

Paper Number(s): **E2.7A**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2002

EEE PART II: B.Eng., M.Eng. and ACGI

PRINCIPLES OF COMPUTERS AND SOFTWARE ENGINEERING

Friday, 7 June 2:00 pm

Corrected Copy

There are THREE questions on this paper.

Answer TWO questions.

None

This exam is OPEN BOOK.

Time allowed: 1:30 hours.

Examiners responsible:

First Marker(s): Shanahan, M.P.
Second Marker(s): Demiris, Y.K.

Information for Invigilators:

Students may bring any written or printed aids into the exam.

Information for Candidates:

None.

QUESTION ONE

Here is the type definition for a binary tree of strings.

```
TTree = ^TNode;  
TNode =  
record  
    Node : string;  
    Left : TTree;  
    Right : TTree;  
end;
```

To answer the following questions, you can assume the existence of access procedures for the type TTree called Empty, Left, Right, and Root with the obvious meanings.

a) Write a Pascal function that takes a binary tree and returns the difference between the number of nodes in its left sub-tree and the number of nodes in its right sub-tree. For example, given the tree of Fig. 1 below, the function would return 2, since there are 5 nodes in the left sub-tree and 3 in the right sub-tree.

[6]

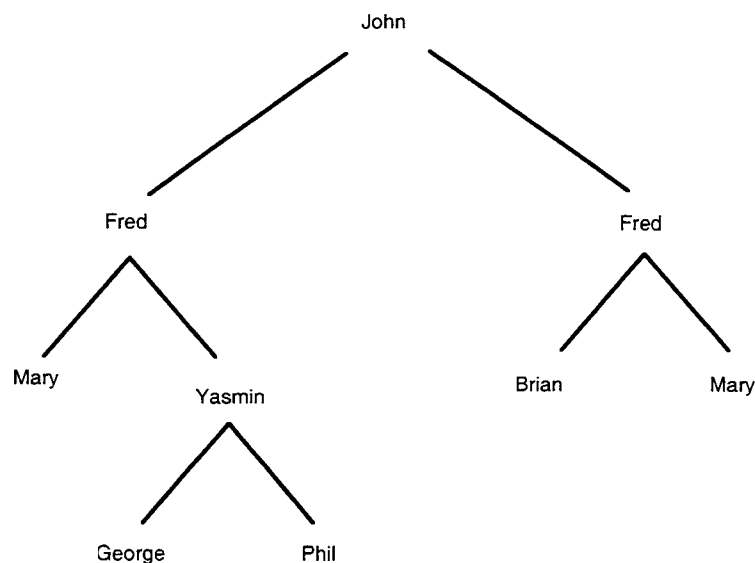


Fig. 1

b) Write a Pascal function that returns the height of a binary tree. This corresponds to the number of nodes in the longest branch of the tree from root to leaf. For example, the height of the tree in Fig. 1 is 4, the longest branch being the one that stretches from John to Phil (or George).

[7]

c) A binary tree is *symmetrical* if its left sub-tree is a mirror image of its right sub-tree. For example, the tree in Fig. 1 above is not symmetrical. But the tree in Fig. 2 below is symmetrical.

Continued on next page

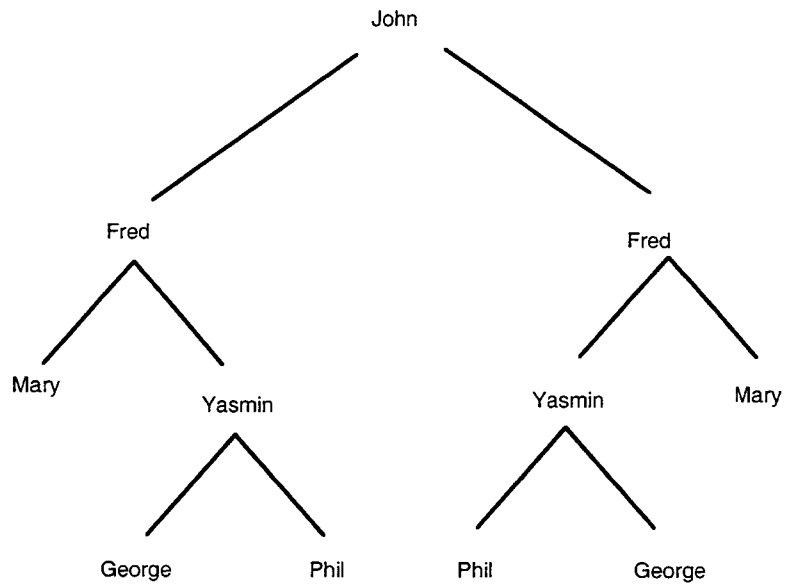


Fig. 2

Write a Pascal function that takes a binary tree and returns True if the tree is symmetrical and False if it is not.

[7]

QUESTION TWO

Consider the following Pascal program. (Note: the function `sqrt(x)` returns the square root of `x`, and the function `round(x)` rounds `x` to the nearest integer.)

```
program Compute;  
const n = 100;  
var  
    i, j : integer;  
    A : array [1..n] of boolean;  
begin  
    // Initialise array  
    for i := 1 to n do  
    begin  
        A[i] := true;  
    end;  
    // Fill in array  
    for i := 2 to round(sqrt(n)) do  
    begin  
        j := 2 * i;  
        while j <= n do  
        begin  
            A[j] := false;  
            j := j + i;  
        end;  
    end;  
end.
```

a) Simulate the first three iterations of the second for loop, and show the contents of the first 15 elements of the array A after each iteration. (You can abbreviate true to T and false to F.)

[8]

b) What does the program do? Explain how it does it?

[8]

c) Suggest one way of improving the efficiency of the program.

[4]

QUESTION THREE

Here is the Pascal type declaration for a dynamic linked list of real numbers.

```
type
  TList = ^TLink;
  TLink =
    record
      First : real;
      Rest  : TList;
    end;
```

a) Write a function `Middle` that takes a `TList` and returns the real number half way along that list. In other words, if the list is of length N , your function must return the $N/2^{\text{th}}$ element of the list. (If the list has an odd number of elements, return the $(N+1)/2^{\text{th}}$ element.) Use the following method. Count the number of elements N in the list, then start from the beginning of the list again and work along it until the $N/2^{\text{th}}$ element is reached.

[9]

b) Write a second version of `Middle` that uses the following method. Starting from the beginning of the list, work along it maintaining two pointers. The first pointer advances one element at a time, and the second pointer advances two elements at a time. When the second pointer reaches the end of the list, the first pointer will point to the element required.

[9]

c) In terms of loop iterations and/or recursive calls, which function is most efficient, and by how much? Explain your answer.

[2]

Model Answers

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QUESTION ONE

a)

```
function Difference(T : TTree): integer;
begin
    Difference :=
        Abs(Count(Left(T)) - Count(Right(T)));
end;

function Count(T : TTree): integer;
begin
    if T = Empty
    then Count := 0
    else Count := Count(Left(T)) + Count(Right(T));
end;
```

b)

```
function Height(T : TTree): integer;
begin
    if T = Empty
    then Height := 0
    else begin
        L := Height(Left(T));
        R := Height(Right(T));
        if L > R
        then Height := L + 1
        else Height := R + 1;
    end;
end;
```

c)

```
function Symmetrical(T : TTree): boolean;
begin
    if (T = Empty) or Mirrors(Left(T), Right(T))
    then Symmetrical := true
    else Symmetrical := false;
end;
```

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```
function Mirrors(T1, T2 : TTree): boolean;
var F : boolean;
begin
    if (T1 = Empty) and (T2 = Empty)
    then Mirrors := true
    else begin
        F := Mirrors(Left(T1),Right(T2));
        Mirrors :=
            F and Mirrors(Right(T1),Left(T2));
    end;
end;
```


QUESTION TWO

a)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	T	F	T	F	T	F	T	F	T
T	T	T	F	T	F	T	F	F	F	T	F	T	F	F
T	T	T	F	T	F	T	F	F	F	T	F	T	F	F

b) The program computes the first 100 prime numbers (using a simple version of the sieve of Eratosthenes). When the program terminates, the i^{th} element of A will be true if i is a prime number and false if it isn't. Initially all the elements of A are true (first for loop). The second for loop knocks out successive multiples of the natural numbers (by setting the corresponding element in A to false). First it eliminates multiples of 2, then multiples of 3, and so on. The algorithm only needs to go as far as multiples of \sqrt{n} to get all the primes up to n.

c) The second for loop can be made more efficient by including a check to see whether A[i] is false. If so, all multiples of i will already be false, so there's no need to execute the while loop.

```

for i := 2 to round(sqrt(n)) do
  begin
    if A[i] = true
      then begin
        j := 2 * i;
        while j <= n do
          begin
            A[j] := false;
            j := j + i;
          end;
        end;
      end;
    end;
  end;

```

QUESTION THREE

a)

```
function Middle(L : TList): real
var N, M : integer;
begin
  N := Length(L);
  if N mod 2 = 0
  then N := N div 2
  else N := (N+1) div 2;
  if N <> 0
  then begin
    M := 1;
    while M <> N do
    begin
      L := L^.Rest;
      M := M+1;
    end;
    Middle := L^.First;
  end
  else Middle := 0;
end;

function Length(L : TList): integer;
begin
  if L = nil
  then Length := 0
  else Length := Length(L^.Rest) + 1;
end;
```

b)

```
function Middle(L : TList): real;
begin
  if L = nil
  then Middle := 0
  else begin
    Ptr := L^.Rest;
    while Ptr <> nil and Ptr^.Rest <> nil do
    begin
      Ptr := Ptr^.Rest^.Rest;
      L := L^.Rest;
    end;
    Middle := L^.First;
  end;
end;
```

c) The second function is more efficient. For a list of length n , it will execute $n/2$ iterations of the while loop. The first function executes n recursive calls to Length plus $n/2$ iterations of the while loop. So the first function will take around three times as long.