controlled rectifier in all of the above ways except that the thyristor rectifier can control the output voltage and can deal with re-generative loads

[6]

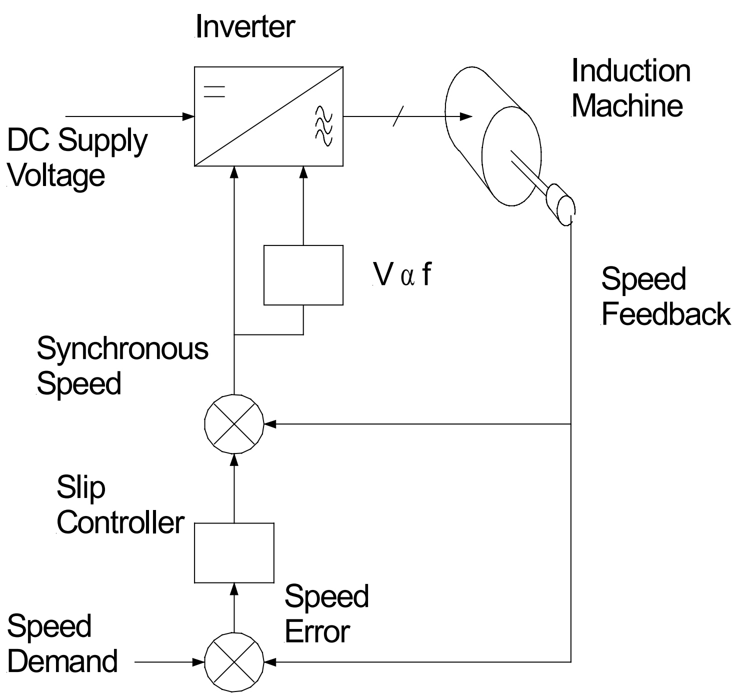
5) (a) Describe (using sketches) the main elements of a drive system based on an induction machine. Include a control loop that will control the slip in order to regulate the speed.

[7]

In the simplest case the speed demand is translated directly to a supply frequency reference for the inverter (taking account of the number of poles of the machine). It is assumed that the slip of the machine is small and that this small speed error is unimportant. In order to keep the air-gap flux magnitude at approximately its design value, the voltage magnitude is varied in proportion to the frequency. Neglecting the effect of the stator winding impedance, this will keep the magnetising current constant.



If the error introduced by the slip is unacceptable then speed feedback is used. The speed error is calculated and from this a slip demand established to increase or decrease the torque. The desired synchronous speed is calculated by adding the slip to the measured speed. The inverter frequency is then set. The applied voltage is again calculated in proportion to the frequency.



(b) For the induction machine model in **Figure 10** show that the torque equation $T = 3P\Psi I_R \sin(\delta)$ where P is the number of pole-pairs and Ψ is the magnitude of the air-gap flux-linkage, becomes:

$$T = \frac{3P\Psi^2}{R_R}\left(\omega_S - \omega_R\right)$$

[5]

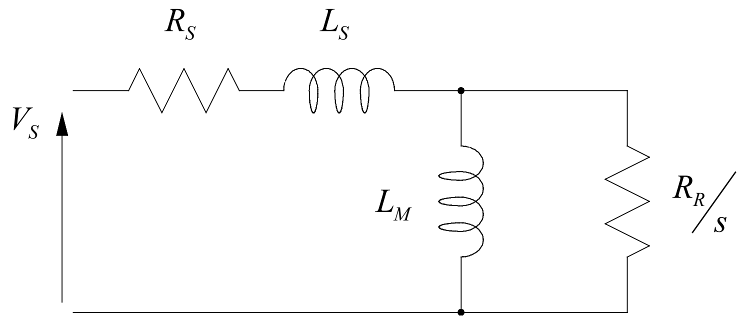


Figure 10, Per-phase equivalent circuit of an induction machine (with negligible rotor leakage inductance)

Defining E as the voltage across L_M.

$$\overline{\Psi} = \overline{I_M} \cdot L_M$$

$$\overline{E} = j\omega_E \overline{\Psi}$$

$$\overline{I_R} = \frac{\overline{E}}{R_R/s} = \frac{s\overline{E}}{R_R}$$

$$T = 3P\left|\overline{\Psi}\right| \cdot \left|\overline{I_R}\right| \sin(\delta)$$

$$= 3P\left|\overline{\Psi}\right| \cdot \frac{s\omega_E\left|\overline{\Psi}\right|}{R_R} \sin\left(\angle j\overline{\Psi} - \angle\overline{\Psi}\right)$$

$$= \frac{3P\Psi^2}{R_R} s\omega_E$$

$$= \frac{3P^2\Psi^2}{R_R}\left(\omega_S - \omega_R\right)$$

(c) The induction machine shown in figure 3 has the following data:

L_M = 0.035 H
L_S = 0.002 H
R_S = 0.1 Ω
R_R = 0.1 Ω
P = 1
I_R (max) = 40 A
ω (base) = 100π rad/s
Ψ(max) = 0.65 Wb

(i) For operation at the base speed and maximum flux linkage calculate: the torque, the difference between synchronous and rotor speeds, the rotor speed, the output power and the necessary stator voltage.

[5]

$$T = 3P\Psi I_R \sin(\delta) \quad \sin(\delta) = 1$$
$$= 3 \times 1 \times 0.65 \times 40$$
$$= 78\ Nm$$

$$\omega_R - \omega_S = \frac{I_R R_R}{\omega_E \Psi}\omega_S$$
$$= \frac{40 \times 0.1}{100\pi \times 0.65} \times 100\pi$$
$$= 6.15\ rad/s$$

$$\omega_R = 100\pi - 6.15$$
$$= 308\ rad/s$$

$$P = T\omega_R$$
$$= 78 \times 308$$
$$= 24\ kW$$

$$\overline{V} = \overline{E} + (R_S + j\omega_E L_S)(\overline{I_R} - j\overline{I_M})$$
$$= 100\pi \times 0.65 + (0.1 + j100\pi \times 0.002)(40 - j18.6)$$
$$= 220 + j23$$
$$V = 221\,V$$

(ii) Recalculate the same quantities for operation at twice the base speed and half the maximum flux linkage and justify the changes found.

[3]

At $\omega = 2\omega_B$ and $\Psi = \Psi_B/2$

$$T = 3P\Psi I_R \sin(\delta) \quad \sin(\delta) = 1$$
$$= 3 \times 1 \times 0.65/2 \times 40$$
$$= 39\ Nm$$

$$\omega_R - \omega_S = \frac{I_R R_R}{\omega_E \Psi}\omega_S$$
$$= \frac{40 \times 0.1}{100\pi \times 0.65/2} \times 100\pi$$
$$= 12.30\ rad/s$$

$$\omega_R = 2 \times 100\pi - 12.30$$
$$= 616\ rad/s$$

$$
\begin{aligned}
P &= T\omega_R \\
&= 39 \times 616 \\
&= 24\ kW \\
\overline{V} &= \overline{E} + \left(R_S + j\omega_E L_S\right)\left(\overline{I_R} - j\overline{I_M}\right) \\
&= 2 \times 100\pi \times {0.65}/{2} + \left(0.1 + j2 \times 100\pi \times 0.002\right)\left(40 - j9.3\right) \\
&= 220 + j48 \\
V &= 225\ V
\end{aligned}
$$

6) (a) Explain why the physical size of a power semiconductor is in approximate proportion to its power rating.

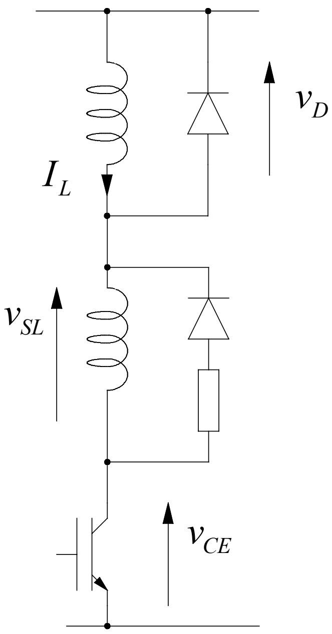
[4]

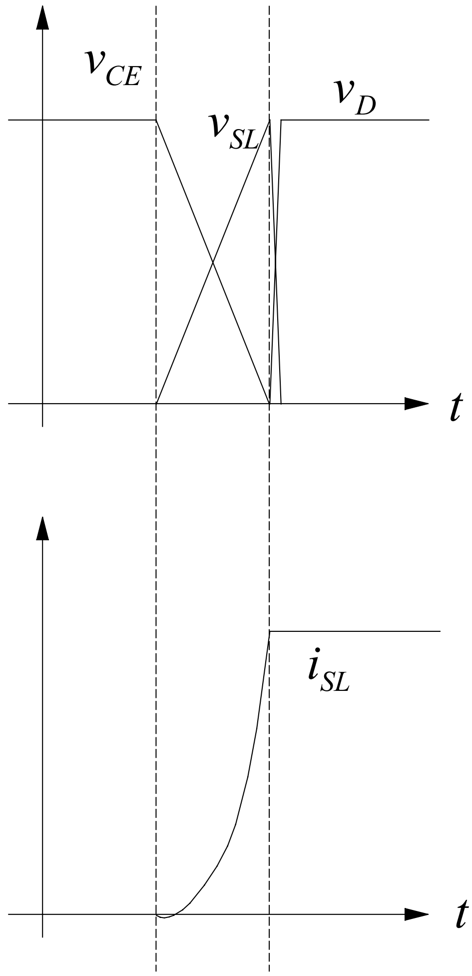
Cross-sectional area given by current rating divided by current density. Current density is limited by heating effects that would cause hot-spots and local melting. Depth of semiconductor is related to reverse breakdown voltage of the principal junction. One side of junction must be lightly doped to control peak electric field therefore the depletion layer grows deep and device must be physically deep enough to support this before punch-through or avalanche occurs.

(e) A snubber is required to reduce the turn-on power loss of a transistor switching an inductive load of 50 A from a supply voltage of 500 V. Sketch a suitable circuit and make component choices given that the snubber should complete its action over the fall time of the transistor voltage, t_{fv} which is 200 ns and that the snubber must reset within 5 μs.

[6]

The snubber should consist of an inductor in series with the transistor to control the rate of rise of current. A resistive path must be available to reset the snubber at turn-off.





$$v_{CE} = V_{DC}\left(1 - \frac{t}{t_{fv}}\right)$$

$$v_{SL} = V_{DC}\frac{t}{t_{fv}}$$

$$i_{SL} = \frac{V_{DC}}{L_S}\int_0^{t_{fv}} \frac{t}{t_{fv}} \cdot dt = \frac{V_{DC}}{L_S}\left[\tfrac{1}{2}\frac{t^2}{t_{fv}}\right]_0^{t_{fv}} = \tfrac{1}{2}\frac{V_{DC}}{L_S}t_{fv}$$

$$L_S = \tfrac{1}{2}\frac{V_{DC}}{I_L}t_{fv} = \frac{500 \times 200 \times 10^{-9}}{2 \times 50} = 1\,\mu H$$

Choose snubber resistor to allow 5 time-constants of discharge within stated reset period.

$$\frac{L_S}{R_S} = \tau = \frac{t_{Reset}}{5}$$

$$R_S = \frac{5L_S}{t_{Resset}} = \frac{5 \times 1 \times 10^{-6}}{5 \times 10^{-6}} = 1\Omega$$

(f) Calculate the power dissipation in the transistor and the snubber components.

[5]

The voltage and current product must be integrated over the fall time. The voltage is a linear fall and the current is a square-law rise.

$$E_T = \int_0^{t_{fv}} V_{DC}\left(1-\frac{t}{t_{fv}}\right)\times I_L\left(\frac{t}{t_{fv}}\right)^2 \cdot dt = V_{DC} I_L\left[\frac{1}{3}\frac{t^3}{t_{fv}{}^2}-\frac{1}{4}\frac{t^4}{t_{fv}{}^3}\right]_0^{t_{fv}}$$

$$= \frac{V_{DC} I_L}{12} t_{fv} = \frac{500\times 50\times 200\times 10^{-9}}{12} = 417\,\mu J$$

All of the stored energy in the inductance is dissipated in the resistor

$$E_R = \tfrac{1}{2} L_S I_L{}^2 = \tfrac{1}{2}\times 1\times 10^{-6}\times 50^2 = 1.25\,mJ$$

The power is found my multiplying by the frequency

$$P_T = f\, E_T = 2.08\,W$$
$$P_R = f\, E_R = 6.25\,W$$

(g) The transistor and snubber reset resistor are placed on the same heat-sink. Find the temperature of the transistor junction if the transistor operates at 5 kHz and has additional losses of 2 W. The ambient temperature is 25°C and the various thermal resistances are as follows:

Junction to heat-sink, $R_{th\text{-}JS}$ = 1 K/W
Resistor to heat-sink, $R_{th\text{-}RS}$ = 0.5 K/W
Heat-sink to air, $R_{th\text{-}SA}$ = 3 K/W
Resistor to air direct, $R_{th\text{-}SA}$ = 12 K/W

[5]

There are two paths for heat from the resistor to air – direct and via the heat-sink which is shared with the transistor. Find fraction of heat through each path using simultaneous equations.

$$\theta_R = \theta_A + \alpha\, P_R\, R_{th-RA}$$
$$\theta_R = \theta_A + (1-\alpha) P_R\, R_{th-RS} + (1-\alpha) P_R\, R_{th-SA} + P_T\, R_{th-SA}$$

$$\alpha = \frac{P_R\left(R_{th-RS}+R_{th-SA}\right)+P_T R_{th-SA}}{P_R\left(R_{th-RA}+R_{th-RS}+R_{th-SA}\right)} = \frac{6.25\times(0.5+3)+(2.08+2)\times 3}{6.25\times(12+0.5+3)} = 0.352$$

$$\theta_R = 25 + 0.352\times 6.25\times 12 = 51.4°\,C$$

$$\theta_T = \theta_A + P_T\left(R_{th-JS}+R_{th-SA}\right)+(1-\alpha)P_R R_{th-SA}$$
$$= 25 + (2.08+2)\times(1+3)+(1-0.352)\times 6.25\times 3 = 53.5°\,C$$