Paper Number(s): **E2.4**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2001

EEE PART II: M.Eng., B.Eng. and ACGI

COMMUNICATIONS II

Friday, 22 June 2:00 pm

There are FIVE questions on this paper.

Answer ANY THREE questions.

All questions carry equal marks.

Please use a separate answer book for Sections A and B.

Time allowed: 2:00 hours

Examiners: Ward,D.B., Barria,J.A., Gurcan,M.K.
and Gurcan,M.K.

SECTION A Communications Principles (Please use separate answer book)

1. (a) Justify the representation:

$$n(t) = \sum_k a_k \cos(2\pi f_k t + \theta_k)$$

for band-limited white noise of which a representative frequency is f_k, and the θ_k are random phases which are independent and uniformly distributed over 0 to 2π.

[5]

(b) Show that this bandpass noise can be written as

$$n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

and explain what band of frequencies are present in each of $n_c(t)$ and $n_s(t)$.

[5]

(c) Derive expressions for the average power in each of $n_c(t)$ and $n_s(t)$.

[5]

(d) By deriving a phasor representation for $n(t)$, explain why $n_c(t)$ and $n_s(t)$ are commonly referred to as the in-phase and quadrature terms, respectively.

[5]

2. (a) Describe the process of pulse code modulation (PCM) of an analog signal, and state what is meant by quantization noise.

[4]

(b) For a uniform quantizer, derive an expression for the mean-square quantization error in terms of the step size.

[4]

(c) If the input to a uniform n-bit quantizer is the sine wave $A_m \sin(2\pi f_m t)$, derive an expression for the signal-to-noise ratio (in decibels) at the output of the quantizer. Assume that the dynamic range of the quantizer is $-A_m$ to A_m.

[4]

(d) Consider a binary source alphabet where a symbol 0 is represented by 0 volts, and a symbol 1 is represented by 1 volt. Assume these symbols are transmitted over a baseband channel having uniformly distributed noise with a probability density function:

$$f(n) = \begin{cases} \frac{1}{2}, & |n| < 1 \\ 0, & \text{otherwise.} \end{cases}$$

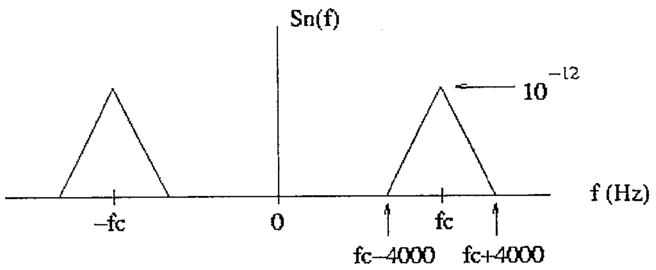
Assume that the decision threshold T is within the range of 0 to 1 volt.

i. If the symbols are equally likely, show that the probability of error is independent of the choice of threshold.

ii. If the probability of occurrence of symbol 0 is p, derive an expression for the probability of error. Hence determine the optimum value of the threshold if $p = 1/3$.

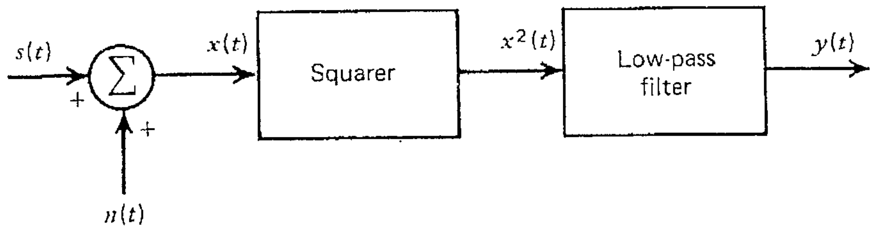
[8]

3. (a) A signal $m(t)$ with a uniform power spectral density (PSD) and a bandwidth of 4 kHz, is modulated with a carrier of frequency $f_c = 500$ kHz to produce a DSB-SC signal. The modulated signal is transmitted over a channel with a noise PSD of $S_n(f)$ as shown below. The power of the DSB-SC signal is 1μ W at the receiver input. The received signal is bandpass filtered, multiplied by $2\cos(2\pi f_c t)$, and then lowpass filtered to obtain the output. Determine the output signal-to-noise ratio.



[8]

(b) Consider an AM receiver using a square-law detector whose output is proportional to the square of the receiver input $x(t)$, as indicated in the following figure.



The AM waveform is :

$$s(t) = A[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$$

where μ is the modulation index. Assume that the additive noise at the receiver input is white Gaussian bandpass noise with zero mean. Show that the output signal-to-noise ratio of the receiver is given by:

$$SNR_{\text{out}} = \frac{2\mu^2\rho^2}{1+\rho(2+\mu^2)}$$

where ρ is the carrier-to-noise ratio at the input to the receiver. Assume that a capacitor is included at the output of the receiver to block DC.

You may find the following identities useful:

$$\cos^2(A) = \tfrac{1}{2}(1+\cos(2A))$$
$$\sin^2(A) = \tfrac{1}{2}(1-\cos(2A))$$
$$\cos(A)\sin(A) = \tfrac{1}{2}\sin(2A)$$

[12]

4. (a) Using the Huffman coding procedure, construct a coding scheme for an alphabet whose symbols occur independently with probabilities 0.1, 0.4, 0.25, 0.1, 0.15. Calculate the average codeword length of the resulting source code and compare it with the source entropy.

[8]

(b) State the source coding theorem, explaining and giving units for all terms used.

[3]

(c) What is the channel capacity of an additive Gaussian noise channel with bandwidth of 10 kHz, if the required signal-to-noise ratio at the receiver input is 20 dB? Hence, for a three-symbol alphabet having probabilities 0.2, 0.35, 0.45, calculate the maximum symbol rate that allows reliable communication over this channel.

[5]

(d) A discrete source produces the symbols A and B with probabilities $p_A = \frac{3}{4}$ and $p_B = \frac{1}{4}$ at a rate of 100 symbols/second. The symbols are grouped in blocks of two and encoded as follows:

Grouped symbols	Binary code
AA	1
AB	01
BA	001
BB	000

Is this code optimum (justify your decision)? If not, how efficient is it?

[4]

SECTION B Networks (Please use separate answer book)

5. Answer any two of the following subsections (a), (b) and (c).

(a) Discuss the principle of connection-oriented services and connectionless services. Discuss the relevance of characterising a service by means of its quality of service (QoS) features. [10]

(b) Describe and discuss the importance of Media Access Control (MAC) techniques for broadcast communications networks. Describe the main advantages and disadvantages of a centralised MAC scheme. [10]

(c) Describe and discuss the importance of the network layer in the OSI protocol reference model and routing in packet switched networks. [10]

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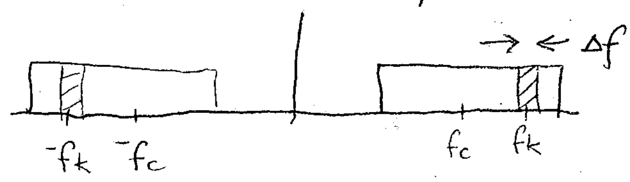
1.(a) $n(t) = \sum_k a_k \cos(2\pi f_k t + \theta_k)$

- white noise has a flat power spectral density:



- for Δf small, the shaded component can be represented by a randomly-phased sinusoid of frequency f_k, and random phase θ_k, and amplitude a_k.
- summing these random sinusoids over the entire band gives the representation required. **5**

1.(b) - let $f_k = (f_k - f_c) + f_c$

$$\therefore n_k(t) = a_k \cos\left[2\pi(f_k - f_c)t + \theta_k + 2\pi f_c t\right]$$

but $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$\therefore n_k(t) = a_k \cos(2\pi(f_k - f_c)t + \theta_k)\cos(2\pi f_c t) - a_k \sin(2\pi(f_k - f_c)t + \theta_k)\sin(2\pi f_c t)$$

$$\therefore n(t) = \sum_k n_k(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

where $n_c(t) = \sum_k a_k \cos(2\pi(f_k - f_c)t + \theta_k)$

$n_s(t) = \sum_k a_k \sin(2\pi(f_k - f_c)t + \theta_k)$

Each of $n_c(t)$ & $n_s(t)$ contain frequencies $(f_k - f_c)$

Since f_k are centred around f_c, hence frequencies $(f_k - f_c)$ present in $n_c(t)$ & $n_s(t)$ are centred around 0, ie they are baseband **5**

1. (c) Power in $n_c(t)$ is:

$$E\{n_c^2(t)\} = E\Big\{\sum_k\sum_\ell a_k a_\ell \cos(2\pi(f_k-f_c)t+\theta_k) \times \cos(2\pi(f_\ell-f_c)t+\theta_\ell)\Big\}$$

$$= \sum_k\sum_\ell E\{a_k a_\ell \cos(2\pi(f_k-f_c)t+\theta_k)\cos(2\pi(f_\ell-f_c)t+\theta_\ell)\}$$

Since phase terms are independent, it follows that

$$E\{a_k a_\ell \cos(2\pi(f_k-f_c)t+\theta_k)\cos(2\pi(f_\ell-f_c)t+\theta_\ell)\}=0, \quad k\neq \ell$$

$$E\{\cdot\} = E\{a_k^2 \cos^2(2\pi(f_k-f_c)t+\theta_k)\}, \quad k=\ell$$

$$= E\{a_k^2\}\, E\{\cos^2(2\pi(f_k-f_c)t+\theta_k)\}, \quad k=\ell$$

since a_k & θ_k are independent.

But $E\{\cos^2(2\pi(f_k-f_c)t+\theta_k)\} = \frac{1}{2}$

$\therefore$ Power in $n_c(t)$ is $\sum_k \frac{E\{a_k^2\}}{2}$

For $n_s(t)$, similarly we have

$$E\{n_s^2(t)\} = \sum_k\sum_\ell E\{a_k a_\ell \sin(2\pi(f_k-f_c)t+\theta_k)\sin(2\pi(f_\ell-f_c)t+\theta_\ell)\}$$

Again, we have:

$$E\{\sin(2\pi(f_k-f_c)t+\theta_k)\sin(2\pi(f_\ell-f_c)t+\theta_\ell)\}=0, \quad k\neq\ell$$

since θ_k are independent

$$E\{\sin^2(2\pi(f_k-f_c)t+\theta_k)\} = \frac{1}{2}, \quad k=\ell.$$

$\therefore$ Power in $n_s(t)$ is $\sum_k \frac{E\{a_k^2\}}{2}$ also.

5.

1.(a)

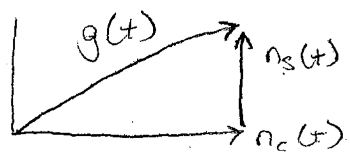
Let $g(t) = n_c(t) + j n_s(t)$

$$g(t) e^{j2\pi f_c t} = n_c(t)\left(\cos 2\pi f_c t + j \sin 2\pi f_c t\right) + j n_s(t)\left(\cos 2\pi f_c t + j \sin 2\pi f_c t\right)$$

$$= n_c(t)\cos 2\pi f_c t + j n_c(t) \sin 2\pi f_c t - n_s(t) \sin 2\pi f_c t + j n_s(t) \cos 2\pi f_c t$$

$$\therefore n(t) = Re\left\{ g(t) e^{j2\pi f_c t} \right\}$$

Phasor diagram is:



where we see that $n_c(t)$ is in-phase ~~xxxx~~ component of rotating phasor & $n_s(t)$ is 90° out of phase (in quadrature) with rotating phasor. 5

2.(a) PCM consists of:

1. sampling at or above Nyquist rate
2. quantizing each sample into discrete levels 3.
3. encoding into a digital stream.

Quantization noise is introduced in step 2. It is caused by the fact that errors are introduced when amplitude is rounded to the nearest quantization level. 1.

2(b) For a uniform quantizer with separation of Δ volts between levels, quantization error is a random variable bounded by $-\Delta/2 \le q \le \Delta/2$. It is approximately uniformly distributed with pdf:

$$p(q) = \begin{cases} \frac{1}{\Delta}, & -\Delta/2 \le q \le \Delta/2 \\ 0, & \text{otherwise} \end{cases}$$ 1.

Mean square error is thus

$$E\{e^2\} = \int_{-\infty}^{\infty} q^2 \, p(q) \, dq$$

$$= \int_{-\Delta/2}^{\Delta/2} q^2 \frac{1}{\Delta} \, dq$$

$$= \frac{\Delta^2}{12}$$ 3.

2.(c) For a sine wave $A_m \sin(2\pi f_m t)$, the average power is:

$$P_S = \frac{A_m^2}{2}$$ 1.

Average noise power (from (b)) is:

$$P_N = \frac{\Delta^2}{12}$$

The range of the quantizer is $2A_m = L\Delta$ where L is the no. of levels, which for a n-bit quantizer is $L = 2^n - 1 \approx 2^n$

$$\therefore \Delta = \frac{2A_m}{2^n} \quad \& \quad \Delta^2 = \frac{4A_m^2}{2^{2n}}$$ 1.

$$\therefore SNR = \frac{P_S}{P_N} = \frac{A_m^2}{2} \times \frac{12 \times 2^{2n}}{4A_m^2}$$

$$= 3/2 \times 2^{2n}$$ 1.

In decibels,

$$SNR_{dB} = 10 \times 2n \log_{10} 2 + 10 \log_{10} 3/2$$
$$= 6.02n + 1.8 \text{ dB.}$$ 1.

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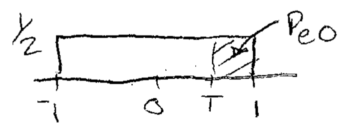
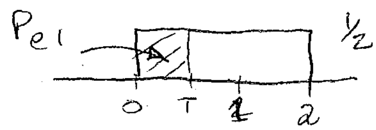
2(c) PDF's of received signals :

"0" transmitted



$P_{e0} = \frac{1}{2}(1-T)$

"1" transmitted



$P_{e1} = \frac{1}{2}T$

(i) for equally-likely symbols:

$$P_e = p_0 P_{e0} + p_1 P_{e1} = \tfrac{1}{2}(P_{e0} + P_{e1})$$
$$= \tfrac{1}{2}\left(\tfrac{1}{2}(1-T) + \tfrac{1}{2}T\right)$$
$$= \tfrac{1}{4}(1-T+T) = \tfrac{1}{4}$$ is independent of T — 3.

(ii) if $p_0 = p$ $\therefore p_1 = 1-p$

$$\therefore P_e = p\,\tfrac{1}{2}(1-T) + (1-p)\,T/2$$
$$= p/2 - Tp/2 + T/2 - Tp/2$$
$$= \tfrac{1}{2}(p + T - 2pT)$$ — 3.

For $p = \frac{1}{3}$ $\therefore P_e = \frac{1}{2}\left(\frac{1}{3} + T - \frac{2}{3}T\right)$
$$= \tfrac{1}{2}\left(\tfrac{1}{3} + \tfrac{1}{3}T\right)$$ — 1.

for $T \in [0,1]$, P_e is minimum for $T = 0$ — 1.

$\therefore P_{e_{min}} = \frac{1}{6}$

3(a) For DSB-SC, transmitted signal is $A m(t) \cos \omega_c t$ 1.

& received signal is: 1.

$$r(t) = [A m(t) + n_c(t)] \cos \omega_c t - n_s(t) \sin \omega_c t$$

After coherent detection this becomes

$$y(t) = A\, m(t) + n_c(t)$$ 1.

Signal power is: $P_S = A^2 \overline{m^2(t)}$

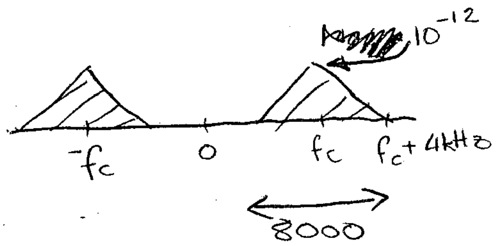
Noise power is: $P_N = \overline{n_c^2(t)}$

Power in DSB-SC is: $\overline{(A m(t) \cos \omega_c t)^2} = A^2/2 \; \overline{m^2(t)}$ 2.

but this equals $1 \mu W$

$\therefore A^2 \overline{m^2(t)} = 2 \mu W$

Power in $n_c(t)$ is same as in bandpass noise $n(t)$ which is given by shaded area



ie $P_N = \int S_n(f)\, df$

$= 2 \times \frac{1}{2} (2 \times 4000 \times 10^{-12})$ 2.

$= 8 \times 10^{-9}$

$$SNR = P_S / P_N = \frac{2 \times 10^{-6}}{8 \times 10^{-9}} = 250 = 24\,dB$$ 1.

3(b) Received signal is:

$$x(t) = (A(1+\mu \cos\omega_m t) + n_c)\cos\omega_c t - n_s \sin\omega_c t$$ 1.

Squared signal is:

$$x^2(t)$$

$$= [A(1+\mu\cos\omega_m t) + n_c]^2 \cos^2\omega_c t - n_s^2 \sin^2\omega_c t$$

$$- 2(A(1+\mu\cos\omega_m t) + n_c) n_s \cos\omega_c t \sin\omega_c t$$ 1.

But $\cos^2 A = \frac{1}{2}(1+\cos 2A)$

$\sin^2 A = \frac{1}{2}(1-\cos 2A)$

$\cos A \sin A = \frac{1}{2}\sin 2A$

Hence, after the LPF we are left with:

$$y(t) = \tfrac{1}{2}[A(1+\mu\cos\omega_m t) + n_c]^2 - \tfrac{1}{2} n_s^2$$

$$= \tfrac{1}{2}\left\{A^2 + 2A^2\mu\cos\omega_m t + \frac{A^2\mu^2}{2} + 2A_c n_c + 2A\mu n_c \cos\omega_m t + n_c^2 + n_s^2\right\}$$ 2.

After removing DC terms we have:

$$y_o(t) = A^2\mu\cos\omega_m t + A n_c(t) + A\mu n_c(t)\cos\omega_m t + \tfrac{1}{2}n_c^2(t) + \tfrac{1}{2}n_s^2(t)$$ 1.

Signal terms: $A^2\mu \cos\omega_m t$

$$\therefore P_S = \frac{A^4\mu^2}{2}$$ 1.

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3(b) Noise terms: $A n_c(t) + A\mu n_c(t) \cos\omega_m t + \frac{1}{2} n_c^2(t) + \frac{1}{2} n_s^2(t)$

$$\therefore P_N = A^2 \sigma_N^2 + \frac{A^2\mu^2}{2} \sigma_N^2 + \sigma_N^4$$

where $\sigma_N^2 = E\{n_c^2(t)\} = E\{n_s^2(t)\}$

$$\therefore P_N = \tfrac{1}{2} A^2 (2+\mu^2) \sigma_N^2 + \sigma_N^4$$ 2.

Carrier power is: $A^2/2$

$\therefore$ Carrier to noise ratio is $\rho = \dfrac{A^2/2}{\sigma_N^2}$ 1.

Subing into P_N gives:

$$P_N = \sigma_N^4 \left[\rho(2+\mu^2) + 1\right]$$

Output SNR is:

$$SNR_o = \frac{P_S}{P_N} = \frac{A^4\mu^2}{2\sigma_N^4} \frac{1}{1+\rho(2+\mu^2)}$$

but $\dfrac{A^4}{2\sigma_N^2} = 2\rho^2$

$$\therefore SNR_o = \frac{2\rho^2\mu^2}{1+\rho(2+\mu^2)}$$ 3.

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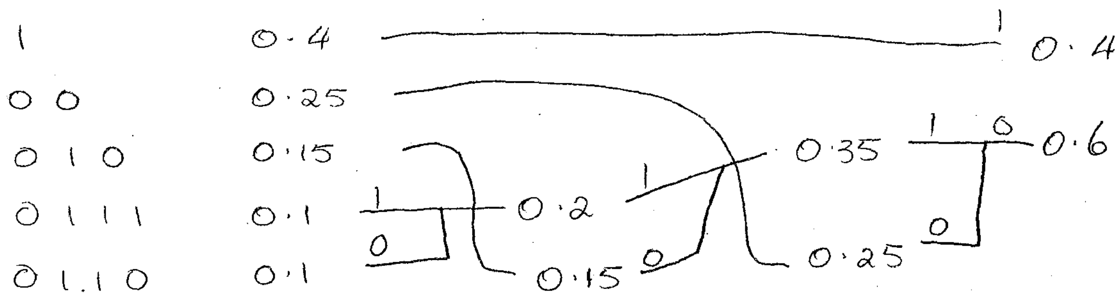
4(a)



4.

Avg codeword length, $L = \Sigma p_k l_k$

$= 0.4 \times 1 + .25 \times 2 + .15 \times 3 + .1 \times 4 + .1 \times 4$ 2.

$= 2.15$ bits/symbol

Entropy, $H = -\sum_k p_k \log_2 p_k$

$= 2.1037$ bits/symbol 2.

4(b) Source coding theorem:

Given a discrete memoryless source of entropy H (bits/symbol) the average codeword length L (bits/symbol) for any coding scheme is bounded as $L \geq H$. 3.

4(c)

$$C = B \log_2(1+SNR)$$
$$= 10\times10^3 \log_2(1+100)$$
$$= 66\,582 \text{ bits/sec.}$$ 1.

Information rate is: $R = rH$ bits/sec.

where r is symbol rate.

By the channel capacity theorem, $R \leq C$

$$\therefore rH \leq C$$
$$\therefore r \leq C/H$$ 2.

Entropy for given alphabet,

$$H = -\sum p_k \log_2 p_k$$
$$= 1.5129 \text{ bits/symbol}$$ 1.

$\therefore$ Maximum symbol rate is:

$$r_{max} = C/H = \frac{66\,582}{1.5129}$$ 1.

$$= 44\times10^3 \text{ symbols/sec.}$$

4(d) Each group of 2 source symbols is considered as a new symbol.
The new alphabet has probabilities

$AA: \ 3/4 \times 3/4 = .5625$

$AB: \ 3/4 \times 1/4 = .1875$

$BA: \ 1/4 \times 3/4 = .1875$

$BB: \ 1/4 \times 1/4 = .0625$

Entropy of new alphabet,

$$H = -\sum_k p_k \log_2 p_k$$

$$= 1.6226 \text{ bits}/(2 \text{ symb}) = 0.8113 \text{ bits/symb.}$$

Avg. code length,

$$L = \sum p_k l_k$$

$$= .5625\times1 + .1875\times2 + .1875\times3 + .0625\times3$$

$$= 1.6875 \text{ bits}/(2 \text{ symb})$$

$$= 0.84375 \text{ bits/symb.}$$ 3.

Since $L > H$, this code is not optimum

Efficiency is $\frac{.8113}{.84375} = 96\%$ efficient 1.

5(a) In a layer based Network Architecture, layers can offer at least two different type of services to the layer above them: connection-oriented and connectionless. Connection-oriented service is modelled after the telephone system. The service user first establishes a connection, uses the connection, and then terminates the connection. In contrast, connectionless service is modelled after the postal system. Each message carries the full destination address, and each one is routed through the system independent of all the others. 5

Each service can be characterised by a quality of service (QoS). Some services are reliable in the sense that they never lose data. Usually a reliable service is implemented by having the receiver acknowledge the receipt of each message, so the sender is sure that it arrived. The ack. process introduces overhead and delays, which are often worth it, but are sometimes undesirable. One application in which delay is not acceptable is digitised voice traffic. It is preferable for telephone users to hear a bit of noise on the line from time to time than to introduce a delay (silence gap) to wait for acknowledgements. 5

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5(?) Media Access Control (MAC)

In a broadcast network, only one device can successfully transmit on the shared medium at a time. Therefore an access control technique is required 3

In a centralised scheme, a controller is designated to grant access to the network
In a decentralised network, the station collectively perform a MAC function to dynamically determine the order in which station transmit 1

A centralised network scheme has certain advantages such as:
- it may afford greater control over access by providing such things as priorities and guaranteed bandwidth
- it allows the logic at each station to be as simple as possible
- it avoids problems of co-ordination 3

On the other hand its principal disadvantages include
- it results in a single point of failure
- it may act as a bottleneck, reducing efficiency
- if propagation delay is high, the overhead may be unacceptable 3

5(c) The network layer: provide upper layers with independence from the data transmission and switching technologies used to connect the systems. A key design issue is determining how packets are routed from source to destination. If too many packets are presented in the network at the same time, they may form bottlenecks. The control of such congestion belongs to the network layer. It is also up to the network layer to overcome the problems related to heterogeneous networks that are interconnected (e.g. addressing). 5

Packet switching represents an attempt to combine the advantage of message and circuit switching while minimising the advantages of both. There are two ways for the network to handle a stream of packets: datagram and virtual circuit

(i) in the datagram approach exact packet is treated independently (this is a connectionless service). Therefore, there is a possibility that the packets could be delivered to its destination in a different sequence from the one in which they were sent. It is up to the destination node to figure out how to reordered them.

5(c) (ii) In a virtual circuit approach, a logical connection is established before any packet are sent (this is a connection-oriented service). Therefore the originating node sends a call request to the destination node (during this phase a route is created). Once the call request is been accepted by the destination node a call accept packet is sent to the originating node. At this point, both terminal stations may exchange data over the established logical connection or virtual circuit. Eventually one of the stations terminates the connection