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ISE3.9

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

CONTROL ENGINEERING

Tuesday, 29 April 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

Corrected Copy

Any special instructions for invigilators and information for candidates are on page 1.

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Second Marker(s) : A. Astolfi

No special instructions for invigilators or instructions for candidates for this paper.

1(a). State Nyquist's encirclement theorem, concerning the number of 'unstable' closed loop poles of unity feedback control system. You should take account of the (possible) presence of unstable open loop poles, and also specify the direction in which encirclements are considered positive. [2]

The unity feedback control system of *Figure 1* has forward path transfer function

$$G(s) = \frac{K(s+1)}{(s-1)^2}.$$

in which the gain K is an adjustable parameter.

Sketch the Nyquist diagram of $G(s)$ for $K = 1$ (You should calculate, and show on the diagram, intercepts with both real and imaginary axes.) [12]

Determine from the diagram the ranges of the gain parameter $K(\geq 0)$ for which the closed loop system is stable and for which it is unstable. [2]

1(b). Choose K such that the closed loop control system of *Figure 1* is critically stable. For this value of K, consider a modified closed loop control system in which the forward path transfer function is replaced by

$$\tilde{G}(s) = (1 + as)^{-1}G(s).$$

(Here $a > 0$ is a small positive constant). By considering how the original Nyquist diagram is changed by the extra pole, assess the stability of the modified control system, *i.e.* determinine whether the presence of a 'small', unmodelled, first order lag is stabilizing or de-stabilizing. [4]

(No calculations are involved in answering this part of the question.)

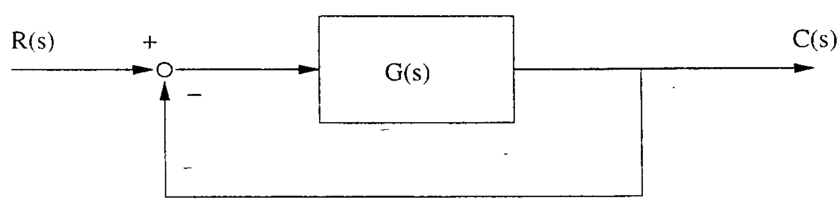

Figure 1

2. *Figure 2* illustrates a control system for controlling a rotating shaft, in which $G_c(s)$ is a proportional + phase advance compensator

$$G_c(s) = K\frac{(1+s/\omega_0)}{(1+s/\omega_1)} \qquad\qquad 0<\omega_0<\omega_1 .$$

($K>0$, ω_0 and ω_1 are design constants.)

(a) Suppose that the compensated control system has gain cross-over frequency $\bar{\omega}$ and phase margin ϕ_m. Show that

$$cos(\theta_1)K|G(j\bar{\omega})| = cos(\theta_0) \qquad \text{and} \qquad \theta_0-\theta_1 = \theta \qquad (1)$$

where

$$\theta_0 = \tan^{-1}\left(\frac{\bar{\omega}}{\omega_0}\right), \quad \theta_1 = \tan^{-1}\left(\frac{\bar{\omega}}{\omega_1}\right) \quad \text{and} \quad \theta = -180^\circ+\phi_m-\angle G(j\bar{\omega}) .$$

[4]

(b) Now assume that

$$G(s) = \frac{4}{s(s+1)(s+2)} .$$

Choose K, ω_0 and ω_1 to meet the following specifications:

(i) (Steady state response)

$$\lim_{t\to\infty} r(t)-c(t) = 0.5 \text{ rs}^{-1}$$

when $r(t)$ is a unit ramp ($r(t)=t$, $t\geq 0$).

(ii) (Speed of response) Gain cross-over frequency $\bar{\omega}=1.7$ rs^{-1}

(iii) (Robustness) Phase margin $\phi_m=40°$. [13]

Briefly comment on the magnitude of ω_1/ω_0 and any expected difficulties in the implementation of this controller. [3]

Hint: In (b), you should use the fact, which you do not have to show, that eqns. (1) can be solved for $\bar{\omega}/\omega_0$ and $\bar{\omega}/\omega_1$ (for fixed K and $\bar{\omega}$) to give

$$\bar{\omega}/\omega_0 = \frac{1-K|G(j\bar{\omega})|\cos(\theta)}{K|G(j\bar{\omega})|\sin(\theta)} \quad \textit{and} \quad \bar{\omega}/\omega_1 = \frac{\cos(\theta)-K|G(j\bar{\omega})|}{\sin(\theta)}$$

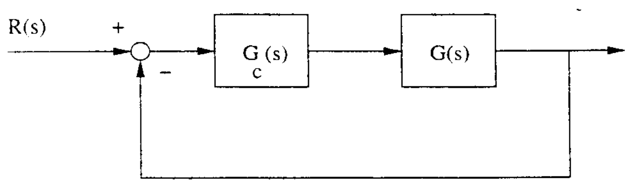

Figure 2

3 Regard the electrical circuit of *Figure 3* as a dynamical system, with zero control input and with output $v_0(t)$, the voltage across the central resistor. All resistances have impedance 1 Ω. Furthermore, $C = 1$ F and $L = 1$ H.

(a) Taking the state variables to be the capacitor voltage v_C and the inductor current i_L, show that the system and output equations are

$$\begin{cases} dx(t)/dt &= Ax(t) \\ y(t) &= c^T x(t) \end{cases}$$

where

$$x(t) = [v_C(t) \quad i_L(t)]^T, \quad y(t) = v_0(t)$$

and

$$A = \begin{bmatrix} -1/2 & -1/2 \\ +1/2 & -3/2 \end{bmatrix} \quad \text{and} \quad c^T = [1/2 \quad -1/2] \ .$$

[14]

(b) Show that the system is not observable. [3]

(c) Show, furthermore, that if the (possibly non-zero) initial values of the state variables satisfy

$$v_C(0) = i_L(0)\,,$$

then

$$v_0(t) = 0 \quad \text{for all } t \geq 0\,.$$

[3]

Hint: Derive a scalar differential equation for $v_0(t)$.

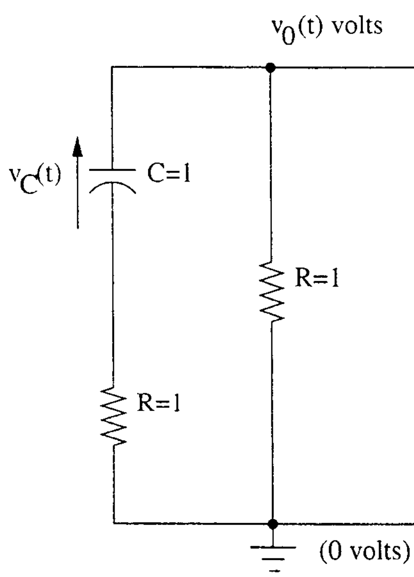

Figure 3

4(a). A system, relating control signals $u(t)$ to the output signals $y(t)$, has transfer function

$$G(s) = \frac{1}{s(s+1)^2}.$$

Provide a state space realisation of this system, in which the state variables are taken to be $y(t)$, $dy(t)/dt$ and d^2y/dt^2. Design a state feedback controller for this realisation, which places the closed loop poles at the locations:

$$-3+0j,\ -2+j,\ -2-j.$$

[12]

Find the controller transfer function $D(s)$ in the control system of *Figure 4(a)* that gives rise to the same closed loop pole locations. [4]

4(b). A proportional + derivative + integral controller, with transfer function

$$D_1(s) = K(1 + T_D s + 1/T_I s),$$

is proposed for a control system with plant transfer function

$$G(s) = \frac{1}{(s+1)^2}$$

to improve transient response and to eliminate state errors for constant reference signals. (See *Figure 4(b)*.) Choose controller parameters K, T_D and T_I to place the closed loop poles at

$$-3+0j,\ -2+j,\ -2-j.$$

[4]

Hint: Use the results of part (a).

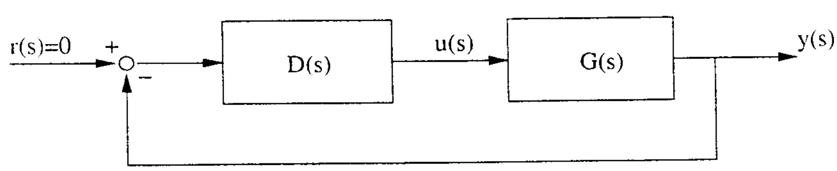

Figure 4(a)

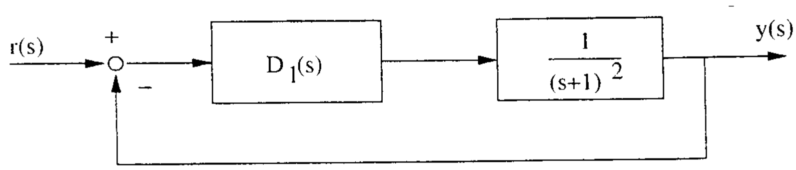

Figure 4(b)

5. Consider the double integrator system

$$d^2y/dt^2 = u(t)$$

relating the displacement $y(t)$ of a unit mass to the applied force $u(t)$.

Determine the constants k_1 and k_2 in the feedback control law

$$u(t) = -(k_1 y(t) + k_2 dy(t)/dt)$$

to minimize the cost

$$\int_0^\infty \left[\alpha y^2(t) + u^2(t)\right] dt\,,$$

for fixed initial values $y(0)$, $\dot{y}(0)$ of the 'state variables' y and $\dot{y}$. Here, $\alpha > 0$ is a fixed design parameter. [18]

To solve this problem, you should reformulate the optimization problem as

$$(OCP) \quad \begin{cases} \text{Minimize } \int_0^\infty \left[\mathbf{x}^T(t)Q\mathbf{x}(t) + u^2(t)\right] dt \\ \text{over control functions } u(.) \text{ and state trajectories } \mathbf{x}(.) \\ \text{such that} \\ d\mathbf{x}(t)/dt = A\mathbf{x}(t) + \mathbf{b}u(t) \\ \mathbf{x}(0) = \mathbf{x}_0\,, \end{cases}$$

for suitably chosen 2×2, 2×2, 2×1, 2×1 matrices Q, A, $\mathbf{b}$, $\mathbf{x}_0$ respectively. (Q is symmetric and positive semi-definite.)

Then use the fact that the solution to (OCP) (in feedback form) is

$$u = -\mathbf{b}^T P\, \mathbf{x},$$

where P is a symmetric, positive definite solution of the Matrix Riccati equation:

$$A^T P + PA + Q - P\mathbf{b}\mathbf{b}^T P = 0\,.$$

Describe the nature of the closed loop transient response of the controlled double integrator system, for the controller you have designed, as $\alpha \to \infty$. [2]

6. A certain nonlinear device, labelled NL, has a parabolic characteristic:

$$f(e) = \begin{cases} e^2 & \text{if } e \geq 0 \\ -e^2 & \text{if } e < 0 \,. \end{cases}$$

Derive the describing function $N(A)$ for this device. [5]

Hint: use the identity $\int_0^\theta \sin^3(\theta')d\theta' = -\cos(\theta) + \frac{1}{3}\cos^3(\theta) + const.$

The nonlinearity device is present in the forward path of the control system of *Figure 6*, in which $G(s)$ is the transfer function

$$G(s) = \frac{(s+4)^2}{s^2(s+1)} \,.$$

Use the describing function method to predict the frequency of a limit cycle oscillation and also its amplitude at the output. [12]

Determine whether the limit cycle is stable or unstable. [3]

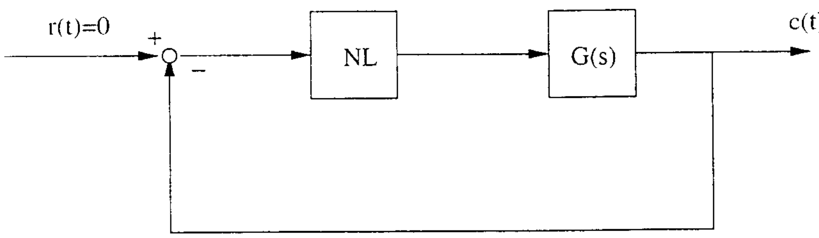

Figure 6