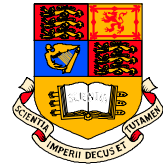


**IMPERIAL COLLEGE  
LONDON**

**[E303/ISE3.3]**



**DEPARTMENT of ELECTRICAL and ELECTRONIC ENGINEERING  
EXAMINATIONS 2004**

**EEE/ISE PART III/IV: M.Eng., B.Eng. and ACGI**

# **SOLUTIONS 2004**

## **COMMUNICATION SYSTEMS**

## **ANSWER to Q1**

- 1)    **A    B    C    D    E**
- 2)    **A    B    C    D    E**
- 3)    **A    B    C    D    E**
- 4)    **A    B    C    D    E**
- 5)    **A    B    C    D    E**
- 6)    **A    B    C    D    E**
- 7)    **A    B    C    D    E**
- 8)    **A    B    C    D    E**
- 9)    **A    B    C    D    E**
- 10)   **A    B    C    D    E**
- 11)   **A    B    C    D    E**
- 12)   **A    B    C    D    E**
- 13)   **A    B    C    D    E**
- 14)   **A    B    C    D    E**
- 15)   **A    B    C    D    E**
- 16)   **A    B    C    D    E**
- 17)   **A    B    C    D    E**
- 18)   **A    B    C    D    E**
- 19)   **A    B    C    D    E**
- 20)   **A    B    C    D    E**

## ANSWER to Q2

a)

$$\left. \begin{array}{l} b_0 = -4V \\ b_4 = +4V \end{array} \right\} \Rightarrow \Delta = \frac{b_4 - b_0}{Q} = 2V$$

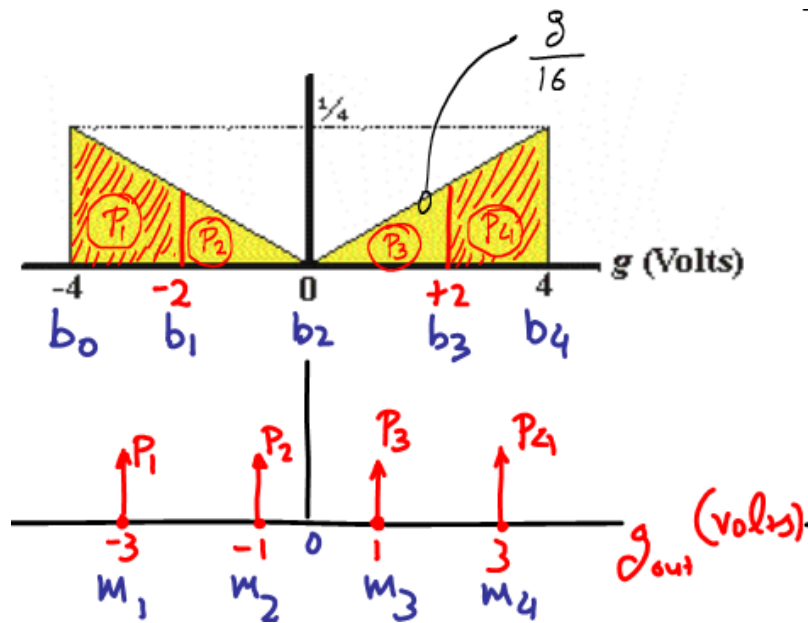
$$Q = 4$$

∴ end points

$b_0 = -4V$	& o/p level	$m_1 = -3V$
$b_1 = -2V$		$m_2 = -1V$
$b_2 = 0V$		$m_3 = 1V$
$b_3 = 2V$		$m_4 = 3V$
$b_4 = 4V$		

[5]

b)



$$P_1 = P_4$$

$$P_2 = P_3$$

$$\begin{aligned}
 P_3 &= \Pr(g_{out} = 1V) = \Pr(0 < g < 2V) \\
 &= \int_0^{2V} p_{df_g}(g) dg \\
 &= \int_0^{2V} \frac{g}{16} dg \\
 &= \frac{1}{16} \left[ \frac{g^2}{2} \right]_0^{2V} = \frac{1}{8} = P_2
 \end{aligned}$$

$$\begin{aligned}
 P_4 &= \Pr(g_{out} = 3V) = \Pr(2 < g < 4V) \\
 &= \int_2^{4V} p_{df_g}(g) dg \\
 &= \int_2^{4V} \frac{g}{16} dg \\
 &= \frac{1}{16} \left[ \frac{g^2}{2} \right]_2^{4V} = \frac{3}{8} = P_1
 \end{aligned}$$

$$\therefore P_{g_{out}} = (-3)^2 \cdot P_1 + (-1)^2 \cdot P_2 + 1^2 \cdot P_3 + 3^2 \cdot P_4 = \frac{56}{8} = 7.$$

$$P_{Mq} = \frac{\Delta^2}{12} = \frac{4}{12} = \frac{1}{3} = 0.333V$$

$$SNR_q = \frac{7}{1/3} = 21$$

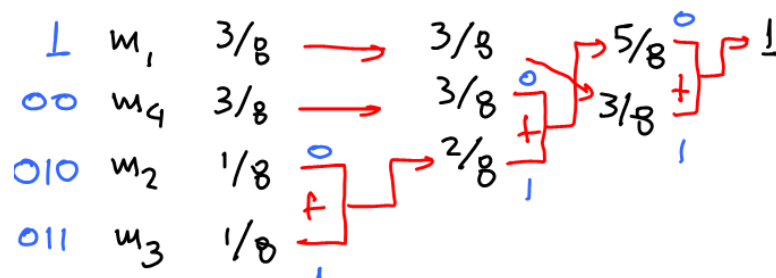
[6]

c)

$$\begin{aligned}
 H_M &= -2 \left( \frac{1}{8} \log_2 \frac{1}{8} + \frac{3}{8} \log_2 \frac{3}{8} \right) \\
 &= 1.8113 \frac{\text{bits}}{\text{symbol}}
 \end{aligned}$$

[3]

d)



[7]

e)

$$\begin{aligned}\bar{\ell} &= 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} + 3 \times \frac{1}{8} \\ &= \frac{15}{8} = 1.8750 \frac{\text{bits}}{\text{symbol}}\end{aligned}$$

[3]

f)

$$r_{inf} = \underbrace{r_m}_{2 \times 8k} \times \underbrace{H_M}_{1.8113} = 28.9804 \frac{\text{bits}}{\text{sec}}$$

[6]

$$r_{data} = r_m \times \underbrace{\bar{\ell}}_{1.8750} = 30 \frac{\text{bits}}{\text{sec}}$$

## ANSWER to Q3

a)

$$P_e = \underbrace{\Pr(r_2|m_1) \cdot \Pr(m_1)}_{\Pr(r_2, m_1)} + \underbrace{\Pr(r_1|m_2) \cdot \Pr(m_2)}_{\Pr(r_1, m_2)} = \underbrace{0.1 \times 0.25}_{0.025} + \underbrace{0.2 \times 0.75}_{0.15} = 0.175 \quad [6]$$

b)

$$\underline{q} = \begin{bmatrix} \Pr(r_1) \\ \Pr(r_2) \end{bmatrix} = \underline{F} \cdot \underline{p} = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix} \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix} = \begin{bmatrix} 0.375 \\ 0.625 \end{bmatrix}$$
$$\underline{J} = \underline{F} \cdot \text{diag}(\underline{p}) = \begin{bmatrix} 0.9 \times 0.25 & 0.2 \times 0.75 \\ 0.1 \times 0.25 & 0.8 \times 0.75 \end{bmatrix} = \begin{bmatrix} 0.225 & 0.15 \\ 0.025 & 0.6 \end{bmatrix}$$

[12]

$$H_{R|M} = -\sum_{\underline{r}} \underline{q} \odot \log_2 \underline{q} \Big|_{\underline{r}} = 0.658695$$

c)

$$H_{Mut} = H_R - H_{R|M}$$
$$H_R = \underline{q}^T \cdot \log_2(\underline{q}) = 0.9544$$
$$\therefore H_{Mut} = 0.9544 - 0.658695 = 0.295705$$

[12]

## ANSWER to Q4

a) code rate : 1/3  
 constraint length 3

[5]

b) generator polynomials

- 1)  $1+D^2$
- 2)  $D^2$
- 3)  $1+D+D^2$

[6]

c)

$$G_c = \begin{bmatrix} 101 & 001 & 111 & 000 & 000 & \dots \\ 000 & 101 & 001 & 111 & 000 & \dots \\ 000 & 000 & 101 & 001 & 111 & \dots \\ \vdots & \text{etc} & \dots & \dots & \dots & \dots \end{bmatrix}$$

[9]

d)

if  $x = \begin{bmatrix} \text{first} & \text{last} \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

then  $x \cdot G_c = [101001010 \text{ etc}]$

[10]