IMPERIAL COLLEGE LONDON **[E303/ISE3.3]**

DEPARTMENT of ELECTRICAL and ELECTRONIC ENGINEERING
EXAMINATIONS 2004

EEE/ISE PART III/IV: M.Eng., B.Eng. and ACGI

COMMUNICATION SYSTEMS

There are FOUR questions (Q1 to Q4)

Answer question ONE (in separate booklet) and TWO other questions.

Question 1 has 20 multiple choice questions numbered 1 to 20, all carrying equal marks. There is only one correct answer per question.

Distribution of marks

Question-1: 40 marks
Question-2: 30 marks
Question-3: 30 marks
Question-4: 30 marks

The following are provided:

- ***A table of Fourier Transforms***
- ***A "Gaussian Tail Function" graph***

Examiner: Dr A. Manikas

2nd Marker: Dr P. L. Dragotti

Information for candidates:

The following are provided on pages 2 and 3:

- a table of Fourier Transforms;
- a graph of the 'Gaussian Tail Function'.

Question 1 is in a separate coloured booklet which should be handed in at the end of the examination.

You should answer Question 1 on the separate sheet provided. At the end of the exam, please tie this sheet securely into your main answer book(s).

Special instructions for invigilators:

Please ensure that the three items mentioned below are available on each desk.

- the main examination paper;
- the coloured booklet containing Question 1;
- the separate answer sheet for Question 1.

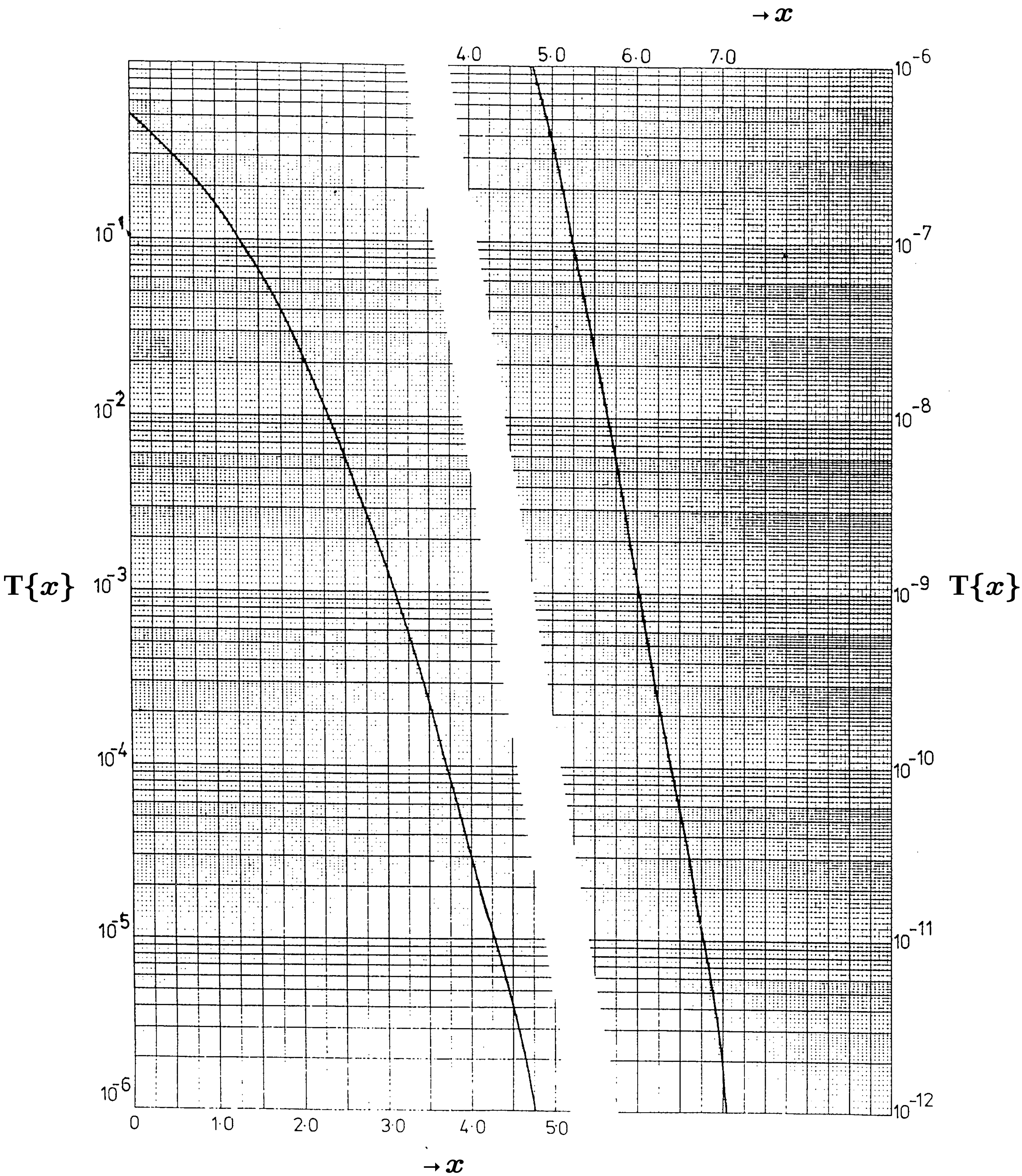
Please remind candidates at the end of the exam that they should tie their Answer Sheet for Question 1 securely into their main answer book, together with supplementary answer books etc.

Please tell candidates they must **NOT** remove the coloured booklet containing Question 1. Collect this booklet in at the end of the exam, along with the standard answer books.

Tail Function Graph

The graph below shows the Tail function $\mathbf{T}\{x\}$ which represents the area from x to ∞ of the Gaussian probability density function N(0,1), i.e.

$$\mathbf{T}\{x\} = \int_x^{\infty} \frac{1}{\sqrt{2\pi}}.\exp\left(-\frac{y^2}{2}\right)dy$$



Note that if $x > 6.5$ then $\mathbf{T}\{x\}$ may be approximated by $\mathbf{T}\{x\} \approx \frac{1}{\sqrt{2\pi}.x}.\exp\left\{-\frac{x^2}{2}\right\}$

FOURIER TRANSFORMS - TABLES

	DESCRIPTION	FUNCTION	TRANSFORM
1	Definition	$g(t)$	$G(f) = \int_{-\infty}^{\infty} g(t).e^{-j2\pi ft}dt$
2	Scaling	$g(\frac{t}{T})$	$\lvert T\rvert \,.\, G(fT)$
3	Time shift	$g(t-T)$	$G(f)\,.\,e^{-j2\pi fT}$
4	Frequency shift	$g(t)\,.\,e^{j2\pi Ft}$	$G(f-F)$
5	Complex conjugate	$g^*(t)$	$G^*(-f)$
6	Temporal derivative	$\frac{d^n}{dt^n}\,.\,g(t)$	$(j2\pi f)^n.\,G(f)$
7	Spectral derivative	$(-j2\pi t)^n.g(t)$	$\frac{d^n}{df^{\,n}}\,.\,G(f)$
8	Reciprocity	$G(t)$	$g(-f)$
9	Linearity	$A\,.\,g(t) + B\,.\,h(t)$	$A\,.\,G(f) + B\,.\,H(f)$
10	Multiplication	$g(t)\,.\,h(t)$	$G(f) * H(f)$
11	Convolution	$g(t) * h(t)$	$G(f)\,.\,H(f)$
12	Delta function	$\delta(t)$	1
13	Constant	1	$\delta(f)$

	DESCRIPTION	FUNCTION	TRANSFORM
14	Rectangular function	$\mathbf{rect}\{t\} \equiv \begin{cases} 1 & \text{if } \lvert t \rvert < \frac{1}{2} \\ 0 & otherwise \end{cases}$	$\mathbf{sinc}(f) = \frac{sin\,\pi f}{\pi f}$
15	Sinc function	$\mathbf{sinc}(t)$	$\mathbf{rect}(f)$
16	Unit step function	$u(t) = \begin{cases} +1, & t > 0 \\ 0, & t < 0 \end{cases}$	$\frac{1}{2}\delta(f) - \frac{j}{2\pi f}$
17	Signum function	$\mathbf{sgn}(t) = \begin{cases} +1, & t > 0 \\ -1, & t < 0 \end{cases}$	$-\frac{j}{\pi f}$
18	Decaying exponential (two-sided)	$e^{-\lvert t\rvert}$	$\frac{2}{1+(2\pi f)^2}$
19	Decaying exponential (one-sided)	$e^{-\lvert t\rvert}.u(t)$	$\frac{1-j2\pi f}{1+(2\pi f)^2}$
20	Gaussian function	$e^{-\pi t^2}$	$e^{-\pi f^2}$
21	Lambda function	$\Lambda\{t\} \equiv \begin{cases} 1-t & \text{if } \quad 0 \le t \le 1 \\ 1+t & \text{if } -1 \le t \le 0 \end{cases}$	$\mathbf{sinc}^2(f)$
22	Repeated function	$\mathbf{rep}_T\{g(t)\} = g(t) * \mathbf{rep}_T\{\delta(t)\}$	$\lvert\frac{1}{T}\rvert.\mathbf{comb}_{\frac{1}{T}}\{G(f)\}$
23	Sampled function	$\mathbf{comb}_T\{g(t)\} = g(t).\mathbf{rep}_T\{\delta(t)\}$	$\lvert\frac{1}{T}\rvert.\mathbf{rep}_{\frac{1}{T}}\{G(f)\}$

The Questions

1. ***This question is bound separately and has 20 multiple choice questions numbered 1 to 20, all carrying equal marks .***

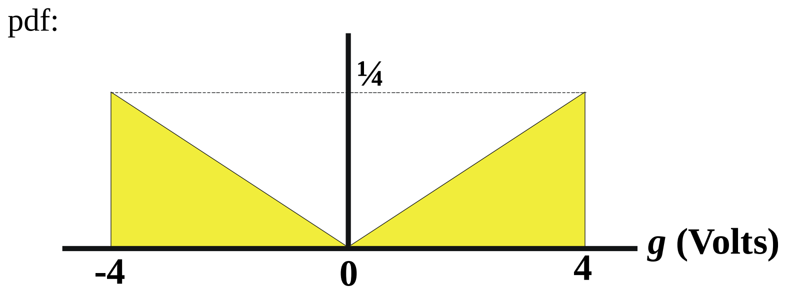
You should answer Question 1 on the separate sheet provided.

Circle the answers you think are correct .

There is only one correct answer per question.

There are no negative marks.

2. A signal $g(t)$ having the probability density function (pdf) shown below is bandlimited to 8 kHz.



The signal is sampled at the Nyquist rate and is fed through a 4-level uniform quantizer.

a) Calculate the *end points* b_i and the quantizer levels m_i of the quantizer. **[5]**

b) Calculate the average signal to *quantization* noise power ratio (SNR_q). **[6]**

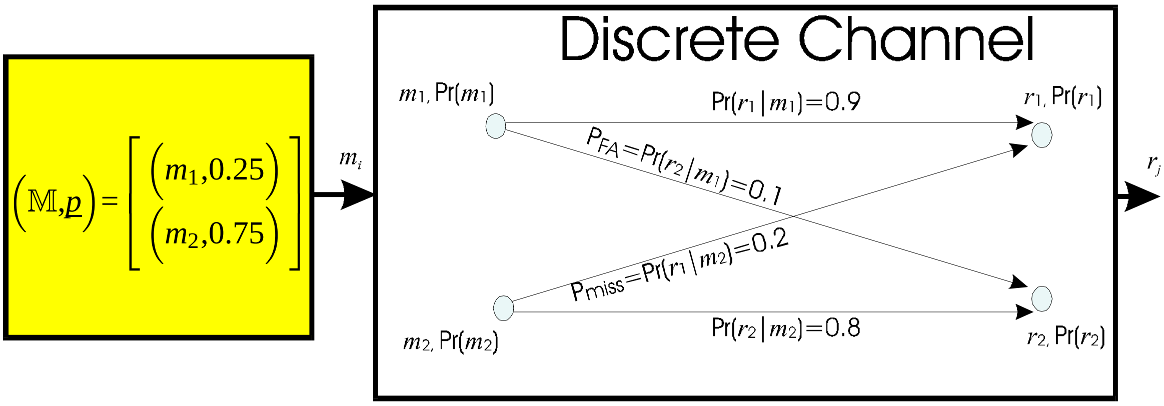
c) Calculate the average information per quantization level. **[3]**

d) Design a prefix source encoder to encode the output levels from the quantizer. **[7]**

e) Find the average codeword length per symbol, i.e. $\overline{l}$, at the output of the source encoder. **[3]**

f) Calculate the information rate and data rate associated with above single-level source encoding approach. **[6]**

3. A discrete channel is modelled as follows:



Estimate:

a) The probability of error at the output of the channel. **[6]**

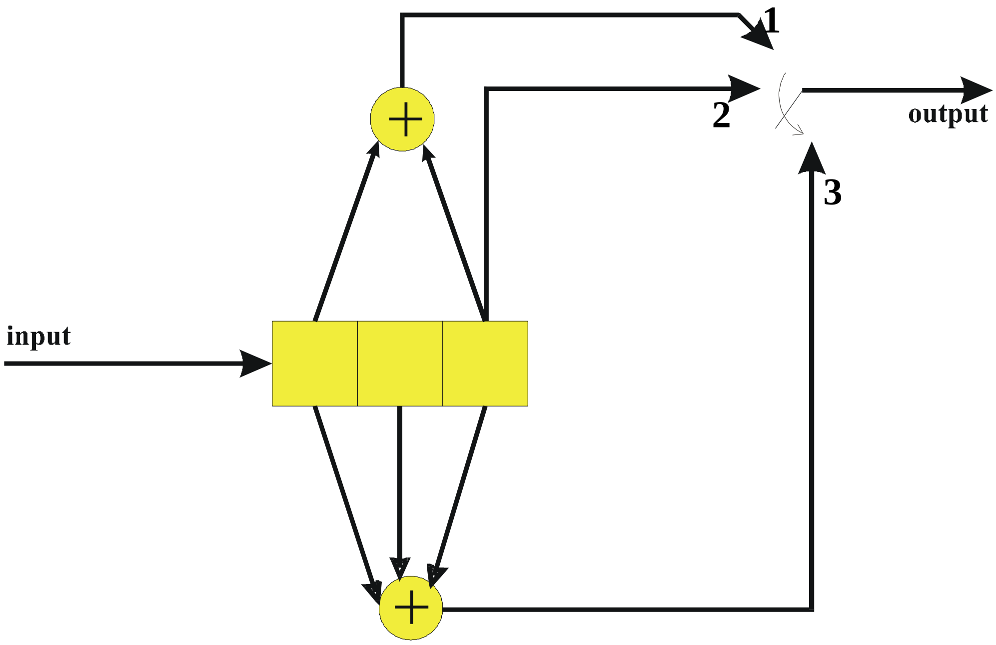
b) The conditional entropy $\mathbf{H}(\underline{\mathbb{R}} \mid \underline{\mathbb{M}})$, where

$$(\underline{\mathbb{R}},\underline{q}) = \left\{ (r_1, \Pr(r_1)), (r_2, \Pr(r_2)) \right\}$$

denotes the ensemble at the channel output. **[12]**

c) The amount of information delivered at the output of the channel. **[12]**

4.



For the convolutional encoder shown in the above figure find

a) the code rate and the constraint length **[5]**

b) the generator polynomials **[6]**

c) the Generator Matrix $\mathbb{G}_c$ **[9]**

d) the encoded output sequence for the input sequence 10110100...
where the first (oldest) input bit is on the left [**10]**

[END]