IMPERIAL COLLEGE OF SCIENCE TECHNOLOGY AND MEDICINE **[E303/ISE3.3]**
UNIVERSITY OF LONDON

DEPARTMENT of ELECTRICAL and ELECTRONIC ENGINEERING
EXAMINATIONS 2002

EEE/ISE PART III/IV: M.Eng., B.Eng. and ACGI

COMMUNICATION SYSTEMS

There are FOUR questions (Q1 to Q4)

Answer question ONE (in separate booklet) and TWO other questions.

Question 1 has 20 multiple choice questions numbered 1 to 20, all carrying equal marks. There is only one correct answer per question.

Distribution of marks

Question-1: 40 marks
Question-2: 30 marks
Question-3: 30 marks
Question-4: 30 marks

The following are provided:

- ***A table of Fourier Transforms***
- ***A "Gaussian Tail Function" graph***

Examiner: Dr A. Manikas

Information for candidates:

The following are provided:

- a table of Fourier Transforms;
- a graph of the 'Gaussian Tail Function'.

Question 1 is in a separate coloured booklet which should be handed in at the end of the examination.

You should answer Question 1 on the separate sheet provided. At the end of the exam, please tie this sheet securely into your main answer book(s).

Special instructions for invigilators:

Please ensure that the five items mentioned below are available on each desk.

- the main examination paper;
- the coloured booklet containing Question 1;
- the separate answer sheet for Question 1;
- a table of Fourier Transforms;
- a graph of the Gaussian Tail Function.

Please remind candidates at the end of the exam that they should tie their Answer Sheet for Question 1 securely into their main answer book, together with supplementary answer books etc.

Please tell candidates they must **NOT** remove the coloured booklet containing Question 1. Collect this booklet in at the end of the exam, along with the standard answer books.

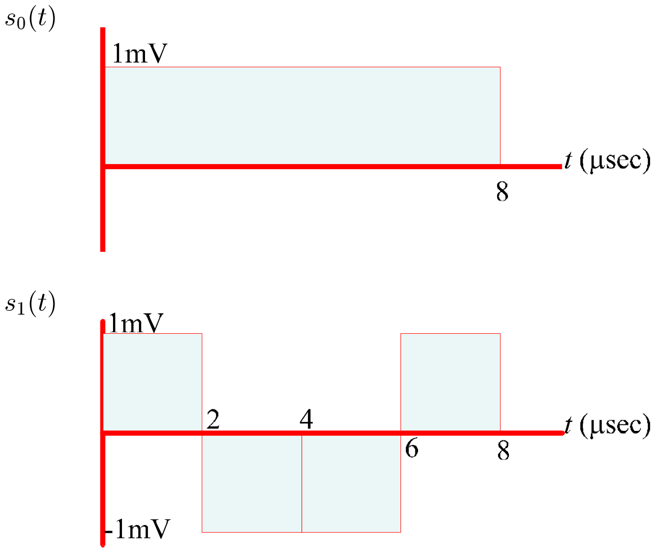
1. ***This question is bound separately and has 20 multiple choice questions numbered 1 to 20, all carrying equal marks .***

You should answer Question 1 on the separate sheet provided.

Circle the answers you think are correct .

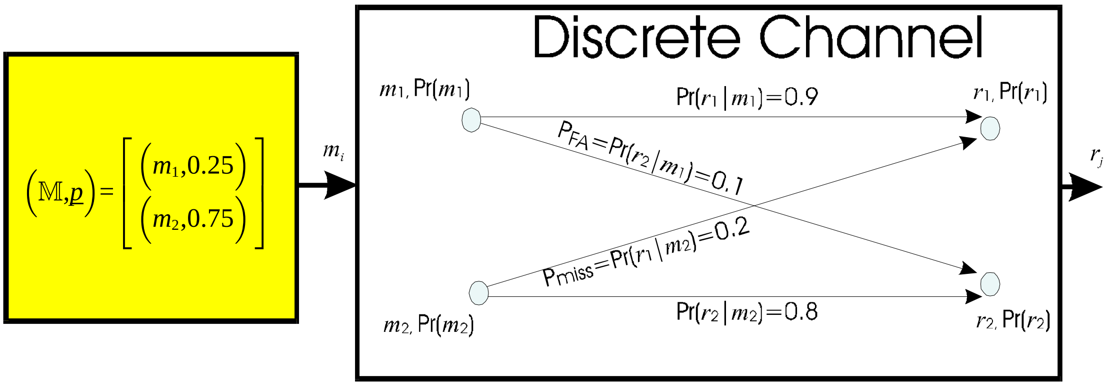
There is only one correct answer per question.

2. a) Consider a *channel encoder* which uses the Hamming (7,4) code. The code is constructed as follows: the information sequence (u_1,u_2,u_3,u_4) is encoded into the codeword $(u_1,u_2,u_3,u_4,u_1 \oplus u_2 \oplus u_4,$ $u_1 \oplus u_3 \oplus u_4,\ u_1 \oplus u_2 \oplus u_3)$, where $\oplus$ denotes modulo-2 addition.
Find the probability that a codeword will be correctly decoded if it is transmitted using a binary digital communication system which employs the following two equally-probable signals



and an optimum digital demodulator. The received signals are corrupted by additive white Gaussian channel noise having a double-sided power spectral density of 10^{-12} W/Hz. **[15]**

b) A discrete channel is modelled as follows:



Estimate:

i) The probability of error at the output of the channel **[3]**
ii) The matrix $\mathbb{J}$ with elements the Joint Probabilities $\Pr(r_i, m_j)$ **[4.5]**
iii)The amount of information delivered at the output of the channel. **[7.5]**

3. a) A high quality music signal with a Crest Factor of 13 dB, occupying a bandwidth from 300 Hz to 15 kHz, is applied to a PCM system. It is required that the Signal-to-Quantization-Noise Ratio (SNR_q) should be better than 42 dB.

Find the minimum number of quantization levels required as well as the transmission rate for a binary code for the following PCM systems:

- a uniform PCM system (i.e. PCM with a uniform quantizer) **[1.5]**
- a μ-*law* PCM with $\mu = 255$ **[1.5]**
- an A-*law* PCM with $A = 87.6$ **[1.5]**
- a differential PCM (i.e. PCM with a differential quantizer) **[1.5]**

Furthermore, for each system, estimate the minimum bandwidth of the transmitted PCM signal. **[6]**

b) A signal *g(t)* having the probability density function (pdf) shown in Figure 3.1 is sampled at twice the Nyquist rate and fed through an 4-level quantizer. The transfer function of the quantizer is shown in Figure 3.2 and the signal *g(t)* is bandlimited to 4 kHz.

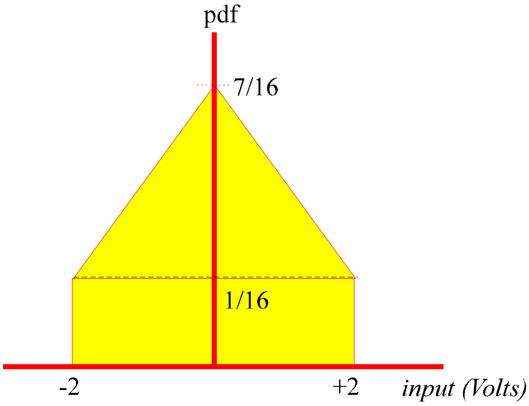


Figure 3.1

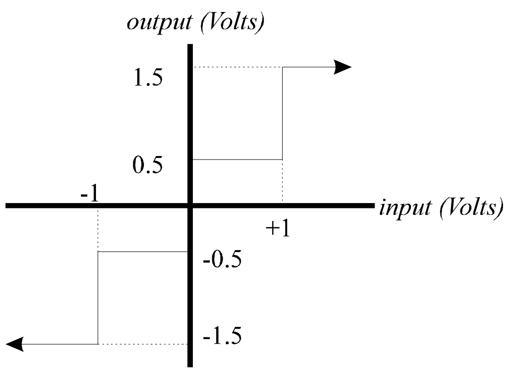


Figure 3.2

Consider the output of the quantizer as the output of a discrete information source $(\underline{\mathbb{M}}, \underline{p})$.

i) Calculate the power of the input signal *g(t)* . **[4.5]**

ii) Calculate and sketch the pdf of the signal (g_{out}) at the output of the quantizer. **[4.5]**

iii) Calculate the average signal-to-quantization-noise ratio. **[3]**

iv) Calculate the symbol rate of the source $(\underline{\mathbb{M}}, \underline{p})$ **[3]**

v) What is the ensemble of the source $(\underline{\mathbb{M}} \times \underline{\mathbb{M}}, \underline{q})$? **[3]**

4. A signal $g(t)$ having the pdf shown in Figure 4.1 is bandlimited to 4 kHz. The signal is sampled at the Nyquist rate and is fed through a 2-level quantizer. The transfer function of the quantizer is shown in Figure 4.2.

pdf :

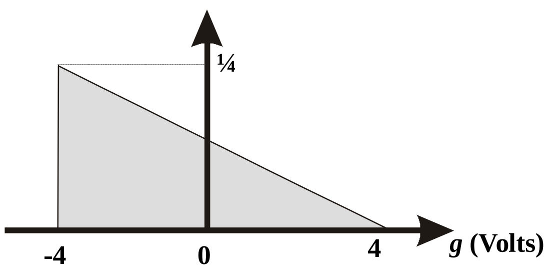


Figure-4.1

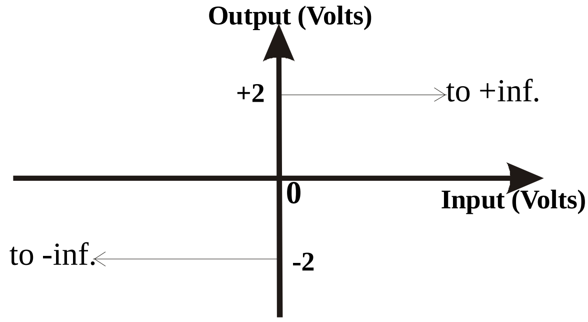
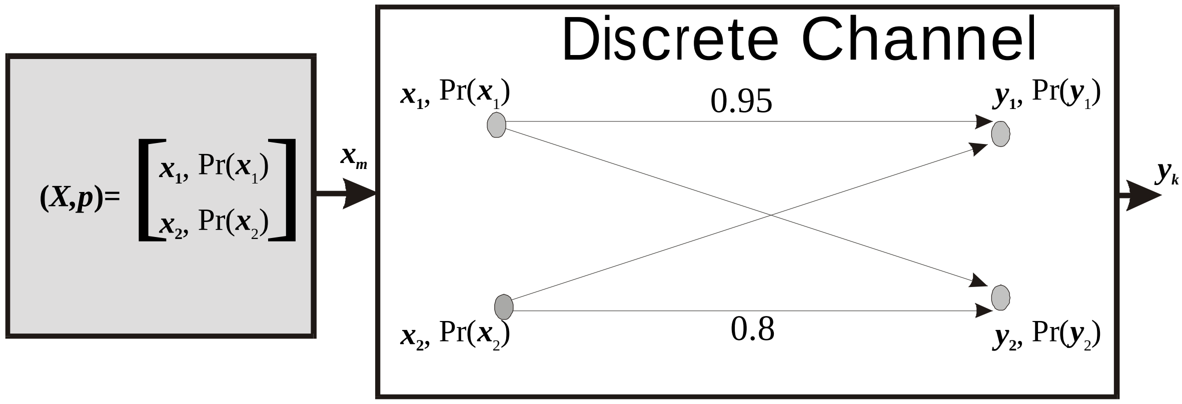


Figure-4.2

A Huffman encoder is used to encode triples of successive output quantization levels while the binary sequence at the output of the Huffman encoder is fed to a Binary on-off Keyed Communication System which employs the following two energy signals

$$s_0(t) = 0$$
$$s_1(t) = 0.5\cos\left(2\pi \frac{5}{T_{cs}} t\right)$$
$$\text{with } 0 < t < T_{cs}$$

and a correlation receiver. The transmitted signals are corrupted by additive white Gaussian channel noise having a double-sided power spectral density of 10^{-3} W/Hz. The whole system is modelled as follows



where the binary information source represents the system up to the output of the Huffman encoder and the discrete channel models the binary on-off keyed system.

In considering the model you should ensure that the output symbols of the source are applied to the channel in such a way as to minimize the error probability at the channel output.

a) Estimate the bit-error probability of the system. **[10.5]**

b) Find the information rate and the bit data rate (symbol rate) at the channel input. **[6]**

c) Estimate the data point (EUE,BUE), where EUE denotes the energy utilization efficiency and BUE represents the bandwidth utilization efficiency of the system. **[6]**

d) Estimate the information point (EUE,BUE), where EUE denotes the information energy utilization efficiency and BUE represents the information bandwidth utilization efficiency of the system. **[4.5]**

e) Is the system a 'realizable' communication system? **[1.5]**

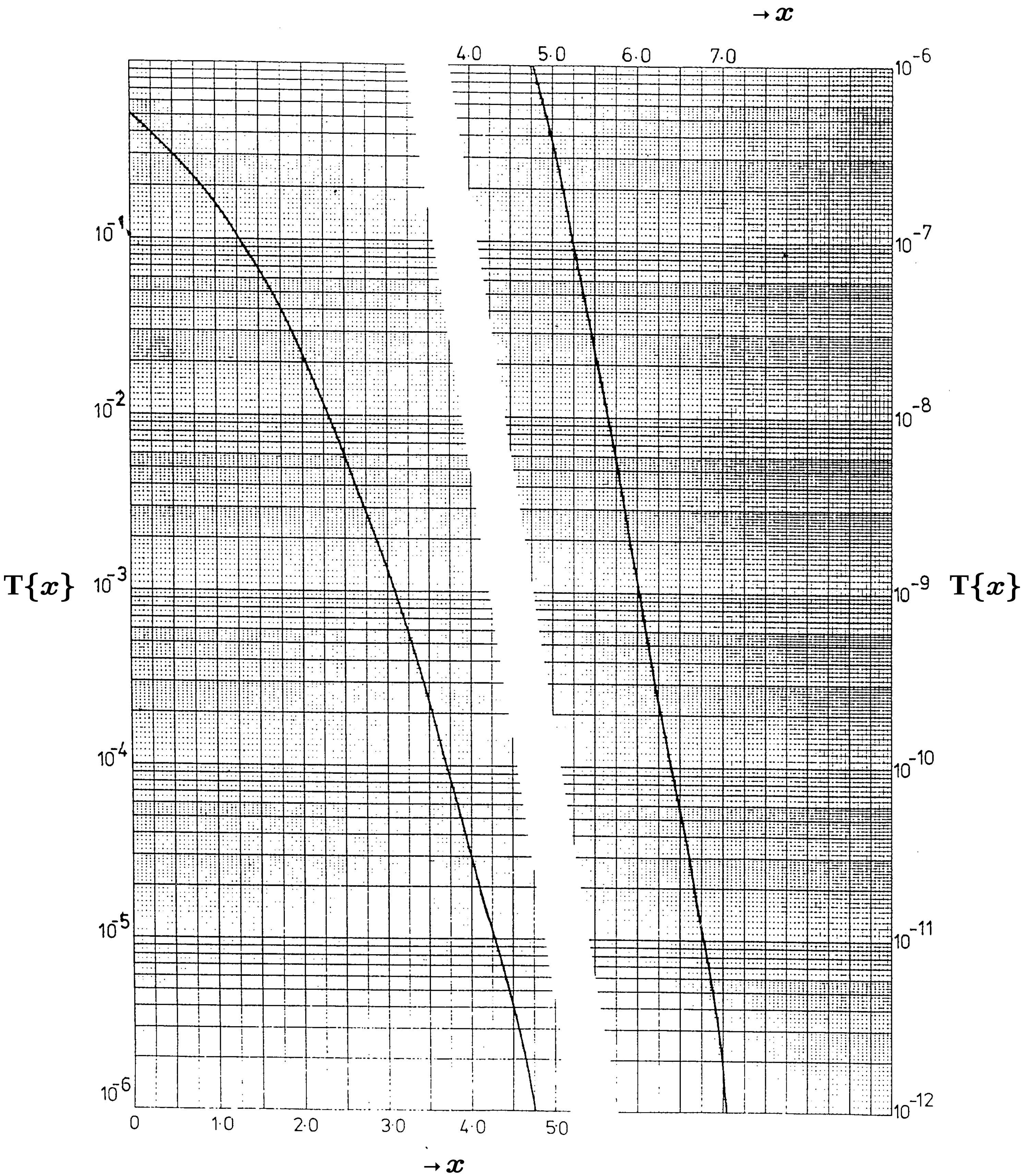
f) What is the signal-to-noise ratio, SNR_{in}, at the receiver's input? **[1.5]**

[END]

Tail Function Graph

The graph below shows the Tail function $\mathbf{T}\{x\}$ which represents the area from x to ∞ of the Gaussian probability density function N(0,1), i.e.

$$\mathbf{T}\{x\} = \int_x^{\infty} \frac{1}{\sqrt{2\pi}}.\exp\left(-\frac{y^2}{2}\right)dy$$



Note that if $x > 6.5$ then $\mathbf{T}\{x\}$ may be approximated by $\mathbf{T}\{x\} \approx \frac{1}{\sqrt{2\pi}\,.\,x}.\exp\left\{-\frac{x^2}{2}\right\}$

FOURIER TRANSFORMS - TABLES

	DESCRIPTION	FUNCTION	TRANSFORM
1	Definition	$g(t)$	$G(f) = \int_{-\infty}^{\infty} g(t).e^{-j2\pi ft}dt$
2	Scaling	$g(\frac{t}{T})$	$\lvert T\rvert \,.\, G(fT)$
3	Time shift	$g(t-T)$	$G(f) \,.\, e^{-j2\pi fT}$
4	Frequency shift	$g(t) \,.\, e^{j2\pi Ft}$	$G(f-F)$
5	Complex conjugate	$g^*(t)$	$G^*(-f)$
6	Temporal derivative	$\frac{d^n}{dt^n} \,.\, g(t)$	$(j2\pi f)^n.\, G(f)$
7	Spectral derivative	$(-j2\pi t)^n.g(t)$	$\frac{d^n}{df^n} \,.\, G(f)$
8	Reciprocity	$G(t)$	$g(-f)$
9	Linearity	$A \,.\, g(t) + B \,.\, h(t)$	$A \,.\, G(f) + B \,.\, H(f)$
10	Multiplication	$g(t) \,.\, h(t)$	$G(f) * H(f)$
11	Convolution	$g(t) * h(t)$	$G(f) \,.\, H(f)$
12	Delta function	$\delta(t)$	1
13	Constant	1	$\delta(f)$

	DESCRIPTION	FUNCTION	TRANSFORM
14	Rectangular function	$\mathbf{rect}\{t\} \equiv \begin{cases} 1 & \text{if } \lvert t\rvert < \frac{1}{2} \\ 0 & otherwise \end{cases}$	$\mathbf{sinc}(f) = \frac{sin\,\pi f}{\pi f}$
15	Sinc function	$\mathbf{sinc}(t)$	$\mathbf{rect}(f)$
16	Unit step function	$u(t) = \begin{cases} +1, & t > 0 \\ 0, & t < 0 \end{cases}$	$\frac{1}{2}\delta(f) - \frac{j}{2\pi f}$
17	Signum function	$\mathbf{sgn}(t) = \begin{cases} +1, & t > 0 \\ -1, & t < 0 \end{cases}$	$-\frac{j}{\pi f}$
18	Decaying exponential (two-sided)	$e^{-\lvert t\rvert}$	$\frac{2}{1+(2\pi f)^2}$
19	Decaying exponential (one-sided)	$e^{-\lvert t\rvert}.u(t)$	$\frac{1-j2\pi f}{1+(2\pi f)^2}$
20	Gaussian function	$e^{-\pi t^2}$	$e^{-\pi f^2}$
21	Lambda function	$\Lambda\{t\} \equiv \begin{cases} 1-t & \text{if } 0 \le t \le 1 \\ 1+t & \text{if } -1 \le t \le 0 \end{cases}$	$\mathbf{sinc}^2(f)$
22	Repeated function	$\mathbf{rep}_T\{g(t)\} = g(t) * \mathbf{rep}_T\{\delta(t)\}$	$\lvert\frac{1}{T}\rvert.\mathbf{comb}_{\frac{1}{T}}\{G(f)\}$
23	Sampled function	$\mathbf{comb}_T\{g(t)\} = g(t).\mathbf{rep}_T\{\delta(t)\}$	$\lvert\frac{1}{T}\rvert.\mathbf{rep}_{\frac{1}{T}}\{G(f)\}$