IMPERIAL COLLEGE
LONDON

[E303/ISE3.3]

DEPARTMENT of ELECTRICAL and ELECTRONIC ENGINEERING
EXAMINATIONS 2003

EEE/ISE PART III/IV: M.Eng., B.Eng. and ACGI

COMMUNICATION SYSTEMS

There are FOUR questions (Q1 to Q4)

Answer question ONE (in separate booklet) and TWO other questions.

Question 1 has 20 multiple choice questions numbered 1 to 20, all carrying equal marks. There is only one correct answer per question.

Distribution of marks

Question-1: 40 marks
Question-2: 30 marks
Question-3: 30 marks
Question-4: 30 marks

The following are provided:

- ***A table of Fourier Transforms***
- ***A "Gaussian Tail Function" graph***

Examiner: Dr A. Manikas

2nd Marker: Dr P. L. Dragotti

Information for candidates:

The following are provided on pages 2 and 3:

- a table of Fourier Transforms;
- a graph of the 'Gaussian Tail Function'.

Question 1 is in a separate coloured booklet which should be handed in at the end of the examination.

You should answer Question 1 on the separate sheet provided. At the end of the exam, please tie this sheet securely into your main answer book(s).

Special instructions for invigilators:

Please ensure that the three items mentioned below are available on each desk.

- the main examination paper;
- the coloured booklet containing Question 1;
- the separate answer sheet for Question 1;

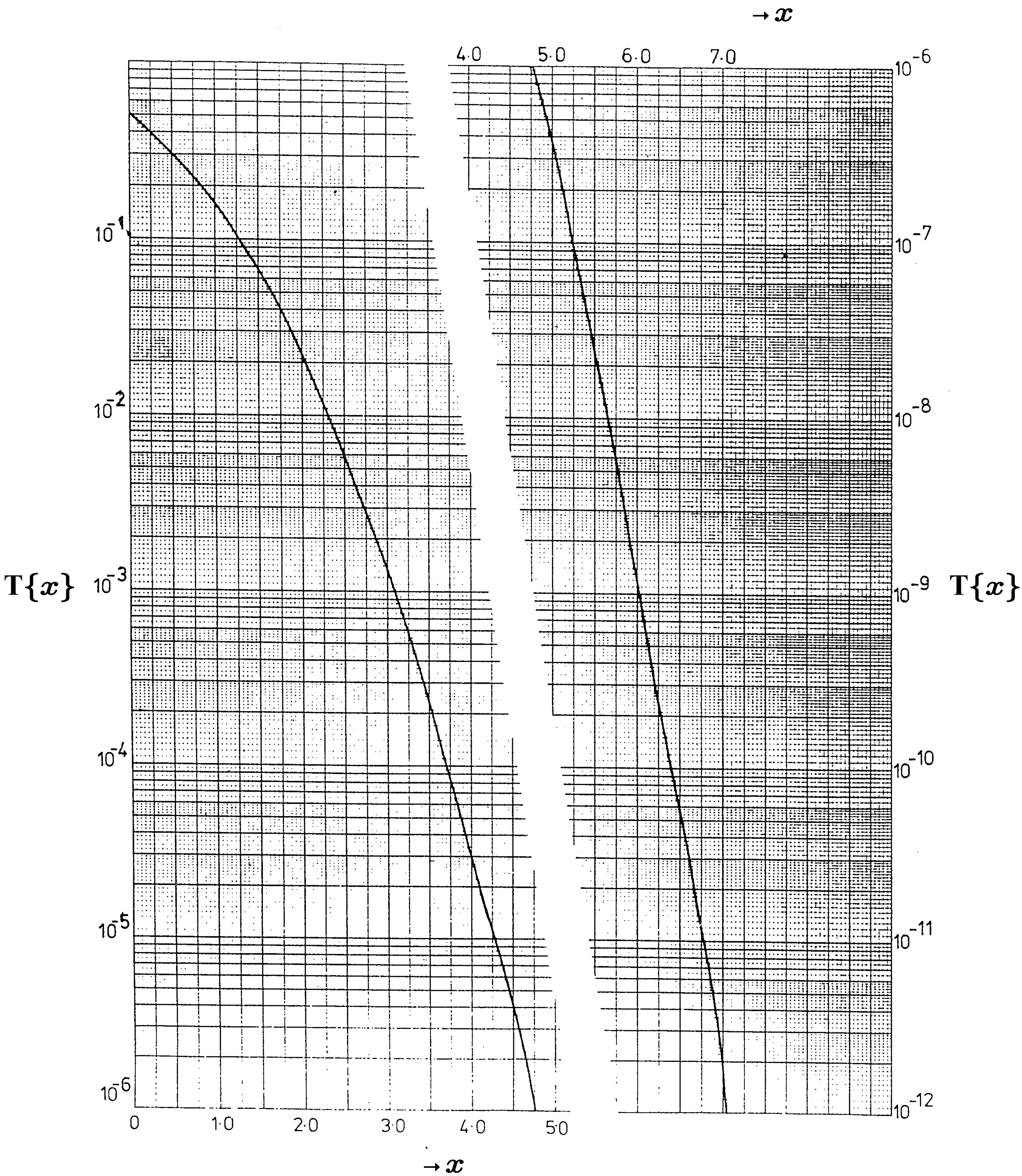
Please remind candidates at the end of the exam that they should tie their Answer Sheet for Question 1 securely into their main answer book, together with supplementary answer books etc.

Please tell candidates they must **NOT** remove the coloured booklet containing Question 1. Collect this booklet in at the end of the exam, along with the standard answer books.

Tail Function Graph

The graph below shows the Tail function $\mathbf{T}\{x\}$ which represents the area from x to ∞ of the Gaussian probability density function N(0,1), i.e.

$$\mathbf{T}\{x\} = \int_x^\infty \frac{1}{\sqrt{2\pi}}.\exp\left(-\frac{y^2}{2}\right)dy$$



Note that if $x > 6.5$ then $\mathbf{T}\{x\}$ may be approximated by $\mathbf{T}\{x\} \approx \frac{1}{\sqrt{2\pi}.x}.\exp\left\{-\frac{x^2}{2}\right\}$

FOURIER TRANSFORMS - TABLES

	DESCRIPTION	FUNCTION	TRANSFORM
1	Definition	$g(t)$	$G(f) = \int_{-\infty}^{\infty} g(t).e^{-j2\pi ft} dt$
2	Scaling	$g(\frac{t}{T})$	$\lvert T\rvert \, . \, G(fT)$
3	Time shift	$g(t-T)$	$G(f) \, . \, e^{-j2\pi fT}$
4	Frequency shift	$g(t) \, . \, e^{j2\pi Ft}$	$G(f-F)$
5	Complex conjugate	$g^*(t)$	$G^*(-f)$
6	Temporal derivative	$\frac{d^n}{dt^n} \, . \, g(t)$	$(j2\pi f)^n . \, G(f)$
7	Spectral derivative	$(-j2\pi t)^n . g(t)$	$\frac{d^n}{df^n} \, . \, G(f)$
8	Reciprocity	$G(t)$	$g(-f)$
9	Linearity	$A \, . \, g(t) + B \, . \, h(t)$	$A \, . \, G(f) + B \, . \, H(f)$
10	Multiplication	$g(t) \, . \, h(t)$	$G(f) * H(f)$
11	Convolution	$g(t) * h(t)$	$G(f) \, . \, H(f)$
12	Delta function	$\delta(t)$	1
13	Constant	1	$\delta(f)$

	DESCRIPTION	FUNCTION	TRANSFORM
14	Rectangular function	$\mathbf{rect}\{t\} \equiv \begin{cases} 1 & \text{if } \lvert t \rvert < \frac{1}{2} \\ 0 & otherwise \end{cases}$	$\mathbf{sinc}(f) = \frac{sin \, \pi f}{\pi f}$
15	Sinc function	$\mathbf{sinc}(t)$	$\mathbf{rect}(f)$
16	Unit step function	$u(t) = \begin{cases} +1, & t > 0 \\ 0, & t < 0 \end{cases}$	$\frac{1}{2}\delta(f) - \frac{j}{2\pi f}$
17	Signum function	$\mathbf{sgn}(t) = \begin{cases} +1, & t > 0 \\ -1, & t < 0 \end{cases}$	$-\frac{j}{\pi f}$
18	Decaying exponential (two-sided)	$e^{-\lvert t\rvert}$	$\frac{2}{1+(2\pi f)^2}$
19	Decaying exponential (one-sided)	$e^{-\lvert t\rvert}.u(t)$	$\frac{1-j2\pi f}{1+(2\pi f)^2}$
20	Gaussian function	$e^{-\pi t^2}$	$e^{-\pi f^2}$
21	Lambda function	$\Lambda\{t\} \equiv \begin{cases} 1-t & \text{if } \quad 0 \le t \le 1 \\ 1+t & \text{if } \, -1 \le t \le 0 \end{cases}$	$\mathbf{sinc}^2(f)$
22	Repeated function	$\mathbf{rep}_T\{g(t)\} = g(t) * \mathbf{rep}_T\{\delta(t)\}$	$\lvert\frac{1}{T}\rvert.\mathbf{comb}_{\frac{1}{T}}\{G(f)\}$
23	Sampled function	$\mathbf{comb}_T\{g(t)\} = g(t).\mathbf{rep}_T\{\delta(t)\}$	$\lvert\frac{1}{T}\rvert.\mathbf{rep}_{\frac{1}{T}}\{G(f)\}$

The Questions

1. ***This question is bound separately and has 20 multiple choice questions numbered 1 to 20, all carrying equal marks .***

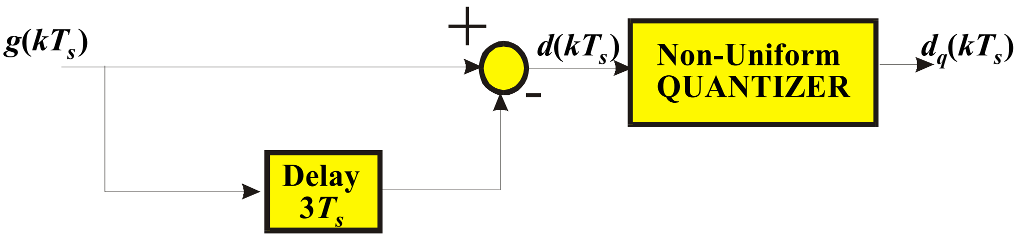
You should answer Question 1 on the separate sheet provided.

Circle the answers you think are correct .

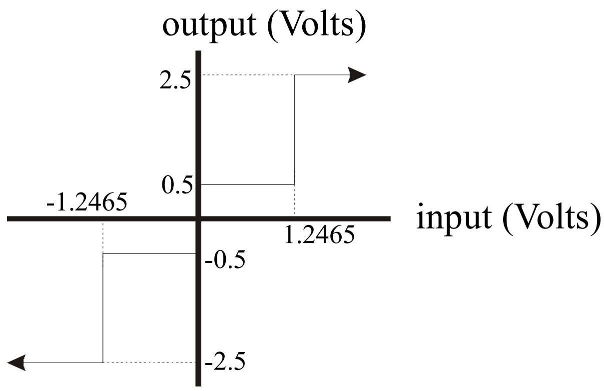
There is only one correct answer per question.

There are no negative marks.

2. Consider a non-white zero-mean Gaussian random source $g(t)$ having an autocorrelation function $R_{gg}(\tau) = \exp(-6000\,|\,\tau\,|)$. The signal is sampled at a rate of 12 ksamples/sec and then is applied at the input of the differential quantizer shown below



where T_s is the sample time, k is an integer. The transfer function of the quantizer is shown below:



a) Calculate the power of the signal $d(kT_s)$. **[5]**

b) Calculate and sketch the pdf of the signal $d_q(kT_s)$ at the output of the quantizer. **[5]**

c) Design a prefix source encoder to encode the output levels from the quantizer. **[10]**

d) Find the information bit rate and the data bit rate at the output of the source encoder. **[10]**

3. Consider a PCM system where its quantizer consists of an *A-law* compander (with $A = 87.6$) followed by a uniform quantizer with "end points" b_i, and "output levels" m_i. The maximum value of the input signal, which is sampled at 18 ksamples/sec, is $5\,Volts$ and the input/output characteristics of the uniform quantizer are given in the following tables:

b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
-5V	-3.75V	-2.5V	-1.25V	0V	1.25V	2.5V	3.75V	5V

m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8
-4.37V	-3.125V	-1.875V	-0.625V	0.625V	1.875V	3.125V	4.375V

a) If the signal at the output of the sampler, at time kT_s, is equal to $-3.7\,Volts$, estimate the instantaneous quantization noise $n_q(kT_s)$ **[20]**

b) Estimate the average Signal-to-Quantization-Noise Ratio (SNR_q). **[5]**

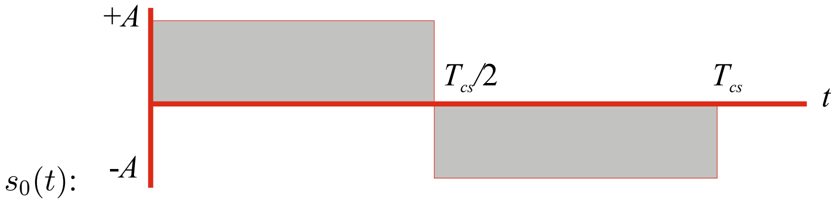
c) How many hours of the audio signal correspond to 2GBytes of PCM data? **[5]**

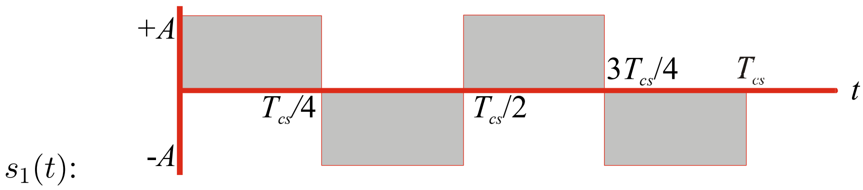
Note that *A-law* compression is defined as follows:

$$\text{output} = \begin{cases} \frac{A.|x|}{1+lnA} & 0 < |x| < \frac{1}{A} \\ \frac{1+ln(A.|x|)}{1+lnA} & \frac{1}{A} < |x| < 1 \end{cases}$$

$$\text{where } x = \frac{\text{input value in } Volts}{\text{maximum input value in } Volts}$$

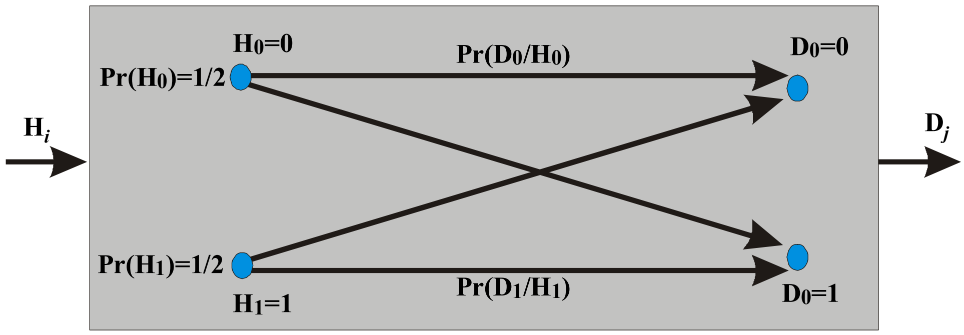
4. Consider a 166.6667 kbits/second binary source of 1's and 0's with the number of ones being equal to the number of zeros. The binary sequence is fed to a triple repetition channel encoder and then to a digital modulator which employs the following two energy signals





with a *one* being sent as the signal (channel symbol) $s_1(t)$ and *zero* being sent as $s_0(t)$. The transmitted signal is corrupted by additive white Gaussian channel noise having a double-sided power spectral density of 10^{-12} W/Hz. The received signal is processed by a matched filter receiver followed by a 'majority logic' channel decoder.

The figure below shows the discrete channel which models the system from the channel encoder's input to the channel decoder's output:



If the bit error rate at the output of the matched filter is 0.3, find:

a) the time duration T_{cs} of a channel symbol **[5]**

b) the amplitude A at the receiver's input. **[10]**

c) the probability that a bit is correctly detected at the output of the channel decoder. **[5]**

d) the forward transition channel-matrix $\mathbb{F}$. **[5]**

e) the joint-probability channel-matrix $\mathbb{J}$, i.e. the matrix with elements the probabilities $\Pr(\mathrm{H}_i, \mathrm{D}_j)\ \forall i,j$ **[5]**

[END]