UNIVERSITY OF LONDON **[E303/ISE3.3]**
IMPERIAL COLLEGE OF SCIENCE TECHNOLOGY AND MEDICINE

DEPARTMENT of ELECTRICAL and ELECTRONIC ENGINEERING
M.Eng, B.Eng and A.C.G.I. EXAMINATIONS 2001
PART III

COMMUNICATION SYSTEMS

— There are FOUR questions (Q1 to Q4)
— Answer question Q1 plus 2 other questions.
— All questions carry equal marks.

Comments for Question Q1:
— Question Q1 has 16 multiple choice questions numbered 1 to 16, all carrying equal marks.
— There is only one correct answer per question.

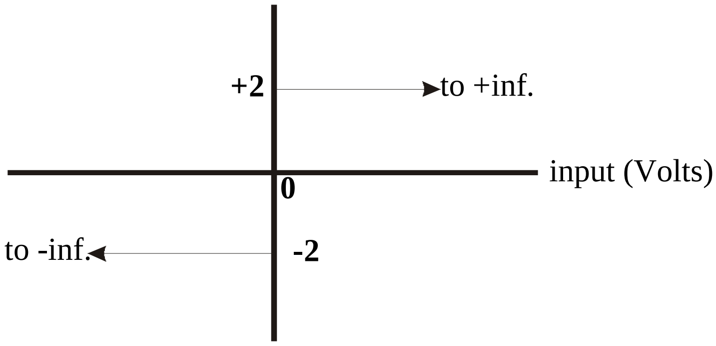
The following are provided:

- ***A table of Fourier Transforms***
- ***A "Gaussian Tail Function" graph***

Examiner responsible: Dr A. Manikas

Question-Q2

A Gaussian signal *g(t)* having mean value of -1 V with std of 2 V, i.e. $\mathcal{N}(-1{,}2)$, is bandlimited to 4 kHz. The signal is sampled at the Nyquist rate and is fed through a 2-level quantizer. The transfer function of the quantizer is shown below.



Consider the output of the quantizer as the output of a discrete information source $(\underline{\mathbb{M}}, \underline{p})$.

a) Calculate and sketch the pdf of the output from the quantizer. **(10%)**

b) Calculate the rms and mean values of the signal at the output of the quantizer. **(10%)**

c) What is the ensemble of the source $(\underline{\mathbb{M}} \times \underline{\mathbb{M}}, \underline{q})$? **(10%)**

d) Design a prefix source encoder to encode pairs of successive output levels from the quantizer. **(20%)**

e) Is the following inequality satisfied?

$$\mathbf{H}(\underline{\mathbb{M}}) \leq \frac{\overline{l_2}}{2} \leq \mathbf{H}(\underline{\mathbb{M}}) + \frac{1}{2}$$

where $\overline{l_2}$ denotes the average length per codeword at the output of the source encoder and $\mathbf{H}(\underline{\mathbb{M}})$ is the entropy of the source $(\underline{\mathbb{M}}, \underline{p})$. **(20%)**

f) State whether the above "double-level" approach (i.e. the one which involves the encoding of pairs of levels) is better than encoding a "single level". **(10%)**

g) Calculate the *information rates* and *data rates* associated with the "single-level" and "double-level" source encoding approaches. **(20%)**

Question-Q3

a) An analogue message signal $g(t)$ with amplitude probability density function $\text{pdf}_g(\text{g}) = 0.5\Lambda\left\{\frac{\text{g}}{2}\right\}$ and a bandwidth of 10kHz, is applied to a PCM system.
Estimate the Signal-to-Quantization-Noise Ratio (SNR_q), as well as the transmission rate, for the following binary PCM communication systems:

- a 256-levels uniform PCM system (i.e. PCM which employs a uniform quantizer of 256 levels) **(20%)**
- an *A-law*/8-bit PCM with $A = 87.6$ **(10%)**

Furthermore, for each system, estimate the minimum bandwidth of the transmitted PCM signal and comment on the performance of the two systems when different message signals with different statistics are applied to the input. **(20%)**

b) Consider a PCM system where its quantizer consists of an A-*law* compander (with $A = 87.6$) followed by a uniform quantizer with "end points" b_i, and "output levels" m_i. The maximum value of the input signal is $10\,Volts$ and the input/output characteristics of the uniform quantizer are given in the following tables:

b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}	b_{13}	b_{14}	b_{15}	b_{16}
-10V	-8.75V	-7.5V	-6.25V	-5V	-3.75V	-2.5V	-1.25V	0V	1.25V	2.5V	3.75V	5V	6.25V	7.5V	8.75V	10V

m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	m_{10}	m_{11}	m_{12}	m_{13}	m_{14}	m_{15}	m_{16}
-9.375V	-8.125V	-6.875V	-5.625V	-4.37V	-3.125V	-1.875V	-0.625V	0.625V	1.875V	3.125V	4.375V	5.625V	6.875V	8.125V	9.375V

- If the signal at the output of the sampler at time kT_s is equal to $3.7\,Volts$, estimate the instantaneous quantization noise $n_q(kT_s)$

- What is the value of $n_q(kT_s)$ if the *A-law* coder is removed?

Note that A-*law* compression is defined as follows:

$$\text{output} = \begin{cases} \frac{A.|x|}{1+lnA} & 0 < |x| < \frac{1}{A} \\ \\ \frac{1+ln(A.|x|)}{1+lnA} & \frac{1}{A} < |x| < 1 \end{cases}$$

$$\text{where } x = \frac{\text{input value in } Volts}{\text{maximum input value in } Volts}$$

(50%)

Question-Q4

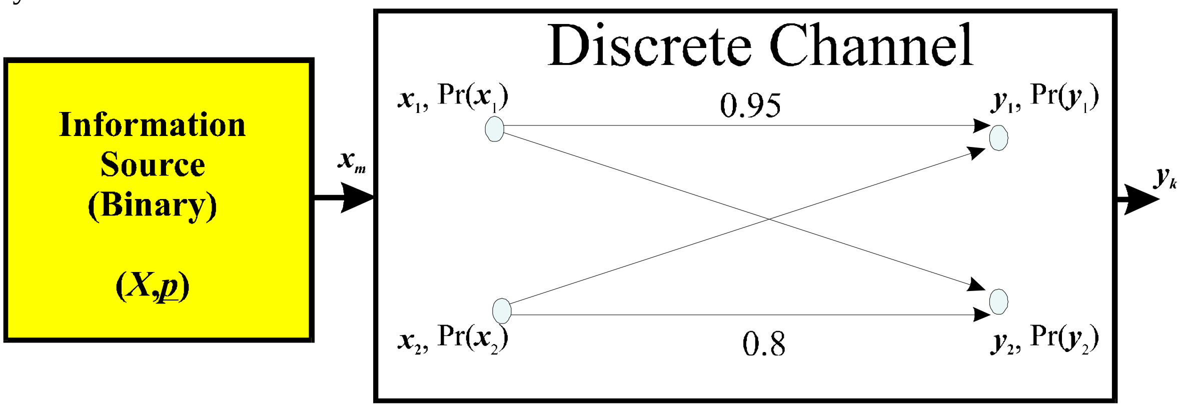
A signal $g(t)$ bandlimited to 4kHz is sampled at the Nyquist rate and is fed through a 2-level quantizer. A Huffman encoder is used to encode triples of successive output quantization levels as follows:

symbols	probs	Huffman
$m_1m_1m_1$	$27/64$	1
$m_1m_1m_2$	$9/64$	001
$m_1m_2m_1$	$9/64$	010
$m_2m_1m_1$	$9/64$	011
$m_1m_2m_2$	$3/64$	00000
$m_2m_1m_2$	$3/64$	00001
$m_2m_2m_1$	$3/64$	00010
$m_2m_2m_2$	$1/64$	00011

while the binary sequence at the output of the Huffman encoder is fed to a Binary on-off Keyed Communication System which employs the following two energy signals of duration T_{cs}

$$s_0(t) = 0$$
$$s_1(t) = \sqrt{\tfrac{3}{8}}\,\mathbf{\Lambda}\left(\frac{t}{0.5\,T_{cs}}\right)$$

and a matched filter receiver. The transmitted signals are corrupted by additive white Gaussian channel noise having a double-sided power spectral density of 10^{-3} W/Hz. The figure below shows a modelling of the whole system where the output of the Huffman encoder is modelled as the output of a binary discrete information source $(X, \underline{p})$ with $X = \{x_1{=}1, x_2{=}0\}$, $\underline{p} = [\Pr(x_1), \Pr(x_2)]^T$ while the binary on-off Keyed system is modelled as a discrete channel as shown below.



a) Find the entropy of the information source (X, p), the information rate and the bit data rate (symbol rate) at the channel input. **(30%)**

b) Estimate the bit-error probability of the system. **(10%)**

c) Estimate the energy utilization efficiency (EUE) and bandwidth utilization efficiency (BUE) using the bit data rate as well as the information rate. **(40%)**

d) Represent the communication system, as a point on the (EUE,BUE) parameter plane. In this plane show also the locus of the system properly labelled. **(10%)**

e) Is the system a 'realizable' communication system? **(5%)**

f) What is the signal-to-noise ratio, SNR_{in}, at the receiver's input? **(5%)**

[END]