



### Fundamental constants

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ m kg/C}^2$$

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.62 \times 10^{-34} \text{ Js}$$

- 1a) The vector wave equation for electromagnetic waves in optical media is

$$\nabla^2 \underline{E} = \mu_0 \epsilon \partial^2 \underline{E} / \partial t^2$$

Here  $\underline{E}(x, y, z, t)$  is the electric field,  $\mu_0$  is the permeability, and  $\epsilon$  is the dielectric constant.

Indicating your assumptions, derive a time-independent scalar wave equation for a plane wave of angular frequency  $\omega$ , polarized in the  $x$ -direction and propagating in the  $z$ -direction. Solve the equation. Assuming that the relative permittivity is  $\epsilon_r = 2.25$ , find the phase velocity, the propagation constant and the wavelength, at a frequency of  $5 \times 10^{14}$  Hz. What would the corresponding values be, in free space? [12]

- b) Show how your equation and solution modifies for a lossy dielectric medium. The relative permittivity of a lossy medium is  $\epsilon_r = 2.25 - j 10^{-5}$ . Find the attenuation coefficient and the distance needed for the wave to travel to decay to  $1/e$  of its original amplitude, at the same frequency. [8]

- 2a). A dielectric waveguide is constructed from three layers of material, with refractive indices  $n_1, n_2$  and  $n_3$ . The core thickness and refractive index are  $d$  and  $n_1$ . The eigenvalue equation for TE modes in such a guide can be obtained as:

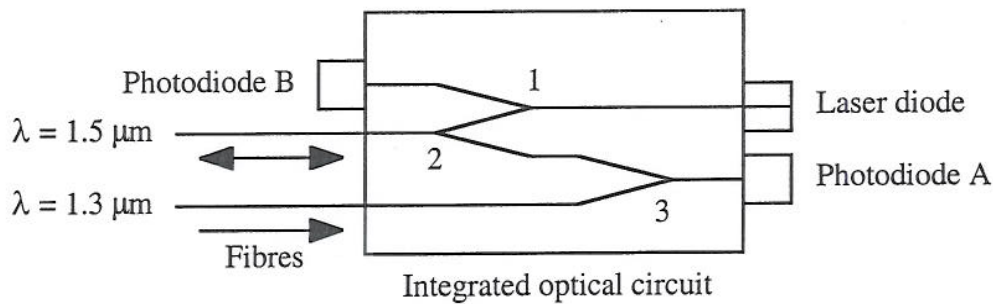
$$\tan(\kappa d) = \kappa(\gamma + \delta) / (\kappa^2 - \gamma\delta)$$

What are  $\kappa$ ,  $\gamma$  and  $\delta$ ? Sketch the guide. On your sketch, provide a pictorial interpretation of  $\kappa$  in terms of rays. [6]

- b) Explain how the eigenvalue equation simplifies for a symmetric guide. Derive a cutoff condition valid for all modes. Find the maximum core thickness  $d$  for single mode operation, assuming that the wavelength is  $0.633 \mu\text{m}$ , the refractive index of the cladding is 1.5, and the refractive index of the core is i) 1.51 and ii) 1.501. [8]
- c) Estimate the effective index of the lowest order mode of a symmetric guide with a core index of 1.51, a core thickness of  $1 \mu\text{m}$  and a cladding index of 1.5, at a wavelength of  $0.633 \mu\text{m}$ . [6]

- 3a) Explain in detail the operation of i) a channel waveguide Y-junction, and ii) a tree-structured Y-junction splitter. What are the advantages of the radiative star, over the latter? [12]

- b) A transmit/receive (TX/RX) module is constructed from a laser diode, two photodiodes and an integrated optical circuit containing three Y-junctions as shown in *Figure 1*. The circuit is connected to two fibres carrying traffic at  $1.3\ \mu\text{m}$  and  $1.5\ \mu\text{m}$  wavelength. Assuming that the Y-junctions are symmetric and lossless, find
- The fraction of RX light at  $\lambda = 1.5\ \mu\text{m}$  detected at photodiode A.
  - The fraction of RX light at  $\lambda = 1.3\ \mu\text{m}$  detected at photodiode A.
  - The fraction of TX light at  $\lambda = 1.5\ \mu\text{m}$  detected at photodiode B.
- In each case, explain the fate of any lost power. [8]



*Figure 1.*

- 4a) Sketch and explain the operation of an arrayed waveguide grating multiplexer (AWG-MUX). Why is this component preferred over other guided wave filters? [10]
- b) Show how AWG-MUX and other components may be used to construct i) an ADD-DROP MUX, and ii) a wavelength switch. If the number of channels is  $N$  in each case, how many MUX, DEMUX and switch components are required? What are these devices used for? [10]

- 5a) In a two-state, lumped-element model, the rate equations governing the internal operation of a LED are:

$$dn/dt = I/ev - n/\tau_e \quad \text{and} \quad d\phi/dt = n/\tau_{rr} - \phi/\tau_p$$

Here  $n$  and  $\phi$  are the electron and photon densities,  $I$  is the current,  $v$  is the active volume,  $\tau_e$  is the electron lifetime,  $\tau_{rr}$  is the radiative recombination lifetime and  $\tau_p$  is the photon lifetime. Explain the physical processes involved in these equations. How should the equations be modified to describe a semiconductor laser? [8]

- b) How is the photon lifetime calculated, for i) a LED, and ii) a semiconductor laser? Assuming typical dimensions and material parameters, estimate  $\tau_p$  in each case. [12]

- 6a) Explain the process of optical absorption in a semiconductor. What are the limitations of photoconductive detectors? Why are PIN diodes preferred to PN diodes? [6]

- b) Explain the importance of the semiconductor  $\text{In}_{1-x}\text{Ga}_x\text{As}_{1-y}\text{P}_y$  in optoelectronics. *Figure 2* shows its energy gap variation. Over what spectral range could lasers operate when constructed using this material? [6]

- c) A sandwich student at an optoelectronics company has returned to their university at the start of term leaving incomplete records in their logbook. *Figure 3* shows their design for a photodiode. Redraw the diagram with full labelling, explaining your reasoning. [8]

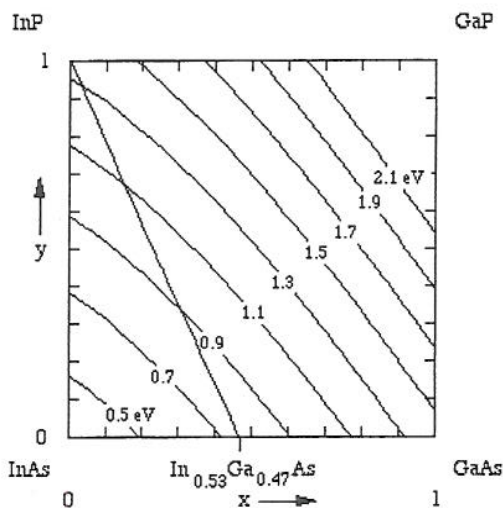
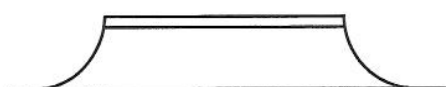


Figure 2.



Materials: InP, InGaAs, Ti/Au  
Doping:  $n^-$ ,  $n^+$ ,  $p^+$

How does the light go? Forgot to ask.  
Must try and finish tomorrow ...

Figure 3.



**E312 Optoelectronics Solutions 2006**

S013

1a) The vector wave equation for electromagnetic waves in optical media is

$$\nabla^2 \underline{\mathbf{E}} = \mu_0 \epsilon \partial^2 \underline{\mathbf{E}} / \partial t^2$$

Here  $\underline{\mathbf{E}}(x, y, z, t)$  is the electric field,  $\mu_0$  is the permeability,  $\epsilon$  is the dielectric constant and  $t$  is time. A wave polarized in the  $x$ -direction only has an  $x$ -component. For  $\underline{\mathbf{E}}_x$ , we have:

$$\partial^2 \underline{\mathbf{E}}_x / \partial x^2 + \partial^2 \underline{\mathbf{E}}_x / \partial y^2 + \partial^2 \underline{\mathbf{E}}_x / \partial z^2 = \mu_0 \epsilon_0 \epsilon_r \partial^2 \underline{\mathbf{E}}_x / \partial t^2$$

Where  $\epsilon = \epsilon_0 \epsilon_r$ . At angular frequency  $\omega$ , we may write  $\underline{\mathbf{E}}(x, y, z, t) = \underline{\mathbf{E}}(x, y, z) \exp(j\omega t)$ . Differentiating and substituting, we obtain:

$$\partial^2 \underline{\mathbf{E}}_x / \partial x^2 + \partial^2 \underline{\mathbf{E}}_x / \partial y^2 + \partial^2 \underline{\mathbf{E}}_x / \partial z^2 = -\omega^2 \mu_0 \epsilon_0 \epsilon_r \underline{\mathbf{E}}_x$$

If the wave is plane and propagating in the  $z$ -direction,  $\partial \underline{\mathbf{E}}_x / \partial x = \partial \underline{\mathbf{E}}_x / \partial y = 0$ . Hence:

$$d^2 \underline{\mathbf{E}}_x / dz^2 = -\omega^2 \mu_0 \epsilon_0 \epsilon_r \underline{\mathbf{E}}_x$$

Substituting the solution  $\underline{\mathbf{E}}_x = \underline{\mathbf{E}}_{x+} \exp(-jkz)$  into the above, we obtain  $k^2 = \omega^2 \mu_0 \epsilon_0 \epsilon_r$ . The propagation constant is then  $k = \omega \sqrt{(\mu_0 \epsilon_0 \epsilon_r)} = k_0 \sqrt{\epsilon_r}$  where  $k_0$  is the value in free space.

The propagation constant is related to the wavelength  $\lambda$  by  $k = 2\pi/\lambda$ , so  $\lambda = 2\pi/k = \lambda_0 / \sqrt{\epsilon_r}$ , where  $\lambda_0$  is the value in free space. The phase velocity is  $v_{ph} = \omega/k = 1/(\mu_0 \epsilon_0 \epsilon_r)^{1/2} = c/\sqrt{\epsilon_r}$ , where  $c$  is the value in free space

[6]

If  $\epsilon_r = 2.25$ , the phase velocity is  $v_{ph} = 1/(4\pi \times 10^{-7} \times 8.85 \times 10^{-12} \times 2.25)^{1/2} = 2 \times 10^8$  m/s. In free space, we obtain  $c = 2 \times 10^8 \times \sqrt{2.25} = 3 \times 10^8$  m/s.

[2]

If the frequency is  $f = 5 \times 10^{14}$  Hz, the propagation constant is  $k = 2\pi \times 5 \times 10^{14} \times (4\pi \times 10^{-7} \times 8.85 \times 10^{-12} \times 2.25)^{1/2} = 15.71 \times 10^6$  m<sup>-1</sup>. In free space we obtain  $k_0 = 15.71 \times 10^6 / \sqrt{2.25} = 10.47 \times 10^6$  m<sup>-1</sup>.

[2]

The wavelength is  $\lambda = 2\pi/k = 2\pi/15.71 \times 10^6$  m = 0.4  $\mu$ m. In free space we obtain  $\lambda_0 = 0.4 \times \sqrt{2.25} = 0.6$   $\mu$ m.

[2]

b) For a lossy medium, the wave equation becomes  $d^2 \underline{\mathbf{E}}_x / dz^2 = -\omega^2 \mu_0 \epsilon_0 [\epsilon_r' - j\epsilon_r''] \underline{\mathbf{E}}_x$ , where  $\epsilon_r'$  and  $\epsilon_r''$  are the real and imaginary parts of the dielectric constant. The propagation constant is now  $k = \omega \sqrt{(\mu_0 \epsilon_0 \epsilon_r')} \sqrt{[1 - j\epsilon_r''/\epsilon_r']}$

Assuming that the loss is small, so that  $\epsilon_r'' \ll \epsilon_r'$ , we may use a binomial approximation, which gives  $k = \omega \sqrt{(\mu_0 \epsilon_0 \epsilon_r')} [1 - j\epsilon_r''/2\epsilon_r']$ . If we write this as  $k = k' - jk''$ , the real and imaginary parts of  $k$  are given by  $k' = \omega \sqrt{(\mu_0 \epsilon_0 \epsilon_r')}$  and  $k'' = k' \epsilon_r''/2\epsilon_r'$ .

[4]

The propagation constant  $k'$  is then as before. The attenuation coefficient is  $k'' = 15.71 \times 10^6 \times 10^{-5} / (2 \times 2.25) = 34.91$  m<sup>-1</sup>.

[2]

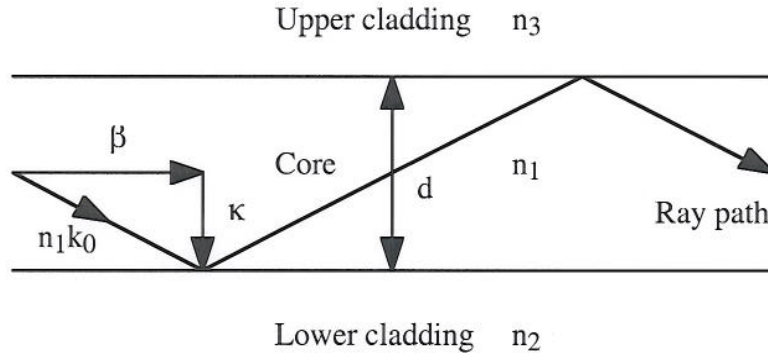
The wave propagates as  $\underline{\mathbf{E}}_x = \underline{\mathbf{E}}_{x+} \exp(-k''z) \exp(-jk'z)$ , and hence falls to 1/e of its original amplitude in a distance  $z = 1/k'' = 1/34.91$  m = 28.64 mm.

[2]

2a) The eigenvalue equation for an asymmetric guide is  $\tan(\kappa h) = \kappa(\gamma + \delta)/(\kappa^2 - \gamma\delta)$ . Here  
 $\kappa = \sqrt{\{n_1^2 k_0^2 - \beta^2\}}$   
 $\gamma = \sqrt{\{\beta^2 - n_2^2 k_0^2\}}$   
 $\delta = \sqrt{\{\beta^2 - n_3^2 k_0^2\}}$   
 $\beta$  is the propagation constant and  $k_0 = 2\pi/\lambda$ , where  $\lambda$  is the wavelength.

[3]

Waveguide, ray path and wave-vectors are as below.



[3]

b) In a symmetric guide,  $\gamma = \delta$ , so that  $\tan(\kappa d) = 2\kappa\gamma / (\kappa^2 - \gamma^2)$   
 To simplify this we make use of the standard expression  $\tan(2A) = 2 \tan(A) / \{1 - \tan^2(A)\}$

Dividing through by  $\kappa^2$ , we get  $\tan(\kappa d) = 2(\gamma/\kappa) / \{1 - (\gamma/\kappa)^2\}$   
 Comparison with the above gives  $\tan(\kappa d/2) = \gamma/\kappa$   
 This is the eigenvalue equation for all symmetric modes

Dividing through by  $\gamma^2$ , we get  $\tan(\kappa d) = 2(\kappa/\gamma) / \{(\kappa/\gamma)^2 - 1\}$   
 Comparison with the above gives  $\tan(\kappa d/2) = -\kappa/\gamma$   
 This is the eigenvalue equation for all anti-symmetric modes

At cutoff, the mode is mainly propagating in layer 2, so  $\beta$  tends to  $n_2 k_0$  and  $\gamma$  tends to zero  
 For symmetric modes we then have  $\tan(\kappa d/2)_{c.o.} = 0$  so  $(\kappa d/2)_{c.o.} = 0, \pi, 2\pi \dots$   
 For antisymmetric modes, the cutoff condition is  $(\kappa d/2)_{c.o.} = \pi/2, 3\pi/2 \dots$   
 So in general we must have  $(\kappa d/2)_{c.o.} = v\pi/2$  where  $v$  is the mode number.

Since  $\kappa = \sqrt{(n_1^2 k_0^2 - \beta^2)}$ , then at cutoff we must have  $\kappa_{c.o.} = k_0 \sqrt{(n_1^2 - n_2^2)}$   
 so the cutoff condition reduces to  $(k_0 d/2) \sqrt{(n_1^2 - n_2^2)} = v\pi/2$

[4]

The 1<sup>st</sup> order mode cuts off when  $(k_0 d/2) \sqrt{(n_1^2 - n_2^2)} = \pi/2$   
 Consequently the condition on  $d$  for single-mode operation is  $d < \lambda / \{2\sqrt{(n_1^2 - n_2^2)}\}$   
 Assuming that  $\lambda = 0.633 \mu\text{m}$ :  
 For  $n_1 = 1.51$  and  $n_2 = 1.5$ , we get  $d < 1.824 \mu\text{m}$   
 For  $n_1 = 1.501$  and  $n_2 = 1.5$ , we get  $d < 5.777 \mu\text{m}$

[4]

c) The effective index  $n_{\text{eff}}$  is related to the propagation constant by  $\beta = 2\pi n_{\text{eff}}/\lambda$ , and must lie in the range  $n_2 < n_{\text{eff}} < n_1$ .

To find  $n_{\text{eff}}$  we must solve the eigenvalue equation  $\tan(kd/2) = \gamma/\kappa$  by a numerical search. One method is to use bisection to locate the zero of the function  $f(n_{\text{eff}}) = \tan(\kappa d/2) - \gamma/\kappa = 0$ .

Using the bisection algorithm, we obtain:

Iteration	$n_{\text{lower}}$	$f(n_{\text{lower}})$	$n_{\text{mid}}$	$f(n_{\text{mid}})$	$n_{\text{upper}}$	$f(n_{\text{upper}})$
1	$1.5+1e^{-6}$	1.1539	1.505	0.885	$1.51-1e^{-6}$	-99.82
2	1.505	0.885	1.5075	-1.269	$1.51-1e^{-6}$	-99.82
3	1.505	0.885	1.50625	-.7058	1.5075	-1.269
4	1.505	0.885	1.505625	-0.491	1.50625	-.7058

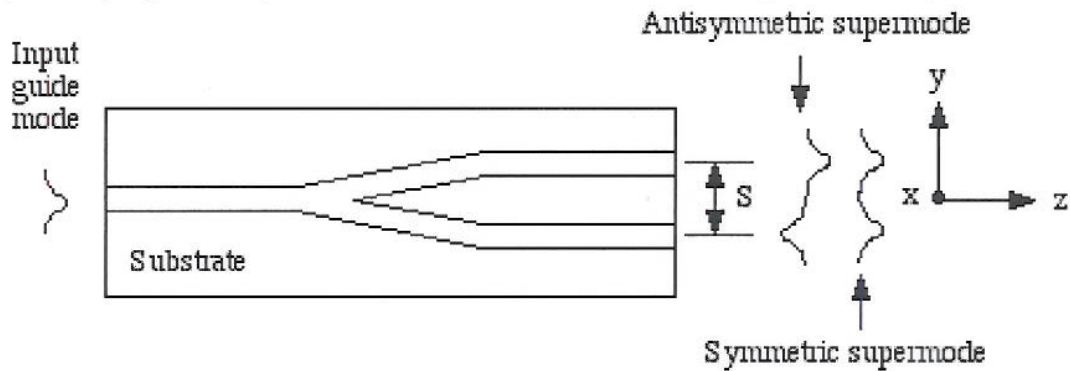
Hence  $n_{\text{eff}}$  is approximately 1.5053

Marks awarded on the basis of the description of the algorithm and the final accuracy.

[6]



3a) The Y-junction is a branching element consisting of a single-moded input guide, which is gradually separated by a forked transition into two similar single-mode outputs as below.



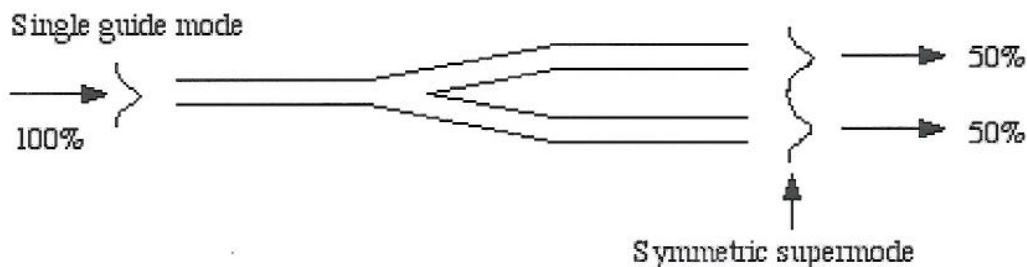
At the left-hand end, the structure consists of one single-moded guide, which supports a characteristic mode with a transverse field distribution  $E_L(x, y)$ , given by (say)  $E_L(x, y) = E(x, y)$ . At the right-hand end, the device consists of two similar single-moded guides, separated by a distance  $S$ . The transverse field patterns of the upper and lower guides are:

$$E_{RU}(x, y) = E(x, y - S/2) \quad \text{and} \quad E_{RL}(x, y) = E(x, y + S/2)$$

The combined structure at the right-hand end can also be thought of as a composite guide, which supports its own set of characteristic modes. Since the structure is inherently symmetric, these supermodes are symmetric and anti-symmetric field patterns (defined as  $E_{RS}$  and  $E_{RA}$ , respectively). At the right-hand end of the device, the combined structure can therefore support two guided supermodes, of the form:

$$E_{RS}(x, y) = E_{RU} + E_{RL} \quad \text{and} \quad E_{RA}(x, y) = E_{RU} - E_{RL}$$

We now consider the response to an input to the single guide the left-hand end, as below.



Near the input, the transverse field can be taken as  $E_{in}(x, y) = a_{in} E_L(x, y)$  where  $a_{in}$  is the input mode amplitude. This input (which is a symmetric distribution) will be gradually converted into the symmetric supermode in the transition region. Because the Y-junction is symmetric, no antisymmetric modes can be excited. Similarly, provided the taper rate is low, no symmetric radiation modes will be generated. In a carefully constructed Y-junction, conversion into the symmetric supermode will therefore be 100% efficient. At the output, the transverse field can therefore be written as  $E_{out}(x, y) = a_{out} E_{RS}(x, y)$ . Assuming 100% efficiency, we can calculate the field amplitudes. The input power is given by  $P_{in} = (\beta/2\omega\mu_0) a_{in}^2 \langle E, E \rangle$  and the output power is  $P_{out} = (\beta/2\omega\mu_0) a_{out}^2 \langle E_{RS}, E_{RS} \rangle$ .

If the two output guides are sufficiently far apart, this can be approximated as:

$$P_{out} = (\beta/2\omega\mu_0) a_{out}^2 \{ \langle E_{RU}, E_{RU} \rangle + \langle E_{RL}, E_{RL} \rangle \} = 2 (\beta/2\omega\mu_0) a_{out}^2 \langle E, E \rangle$$

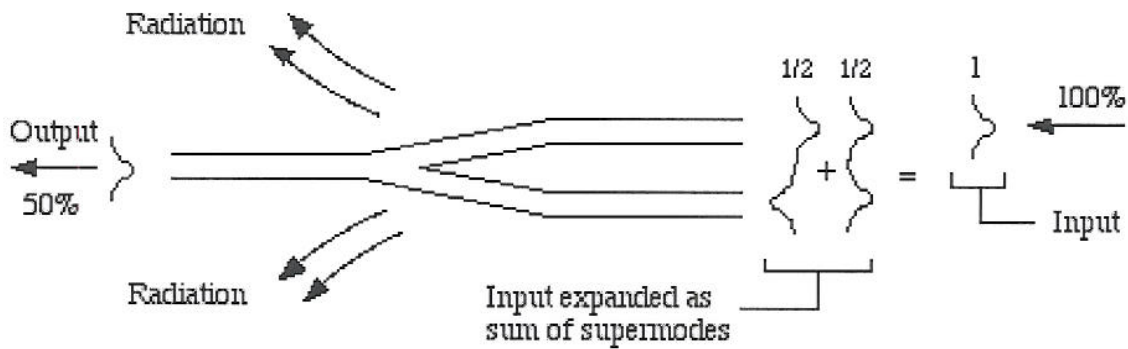
where we have assumed that  $\langle E_{RU}, E_{RL} \rangle \approx 0$ . Equating the results above,  $a_{out} = a_{in}/\sqrt{2}$ .

We now consider the device output in terms of the modes travelling in each of the two right-hand guides individually. Clearly, the output can be rewritten as:

$$E_{out}(x, y) = a_{in}/\sqrt{2} E_{RU}(x, y) + a_{in}/\sqrt{2} E_{RL}(x, y)$$

This result implies that the Y-junction divides the input equally between the two output guides. Each has a modal amplitude of  $1/\sqrt{2}$  times the input, and carries half the power. [4]

We now consider operation in the reverse direction, when light is incident from just one of the two channels on the right.

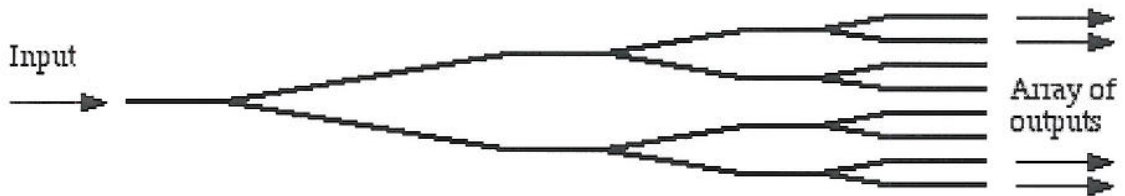


This time the incident beam can be written as  $E_{in}'(x, y) = a_{in}' E_{RU}(x, y)$ . This field can be expanded in terms of the supermodes, as:

$$E_{in}'(x, y) = (a_{in}'/2) [E_{RS}(x, y) + E_{RA}(x, y)]$$

At the junction, the fate of these components is as follows. The symmetric supermode must be gradually converted into the single guide mode. This contribution can be written as  $E_{out}'(x, y) = a_{out}' E(x, y)$ , so that  $a_{out}' = \sqrt{2} (a_{in}'/2)$ . Consequently, the amplitude of the emerging mode is  $1/\sqrt{2}$  times that of the input mode, and hence the power carried by this contribution is half the input power. The antisymmetric supermode cannot be converted into a symmetric distribution by a symmetrically tapered Y-junction, and hence cannot emerge as a guided mode at all. Instead, this component must be converted into antisymmetric radiation modes. Excitation of the Y-junction from the right hand side with a single input therefore results in an automatic loss of 50% of the power to radiation. [4]

Y-junctions can be combined together in binary trees. These allow a single input to be split into  $2N$  outputs, all carrying equal power, with  $N$  levels of two-way splitting. The figure below shows a tree with three levels, which produces eight outputs from a single input. [2]



Radiative stars are advantageous at high port counts, because they are smaller, simpler, have lower loss, and more equal splitting.

[2]

b) Referring to the figure below, and assuming that the Y-junctions are perfect:

i) Fraction of RX light at  $\lambda = 1.5 \mu\text{m}$  detected at photodiode A.

This light must pass Y-junctions 2 and 3. 50% of the light is first diverted to the laser diode by Y-junction 2. Of the remaining 50%, 25% is radiated at Y-junction 3 and 25% reaches photodiode A.

[3]

ii) Fraction of RX light at  $\lambda = 1.3 \mu\text{m}$  detected at photodiode A

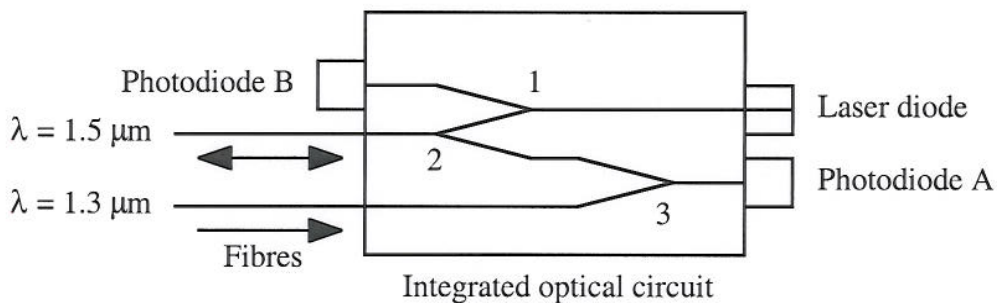
This light must pass only Y-junction 3, where 50% is radiated. The remaining 50% of the light reaches photodiode A.

[2]

ii) Fraction of TX light at  $\lambda = 1.5 \mu\text{m}$  detected at photodiode B.

This light must pass Y-junction 1. 50% reaches photodiode B. Of the remaining 50%, 25% is radiated at Y-junction 2 and 25% emerges from the upper fibre.

[3]

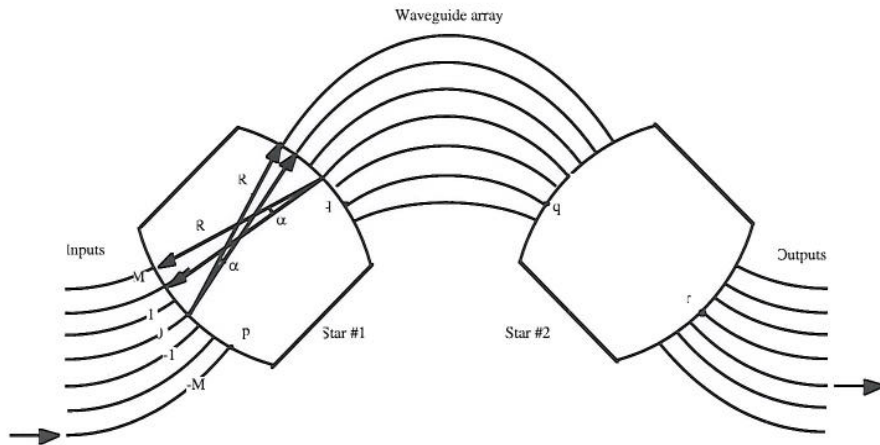




4a). The AWG MUX is constructed from two radiative star components, linked by an array of curved waveguides whose path length varies linearly across the array. The first star distributes the input power (to good approximation) equally across the arrayed waveguide grating. Each component travels through its particular guide to the second star, which performs an amplitude summing operation.

The variation in optical path across the array introduces a linear variation in the phase of the components arriving at the second star. The star itself introduces further linear variations in phase in the sub-components summed at each output port. Different spectral components therefore add in phase for particular output ports. As a result, the spectral components emerge from the ports as a spatially separated array of beams.

[4]



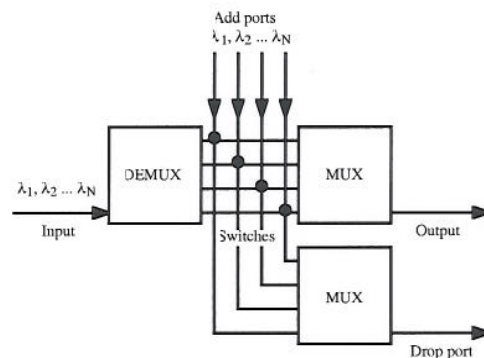
[3]

The AWG-MUX is preferred over other filter types because i) it effectively provides an entire set of similar, closely spaced filters in a single compact device, ii) it can provide both MUX and DEMUX functions, and iii) it can be scaled very simply.

[3]

b) i) An ADD-DROP MUX is needed to add or remove channels from a single, wavelength division multiplexed communication system. An N-channel ADD-DROP MUX requires 3 off AWG MUX components and N off 2 x 2 crosspoint switches, as shown below. The channels are spatially separated using the DEMUX component, then redirected to one of two similar MUX components using an array of crosspoint switches. Spare inputs on the switches allow the ADD channels to be inserted. The MUX components then recombine the different spectral components in an output and a DROP channel.

[3]



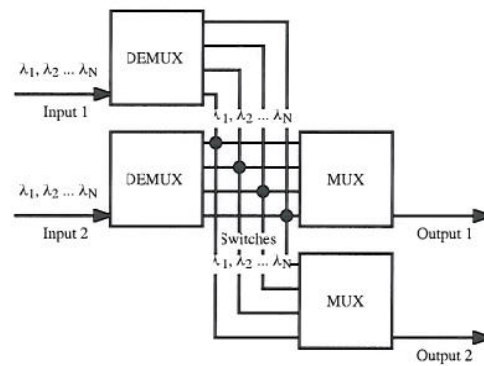
[2]

ii) A wavelength switch is used for arbitrary routing operations, allowing the different wavelengths of two multiplexed channels to be arbitrarily redirected. An N-channel



wavelength switch requires 4 off AWG MUX components and N off 2 x 2 crosspoint switches, as shown below. The channels are spatially separated using the DEMUX components, then redirected to one of two similar MUX components using an array of crosspoint switches.

[3]



[2]

5a) The rate equations for a light-emitting diode are:

$$\begin{aligned} \frac{dn}{dt} &= I/ev - n/\tau_e \\ \frac{d\phi}{dt} &= n/\tau_r - \phi/\tau_p \end{aligned}$$

The physical processes involved are:

$$\begin{aligned} \frac{dn}{dt} & \text{ Rate of change of electron density} \\ \frac{d\phi}{dt} & \text{ Rate of change of photon density} \end{aligned}$$

$$\begin{aligned} I/ev & \text{ Rate of injection of electrons into the active volume} \\ -n/\tau_e & \text{ Rate of loss of electron density by all forms of recombination, including the} \\ & \text{generation of phonons (heat)} \\ n/\tau_r & \text{ Rate of increase in photon density by radiative recombination (band-to-band} \\ & \text{transition involving the loss of an electron-hole pair and the generation of a photon} \\ & \text{whose energy is equal to the difference in energy between the electron and the hole).} \\ -\phi/\tau_p & \text{ Rate of loss of photon density through escape from the LED surface} \end{aligned}$$

[4]

The modifications required to describe a semiconductor laser are:

- i) Inclusion of a coupling term  $\beta$  (of order  $10^{-3}$ ) which describes the fraction of the spontaneously emitted light coupled into the laser cavity
- ii) Inclusion of a term  $G\phi(n - n_0)$  in each equation to describe stimulated emission and absorption. Here  $G$  is the gain constant, and  $n_0$  is the electron concentration needed for transparency.

Modified equations are:

$$\begin{aligned} \frac{dn}{dt} &= I/ev - n/\tau_e - G\phi(n - n_0) \\ \frac{d\phi}{dt} &= \beta n/\tau_r - \phi/\tau_p + G\phi(n - n_0) \end{aligned}$$

[4]

For a LED, the photon lifetime is roughly  $\tau_p \approx Ln/c$ , where  $n$  is now the refractive index (NB: do not confuse with the electron density),  $L$  is the transit distance from the active volume to the surface and  $c$  is the velocity of light.

In a surface emitting LED,  $L$  might be  $1 \mu\text{m}$  and the refractive index might be 3.5, so  $\tau_p \approx 10^{-6} \times 3.5/3 \times 10^8 = 1.2 \times 10^{-14}$  sec.

[4]

For a laser, the photon lifetime is found as follows. Each transit of the cavity takes a time  $T = 2L/v_g$ , where  $v_g$  is the group velocity. The power remaining in the cavity after  $N$  transits is  $(R_1^2 R_2^2)^N$ . The number of transits needed to reduce the power to  $1/e$  of its initial value is found from  $(R_1^2 R_2^2)^N = 1/e$ , or  $N = 1/\{2 \log_e(1/R_1 R_2)\}$ . These transits take a time  $\tau_p = NT$ . The photon lifetime is then  $\tau_p = L/\{v_g \log_e(1/R_1 R_2)\}$ .

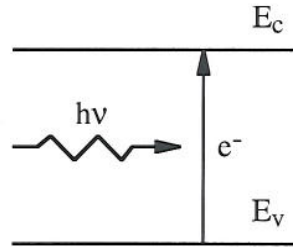
[4]

In a laser,  $L$  might be  $250 \mu\text{m}$  and the refractive index might again be 3.5, so that  $v_g = c/n = 0.857 \times 10^8$  m/s. The end-facet reflectivities would then be  $R_1 = R_2 = (n - 1)/(n + 1) = 0.555$ .

The photon lifetime is then  $\tau_p = 250 \times 10^{-6} / \{0.857 \times 10^8 \log_e(1/0.555^2)\} = 2.477 \times 10^{-12}$  s = 2.5 psec.

[4]

6a) Optical absorption in semiconductors involves the promotion of an electron from the valence band to the conduction band, by a photon whose energy  $hf = hc/\lambda$  is greater than the energy  $eE_g$  required for the band to band transition.

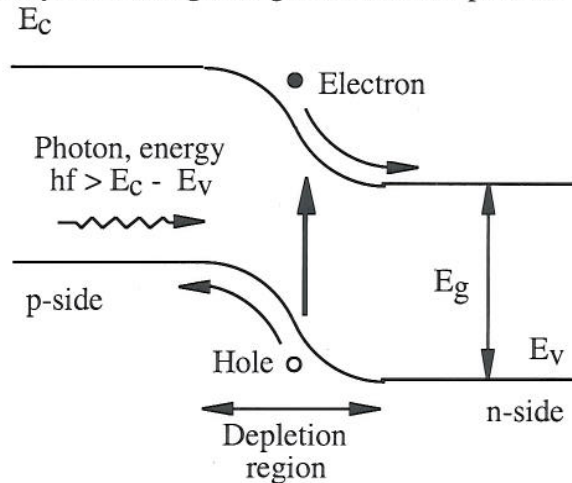


[2]

The main limitation of photoconductive detectors is that the electron-hole pairs thus created can recombine with each other before they reach the contacts. Such detectors are therefore inefficient, unless the recombination time is made very large, when they become slow.

[2]

The limitation above is overcome in diode structures, which use the strong electric field in the depletion layer to separate the electron hole pairs and prevent them recombining. PIN diodes are preferred over simple PN junctions, because the central intrinsic layer effectively stretches the depletion layer, allowing a longer distance for photon capture.



[2]

b) The importance of InGaAsP is that it is a direct gap material, with a wide range of compositions that are lattice matched to InP. Consequently, laser and detector structures operating over a wide spectral range may be constructed by epitaxial growth. The spectral range includes both  $\lambda = 1.3 \mu\text{m}$  and  $\lambda = 1.55 \mu\text{m}$ , the wavelengths used in optical fibre communications for minimum dispersion and minimum loss.

[2]

The locus of lattice matched compositions is the straight line linking InP and  $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$ . InGaAsP therefore allows the construction of devices with bandgaps between  $E_g = 1.35 \text{ eV}$  (InP) and  $0.74 \text{ eV}$  ( $\text{In}_{0.57}\text{Ga}_{0.43}\text{As}$ ).

[2]

The corresponding photon wavelengths are found from

$$eE_g = hc/\lambda, \text{ so that } \lambda = hc/eE_g$$

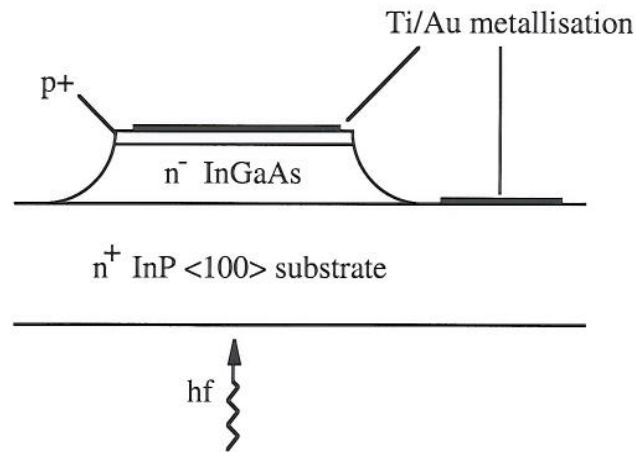
$$\lambda_{\min} = 6.62 \times 10^{-34} \times 3 \times 10^8 / (1.6 \times 10^{-19} \times 1.35) = 9.19 \times 10^{-7} \text{ m, or } 0.92 \mu\text{m}$$

$$\lambda_{\max} = 6.62 \times 10^{-34} \times 3 \times 10^8 / (1.6 \times 10^{-19} \times 0.74) = 1.67 \times 10^{-6} \text{ m, or } 1.67 \mu\text{m}$$

c) The substrate must be the binary compound InP, since this is easiest to grow. The light should be incident through this layer since it has the largest bandgap. [2]

The layer used for detection must be the central layer in a PIN structure. This layer should have the smallest bandgap, for maximum spectral range, and hence should be InGaAs. [2]

P<sup>+</sup> and n<sup>+</sup> layers should be used to allow ohmic (rather than Schottky) contacts to the metallisation. The overall structure should therefore be as below. [2]



[2]