Paper Number(s): **E2.3**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2000

EEE PART II: M.Eng., B.Eng. and ACGI

**POWER, FIELDS AND DEVICES**

Monday, 19 June 2000, 2:00 pm

There are NINE questions on this paper.

There are three sections. Answer FIVE questions including at least ONE question from each of sections A, B and C.

Use a separate answer book for each section.

Time allowed: 3:00 hours

**Corrected Copy**

Qu 8, 3(c), 4

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**Information for Candidates**

Maxwell's equations:

$$\nabla \cdot \mathbf{D} = \rho \quad ; \quad \mathbf{D} = \varepsilon \mathbf{E} \qquad\qquad \nabla \cdot \mathbf{B} = 0 \quad ; \quad \mathbf{B} = \mu \mathbf{H}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad\qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Physical constants and material parameters:

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ H/m

$e = 1.602 \times 10^{-19}$ C

$kT = 0.025$ eV at 290 K

In $SiO_2$ $\quad \varepsilon = 4\varepsilon_0$

In silicon $\varepsilon = 11.7\varepsilon_0$

**Section A**
**Use a separate answer book for each section.**

1. (a) Describe the operation of the switch-mode power supply, SMPS shown in figure 1 and derive an equation for the output voltage as a function of duty-cycle. You may assume that the voltage drops across the semiconductors are negligible and that the capacitor is large enough to ensure that the voltage across it changes little during a switching cycle. [7]

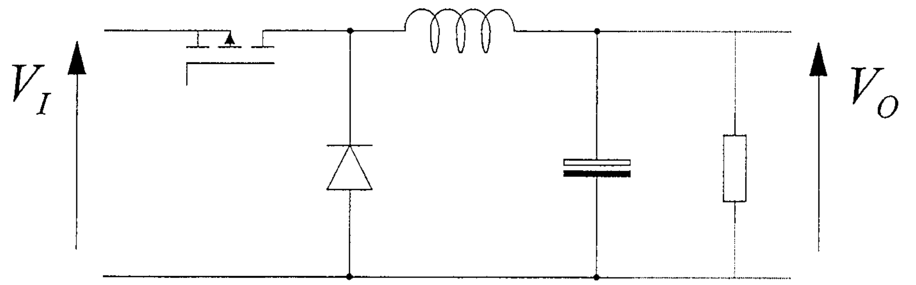


*Figure 1 A Buck SMPS*

(b) Calculate the switching frequency necessary to achieve an output voltage ripple better than 1% at 5V. The SMPS has an input voltage of $V_i$ = 24 V; an inductor of L = 100 μH and a capacitor whose value can be considered large but which has an effective series resistance of $R_{ESR}$ = 20 mΩ. [8]

(c) Estimate the efficiency when supplying 25W to a load if $R_{DS}$(on)=100 mΩ and $V_D$= 0.8 V [3]

(d) In general, will a buck or a boost SMPS provide a lower magnitude of output voltage ripple? [2]

2. (a) Why is a three-phase system adopted for power generation and transmission? [5]

(b) A star connected load composed of three impedances of $30\angle 65^\circ\ \Omega$ is connected to a 3-phase 50Hz supply with a line voltage of 400V. Calculate the total real power consumed by the load. [4]

(c) Draw a diagram showing how two wattmeters may be connected to measure the total power of a three-phase load. Give equations for the readings of each meter and state how these relate to the total power. State any type or connection of load for which the two-wattmeter method is not applicable. [5]

(d) Calculate the readings on the two wattmeters for the $30\angle 65^\circ\ \Omega$ star connected load and compare them with the total power. [4]

(e) For the load considered here one of the wattmeters reads close to zero. State, giving a reason, what load condition would cause one of the wattmeters to read exactly zero.

[2]

3. (a) Describe the principal parts of an induction machine and the materials they are constructed from. Outline the principle of operation. Label and describe the losses on a loss diagram similar to figure 2.

(Corrected copy)

[7]

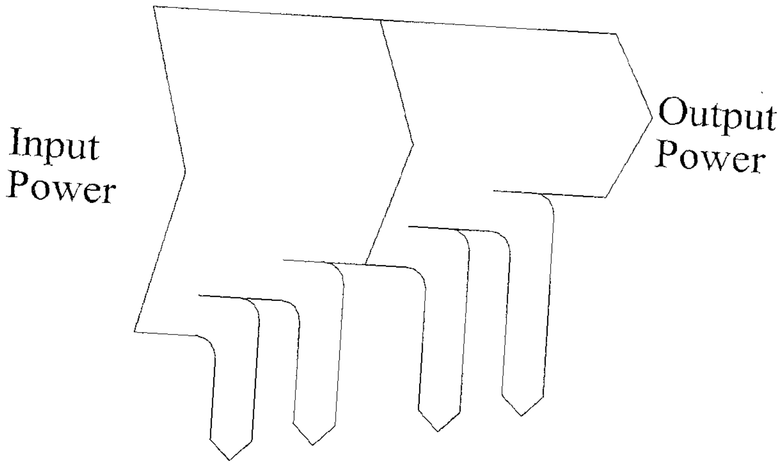


*Figure 2 A Diagram showing various power losses between input and output.*

(b) What is the expected form of the torque speed curve? Give an expression for the torque in terms of rotor current and slip. What is the relationship between rotor speed and slip? Justify the shape of the curve near and at synchronous speed.

[6]

(c) Figure 3 shows an induction machine equivalent circuit that has negligible rotor leakage reactance. It is a three-phase, two pole-pair machine supplied such that the voltage marked $V_A$ is 200V at 50 Hz. The referred rotor resistance is 0.5Ω. Find the value of slip which will give a torque of $T = 50$ Nm.

[7]

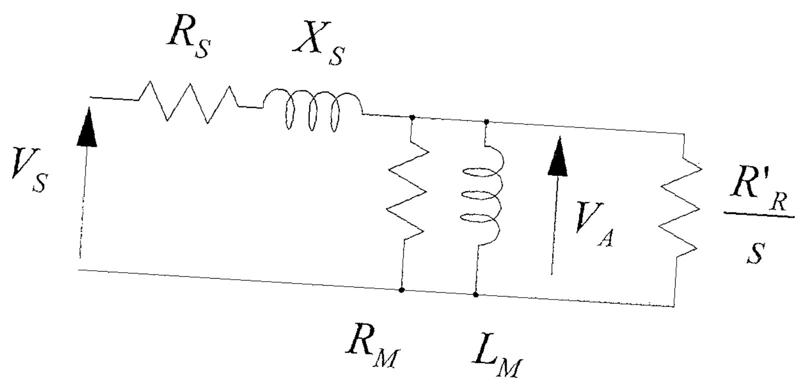


*Figure 3 An induction machine equivalent circuit.*

Assume $\mu_R = 1$ in conductors,

**Section B**

**Use a separate answer book for each section.**

4. Figure 4 shows the cross-sectional view of a coaxial cable, consisting of a solid inner conductor of radios *a*, and an outer conductor with radii *b* and *c*. The space between the conductors is filled with a dielectric.

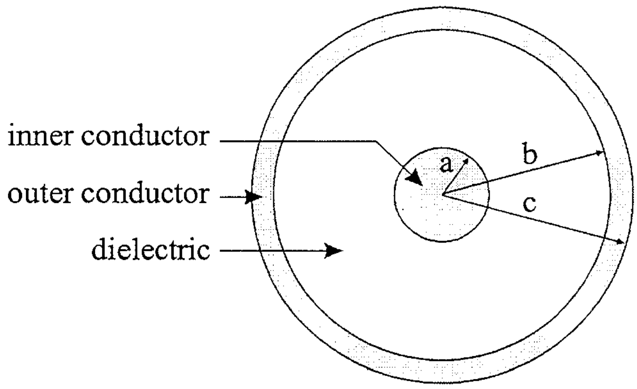


*Figure 4, A coaxial cable*

(a) Assuming that the two conductors are carrying equal and opposite, uniformly distributed currents I, show that the variation of the magnetic field with distance r from the centre of the coax is as follows:

$$H = \begin{cases} \dfrac{I}{2\pi}\dfrac{r}{a^2} \ \mathrm{A/m} & 0 \le r \le a \\ \dfrac{I}{2\pi r} \ \mathrm{A/m} & a \le r \le b \\ \dfrac{I}{2\pi}\dfrac{(c^2 - r^2)}{(c^2 - b^2)r} \ \mathrm{A/m} & b \le r \le c \end{cases}$$

[8]

(b) By considering the energy stored in the magnetic field, show that the so-called 'internal inductance' $L_i$ per unit length associated with the field *inside* the inner conductor is:

$$L_i = \frac{\mu_0}{8\pi} \qquad \mathrm{H/m}$$

Also derive an expression for the normal external inductance L associated with the field in the dielectric. Hence calculate the ratio $L_i/L$ for a coax with an inner conductor diameter of 0.8 mm, an outer diameter of 3.4 mm, and an outer conductor thickness of 0.1 mm. Would you expect the internal inductance $L_o$ of the outer conductor to be significant for this coax? You should justify your answer, but you are not required to calculate $L_0$ explicitly. [10]

(c) Explain why the internal inductance of both conductors can be neglected at high frequencies. [2]

5. (a) The electric field of a particular plane polarized, harmonic electromagnetic wave in a dielectric medium is given by:

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 \exp\{j(\omega t - \mathbf{k}.\mathbf{r})\}$$

where $\mathbf{E}_0$, $\mathbf{k}$ and $\omega$ are real constants.

State the propagation and polarization directions of this wave and, using an appropriate Maxwell equation, show that these two directions must be perpendicular to one another. Also show that the magnetic field associated with the wave may be expressed as:

$$\mathbf{H} = \frac{\hat{\mathbf{k}} \times \mathbf{E}_0}{\eta} \exp\{j(\omega t - \mathbf{k}.\mathbf{r})\}$$

where $\eta$ is the intrinsic impedance of the medium. [12]

Now consider a wave for which:

$$\mathbf{k} = k\hat{\mathbf{z}} \quad , \quad \mathbf{E}_0 = \frac{E_0}{\sqrt{2}}(\hat{\mathbf{x}} - j\hat{\mathbf{y}})$$

(b) By considering the time-variations of the x- and y-components, show that the electric field at any point in space has constant magnitude, and rotates about the direction of propagation. Show also that the sense of rotation is clockwise when looking along the direction of propagation.

The above circularly polarized wave is incident on a perfectly conducting reflector located in the plane z = 0. Show that the reflected wave is also circularly polarized, with opposite rotation sense. [8]

6. (a) Explain what is meant by an electric field line, and show that such lines are normal to equipotential surfaces at all points in an electric field. Show also that lines of electric field are always normal to the boundary between a dielectric and a perfect conductor. What is the behaviour of the magnetic field at such a boundary? [6]

(b) The figure 5 shows a cross-section through a so-called 'stripline' - a transmission line comprising a conducting strip embedded in a dielectric, with upper and lower ground planes.

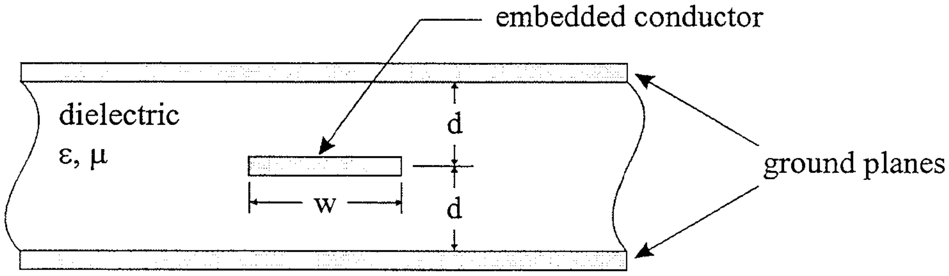


*Figure 5*

Assuming that the stripline supports a pure TEM wave travelling into the page, sketch lines of electric and magnetic field at an instant when the embedded conductor is at a positive potential with respect to the ground planes. What is the speed of propagation of this TEM wave? [6]

(c) Estimate the capacitance per unit length of the line, and hence derive an approximate expression for its characteristic impedance. If the line is to be constructed as a multi-layer printed circuit, by laminating layers of organic material of thickness 100 μm and relative permittivity 2.7, approximately what width of embedded conductor is required for a characteristic impedance of 50 Ω? [8]

**Section C**
**Use a separate answer book for each section.**

7. (a) Draw a labelled diagram of the small-signal equivalent circuit, excluding noise sources, of a bipolar junction transistor (BJT).

A particular BJT has a cut-off frequency $f_T = 60$ MHz and a DC current gain $\beta = 40$. Calculate the forward transit time, and the values of the principal components in the base-emitter equivalent circuit when the collector current is 20 mA. [8]

(b) The same BJT is connected as a common-emitter RF pulse amplifier, and is driven by an input voltage source connected between base and emitter via a resistance of 50 Ω.

Making suitable assumptions, calculate the charge stored in the base:

(i) when the collector current is 20 mA and the transistor is about to enter saturation, and
(ii) when the input voltage is raised further to 3.5V. [6]

(c) Explain why the Miller effect does not operate in a bipolar transistor held in saturation.

The voltage source in part (b) falls instantaneously from 3.5V to zero.
Explain briefly why the base-emitter junction remains forward biased for a time.

Deduce from your equivalent circuit an equation for the resultant rate-of-change of base charge, and solve it to estimate the delay before the transistor collector current begins to fall from its saturation value of 20 mA, i.e. before the transistor enters the active region of operation.
You may neglect the junction capacitances.

[6]

8. The substrate, source and drain of an n-channel enhancement MOSFET, T1 in figure 6, are all connected to earth, and the gate is used as the charge storage electrode in a 1-bit memory cell.

Transistor T1 has gate width x length, WxL, equal to 2µmx2 µm. Transistor T2, which is used to sense the charge stored in T1, has W = L = 1 µm. Both transistors are otherwise identical in construction, having an oxide thickness 0.05 µm and a substrate with a doping concentration of 3x1022/m3.

T1 is designed such that its threshold voltage, 1.8 V, is just reached when the charge placed on the gate is 3x10-15 C.

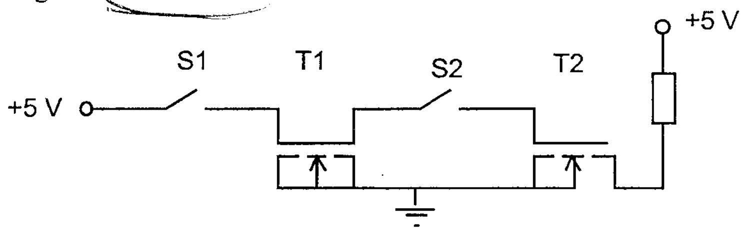


*Figure 6*

Answer ALL of the following questions relating to these transistors.

(a) Explain why a finite amount of charge is needed on the gate to cause T1 to reach threshold.

Where inside the transistor is the counter-charge stored which is delivered to the source/drain terminal when the gate is charged? Estimate the dimensions of the region containing this charge at threshold.

How much charge is needed on the gate of T2 to reach threshold (neglect overlap capacitances)? [10]

(b) Sketch a graph showing how the small-signal capacitance of T1 varies with gate voltage in the range 0-5 V. Explain qualitatively its shape, and its relationship to the so-called oxide capacitance $C_{OX}$. [5]

(c) The gate of T1 alone is charged to 5V by first closing, then opening, switch S1. The presence of the stored charge is subsequently sensed by closing switch S2, whereupon T2, for which $V_{GS}$ was previously zero, is turned on and its drain-source voltage becomes negligible.

Calculate the gate voltage of T2 after the closure of S2. [5]

9. (a) Give an equation for the DC current-voltage relationship in a p-n diode including recombination-generation currents. Why is the full equation usually unnecessary when considering forward current flow? [4]

(b) Draw the small-signal equivalent circuit used in SPICE to model a diode, including noise sources, and give the equations for the values of the noise sources. Explain why the SPICE model includes a conductance GMIN = $10^{-12}$ S in parallel with the other circuit components.

Give TWO possible reasons for the failure of SPICE to converge on a solution to a circuit problem which has been correctly specified. [10]

(c) The equations in SPICE which model a MOSFET's triode and saturation regions of operation are respectively

$$I_D = \frac{K_p}{2}\left(\frac{W}{L}\right)\left(2(V_{GS}-V_{th})V_{DS} - V_{DS}^2\right)(1 + \Lambda V_{DS})$$

$$\text{and } I_D = \frac{K_p}{2}\left(\frac{W}{L}\right)(V_{GS}-V_{th})^2 (1 + \Lambda V_{DS})$$

Why is the expression $(1 + \Lambda V_{DS})$ included in *both* equations?

Which of these equations in the physical model omits that bracketed expression?

Why is the expression NOT compatible with the physical model? [6]

## Answers and marking schedule   Section A Paper E2.3

(1) Describe the operation of the switch-mode power supply, SMPS shown in figure 1 and derive an equation for the output voltage as a function of duty-cycle. You may assume that the voltage drops across the semiconductors are negligible and that the capacitor is large enough to ensure that the voltage across it changes little during a switching cycle. [7]

*Switch on:*
*voltage across the inductor is positive*
*current increases and stores energy in L and C*

*Switch off:*
*Diode forced into conduction to provide path for current*
*Negative voltage imposed across inductor*
*Current decrease releasing energy from L to output (and C)*

*Output voltage is a filtered (average) version of the voltage at the junction of transistor and diode.*

$$\Delta i(on) = \frac{V_I - V_O}{L} t_{on} = \frac{V_I - V_O}{L} \cdot \frac{\delta}{f}$$

$$\Delta i(off) = \frac{-V_O}{L} t_{off} = \frac{-V_O}{L} \cdot \frac{1-\delta}{f}$$

$$\Delta i(on) + \Delta i(off) = 0$$

$$\frac{V_I - V_O}{L} \cdot \frac{\delta}{f} = \frac{V_O}{L} \cdot \frac{1-\delta}{f}$$

$$\frac{V_O}{V_I} = \delta$$

Calculate the switching frequency necessary to achieve an output voltage ripple better than 1% at 5V. The SMPS has an input voltage of $V_i = 24$ V; an inductor of L = 100 μH and a capacitor whose value can be considered large but which has an effective series resistance of $R_{ESR} = 20$ mΩ. [8]

$$v_r = \Delta i \, R_{ESR}$$



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$$\Delta i = \frac{V_I - V_O}{L}\frac{\delta}{f} = \frac{V_I - V_O}{fL}\frac{V_O}{V_I}$$

$$f = \frac{V_I - V_O}{L}\frac{V_O}{V_I}\frac{R_{ESR}}{v_r} = \frac{24-5}{100\times10^{-6}}\cdot\frac{5}{24}\cdot\frac{0.02}{0.01\times5} = 15.83\,\text{kHz}$$

Estimate the efficiency when supplying 25W to a load if $R_{DS}(on)$=100 mΩ and $V_D$= 0.8 V [3]

*For a 25 W output the output current and average inductor current will be 5 A.*

*MOSFET conduction power loss*

$$P_M = {i_{DS}}^2 R_{DS}(on)\delta = 5^2 \times 0.01 \times \frac{5}{24} = 0.52\ \text{W}$$

Diode conduction loss

$$P_D = i_D v_D(on)(1-\delta) = 5\times0.8\times\left(1-\frac{5}{24}\right) = 3.17\ \text{W}$$

$$\text{efficiency} = \frac{P_{out}}{P_{out}+P_D+P_M}\cdot100\% = \frac{25}{25+0.52+3.17}\cdot100\% = 87.1\%$$

In general, will a buck or a boost SMPS provide a lower magnitude of output voltage ripple? [2]

*Buck produces lower voltage ripple because only the inductor ripple current flows in the capacitor whereas the chopped total inductor current of the boost converter flows in the capacitor.*



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(2) Why is a three-phase system adopted for power generation and transmission? [5]

- *Generators are constructed with many coils distributed around their circumference to make good use of the magnetic circuit and to aid cooling.*
- *The many coils are series connected into a smaller number of phase groups.*
- *A large number of phases requires many individual circuits (and switches/breakers etc.) for connections to loads and is not cost effective.*
- *A balanced system is use to allow removal of the neutral conductor and therefore better use of available copper in cables.*
- *A small number of phases gives too much voltage cancellation between coils in almost opposite positions.*
- *A phase number above 1 is required to create rotating flux in AC motors.*

*Best compromise between competing factors is 3 phases*

A star connected load composed of three impedances of $30\angle 65^\circ\ \Omega$ is connected to a 3-phase 50Hz supply of 400V. Calculate the total real power consumed by the load. [4]

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.9\ \text{V}$$
$$I_P = \frac{V_P}{Z} = \frac{230.9}{30} = 7.70\ \text{A}$$
$$P = 3V_P I_P \cos(\phi) = 3 \times 230.9 \times 7.70 \times \cos(65^o) = 2.25\ \text{kW}$$

Draw a diagram showing how two wattmeters may be connected to measure the total power of a three-phase load. Give equations for the readings of each meter and state how these relate to the total power. State any type or connection of load for which the two-wattmeter method is not applicable.

[5]



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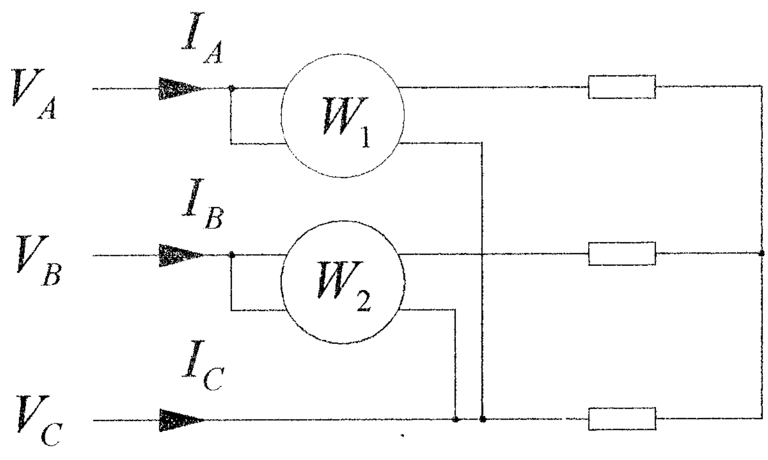


$$W_1 = (V_A - V_C)I_A \cos(\langle V_{AC} - \langle I_A) = \text{Re}\left[V_{AC} I_A^*\right]$$
$$W_2 = (V_B - V_C)I_B \cos(\langle V_{BC} - \langle I_B) = \text{Re}\left[V_{BC} I_B^*\right]$$
$$P_A + P_B + P_C = W_1 + W_2$$

*Two-wattmeter method is applicable only to 3-wire systems. Unbalanced 4-wire systems (in which a neutral current flows) will not give correct readings.*

Calculate the readings on the two wattmeters for the $30\angle 65°\ \Omega$ star connected load and compare them with the total power. [4]

$$W_1 = 400 \times 7.70 \times \cos(-30 - (-65)) = 2522 \text{ W}$$
$$W_2 = 400 \times 7.70 \times \cos(-90 - (-120 - 65)) = -268 \text{ W}$$
$$\text{W}_1 + W_2 = 2254 \text{ W}$$

For the load considered here one of the wattmeters reads close to zero. State, giving a reason, what load condition would cause one of the wattmeters to read exactly zero. [2]

*If the load impedance had an angle of 60° then W2 would observe a current 90° displaced from its voltage and would read zero. W1 would then read the total power.*



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(3)Describe principal parts of an induction machine and the type of material they are constructed from. Outline the principle of operation of an induction machine. Label and describe the losses on the loss diagram similar to figure 3. [7]

*Sketches expected showing:*
*Stator: annular laminations of steel assembled into stack. Slots punched out of inside surface carry a three-phase winding of copper wire.*
*Rotor: circular laminations assembled into a stack on a steel shaft. Slots in rotor surface are filled with aluminium to form rotor bars with shorting rings at each end.*

- *If three-phase stator winding is supplied with three-phase current then a total flux (sum of contributions from each phase) is a constant magnitude but the location of its peak magnitude rotates around the air-gap of the machine as the three-phase currents peak in turn.*
- *The rotating flux wave links with the rotor bars. A voltage is induced due to the rate of change of flux linkage. If the rotor is moving the rate-of-change of flux linkage depends on the velocity of the flux wave with respect to the rotor.*
- *Currents are driven along the shorted rotor bars.*
- *Rotor currents react with the flux to develop torque. The torque will be such a to oppose the relative motion, i.e., the torque acts to accelerate the rotor if its velocity is lower than that of the flux wave.*

*The losses are (from l to r);*
*Stator copper loss* $=3\ I_S^2 R_S$
*Stator iron loss*$=3\ V_S^2/R_m$ *(approx)*
*Rotor copper loss* $=3\ I_R^2 R_R$
*Windage, friction and rotor iron loss(which are speed dependent terms)*

*Also identified is the total power crossing the air-gap*$=3\ I'^2_R R'_R/s$

What is the expected form of the torque speed curve? Give an expression for the torque in terms of rotor current and slip. What is the relationship between rotor speed and slip. Justify the shape of the curve near and at synchronous speed. [6]



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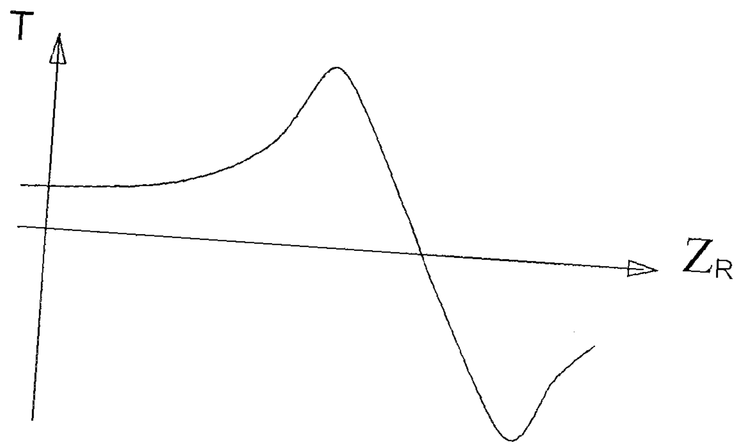


*Torque is expected to be zero at synchronous speed and to increase approximately linearly as the speed is decreased from here. The linear increase is not maintained once the slip frequency is considerable and the torque reaches a peak value and then falls away as the speed is further reduced.*

$$T_{em} = \frac{3 I'^{2}_{R} R'_{R} \frac{1-s}{s}}{Z_R}$$

$$Z_R = (1-s) Z_S$$

$$T_{em} = \frac{3 I'^{2}_{R} R'_{R}}{s Z_s}$$

*The linear relation between slip and torque is based on the voltage across the equivalent rotor resistance ($R_R/s$) being almost constant. Thus the current increases linearly with slip and the torque increases linearly with slip*

Figure 4 shows an induction machine equivalent circuit that has negligible rotor leakage reactance. It is a three-phase, two pole-pair machine supplied such that the voltage marked VA is 200V at 50 Hz. Find value of slip which will give a torque of T=50 Nm. The referred rotor resistance is 0.5:

[7]

$$T \quad \frac{1}{Z_R} 3I'^{2}_{R} R'_{R} \; {}^{1}\!/_{s} \quad 1$$

$$Z_R \quad 1 \quad s\, Z_S$$



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$$I'_R = s\frac{V_A}{R'_R}$$

$$T = \frac{1}{\omega_S(1-s)} 3s^2 \frac{V_A^{\ 2}}{R'^{\ 2}_R} R'_R \left(\frac{1-s}{s}\right)$$

$$s = \frac{\omega_S T R'_R}{3V_A^{\ 2}} = \frac{2\pi \times 50 \times 50 \times 0.5}{3 \times 200^2} = 0.065$$



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4 (a) From symmetry, know $\underline{H}$ is circumferential with $H$ depending only on $r$. ⇒ Ampère's Law for circular loop at any $r$ gives: $H.2\pi r = I_{ENCL}$ where $I_{ENCL}$ : current enclosed.

For $0 \le r \le a$ $\quad I_{ENCL} = \frac{\pi r^2}{\pi a^2} I \Rightarrow H = \frac{1}{2\pi r}\cdot\frac{r^2}{a^2} I = \frac{rI}{2\pi a^2}$ 3

For $a \le r \le b$ $\quad I_{ENCL} = I \quad H = \frac{I}{2\pi r}$ 2

For $b \le r \le c$ $\quad I_{ENCL} = I - \frac{r^2 - b^2}{c^2 - b^2} I = \frac{c^2 - r^2}{c^2 - b^2} I \Rightarrow H = \frac{(c^2 - r^2) I}{2\pi (c^2 - b^2) r}$ 3

(b) Energy density is $U_M = \frac{1}{2}\mu H^2$, so energy/length inside inner conductor is

$$\int_0^a \tfrac{1}{2}\mu H^2 . 2\pi r\, dr = \frac{\mu}{2}\cdot\int_0^a \left(\frac{rI}{2\pi a^2}\right)^2 . 2\pi r\, dr$$

$$= \frac{\mu I^2}{4\pi a^4}\int_0^a r^3 dr = \frac{\mu I^2}{16\pi}$$

Comparing with $\frac{1}{2}L_i I^2$, and assuming $\mu = \mu_0$, we have $L_i = \frac{\mu_0}{8\pi}$ 5

Can use same method for $L$:

$$\tfrac{1}{2}LI^2 = \tfrac{1}{2}\mu\int_a^b \left(\frac{I}{2\pi r}\right)^2 2\pi r\, dr = \frac{\mu I^2}{4\pi}\ln(b/a) \Rightarrow L = \frac{\mu_0}{2\pi}\ln(b/a)$$

Coax in question has $a = 0{\cdot}4$ mm, $b = \frac{3{\cdot}4 - 0{\cdot}1}{2} = 1{\cdot}6$ mm

$L_i/L = [4\ln(b/a)]^{-1} = 0{\cdot}18$ ie not negligible 3

$L_o$ should be much smaller; areas (vols. per unit length) are comparable for two conductors, but $U_M$ is much lower in outer conductor. (Detailed analysis gives $L_o/L \simeq 0{\cdot}014$) 2

(c) At high frequencies, skin effect confines fields in both conductors to surface layers ⇒ no energy storage in body of conductor. 2

5 (a) Propagation + polarization dir'ns are $\hat{k}$ and $\hat{E_0}$ respectively.

M.E. I requires that $\nabla \cdot E = 0$ (in homogeneous dielectric).

But $\nabla \cdot E = \partial/\partial x[E_{0x} \exp j\phi] + \partial/\partial y[E_{0y} \exp j\phi] + \partial/\partial z[E_{0z} \exp j\phi]$

with $\phi = \omega t - \underline{k} \cdot \underline{r}$, and $\frac{\partial \phi}{\partial x} = -k_x$ etc

$\Rightarrow \quad \nabla \cdot E = -j \exp j\phi . [k_x E_{0x} + k_{0y} E_{0y} + k_{0z} E_{0z}] = -j \exp j\phi [\underline{E_0} \cdot \underline{k}]$

$= 0$ everywhere only if $\underline{E_0} \cdot \underline{k} = 0 \Rightarrow \underline{E_0} \perp$ to $\underline{k}$ 5

To get $\underline{H}$, use M.E. III : $\nabla \times E = -\mu \dot{\underline{H}}$

Using fact that $\frac{\partial \phi}{\partial x} \Leftrightarrow -k_x$ etc, can write $\nabla \times \underline{E}$ as :

$$\nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -jk_x & -jk_y & -jk_z \\ E_{0x} & E_{0y} & E_{0z} \end{vmatrix} \exp j\phi = -j(\underline{k} \times \underline{E_0}) \exp j\phi = -\mu \frac{\partial \underline{H}}{\partial t}$$

Integrating wrt $t$ gives :

$$\underline{H} = \frac{j}{\mu} \frac{(\underline{k} \times \underline{E_0}) \exp j\phi}{j\omega} = \frac{\hat{k} \times \underline{E_0}}{\eta} \exp j\phi \quad \text{with } \eta = \frac{\mu\omega}{k}$$ 7

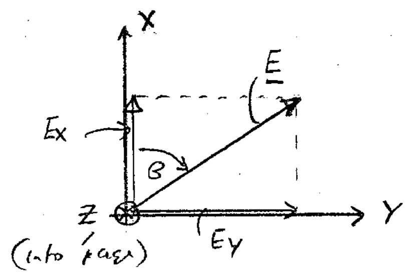
(b) Now have : $\underline{E} = \hat{x} \frac{E_0}{\sqrt{2}} \exp j(\omega t - kz) - \hat{y} \frac{E_0}{\sqrt{2}} \exp j(\omega t - kz + \frac{\pi}{2})$

Consider time-variation of (real) x- and y- components at $z=0$:

$E_x(z=0,t) = \frac{E_0}{\sqrt{2}} \cos \omega t$

$E_y(z=0,t) = -\frac{E_0}{\sqrt{2}} \cos(\omega t + \pi/2)$

$= \frac{E_0}{\sqrt{2}} \sin \omega t$



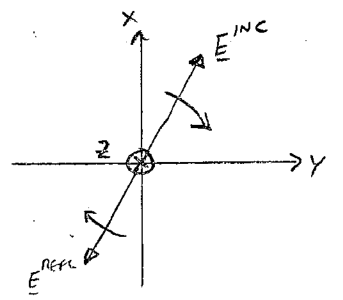

By inspection :

$|E| = E_0$, and $\theta = \omega t \Rightarrow \underline{E}$ has constant magnitude & rotates clockwise about $+z$ dir'n 5

Wave hits reflector at normal incidence $\Rightarrow \underline{k}^{REFL} = -\underline{k}^{INC}$

$\therefore E_{//} = 0$ at reflector surface $\Rightarrow \underline{E}^{REFL}(z=0,t) = -\underline{E}^{INC}(z=0,t)$

[ Follows that $\underline{H}^{REFL}(z=0,t) = \underline{H}^{INC}(z=0,t)$ ]




So: $\underline{E}^{REFL}$ also rotates clockwise when looking along $+z$ dir'n.

But, reflected wave is travelling in $-z$ dir'n, so rotation sense rel to propagation dir'n is reversed.

$\Rightarrow$ circular, anti-clockwise/LH 3



E2.3

6. (a) Electric field line = line that is everywhere tangential to direction of $\underline{E}$.

$\underline{E}$ defined as $\underline{E} = -\nabla V$, where $V$ = potential, so change $dV$ in potential along line element $d\underline{l}$ is $dV = -\underline{E} \cdot d\underline{l}$.

But, for any $d\underline{l}$ lying on an equipotential surface, $dV = 0$

$\Rightarrow$ $\underline{E}$ must be $\perp$ to surface. 2

$\underline{E} \to 0$ inside perfect conductor (otherwise $\underline{J} = \sigma\underline{E}$ would be infinite)

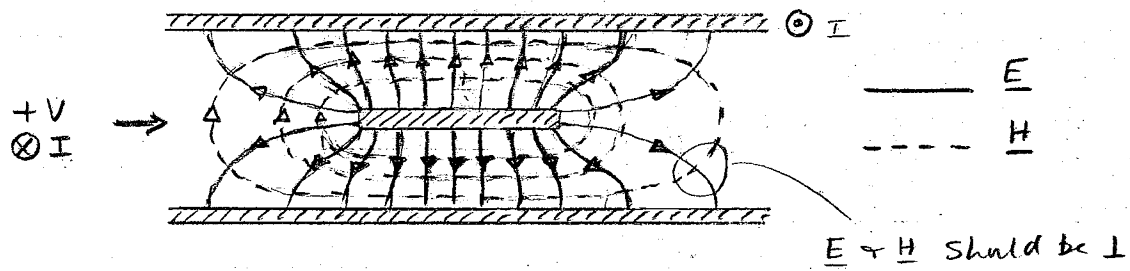
So, continuity of $E_{//}$ at surface implies $E_{tan} = 0$ on surface.

$\Rightarrow$ $\underline{E}$, and hence field lines, must be $\perp$ to boundary 2

$\underline{H}$ also vanishes inside perfect conductor, and continuity of $B_\perp$ at surface implies $H_{norm} = 0$.

$\Rightarrow$ $\underline{H}$ is purely tangential on dielectric side of boundary. 2

(b)




4

Assuming pure TEM, propagation speed is

$$u = \frac{1}{\sqrt{\mu\varepsilon}}$$ 2

To estimate C, neglect fringing, and assume strip is thin. Then $C = \dfrac{2W\varepsilon}{d}$

$$Z = \sqrt{\frac{L}{C}} = \frac{\sqrt{LC}}{C} = \frac{1}{uC} = \frac{\sqrt{\mu\varepsilon}\cdot d}{2W\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}}\cdot\frac{d}{2W} = \eta\cdot\frac{d}{2W}$$ 5

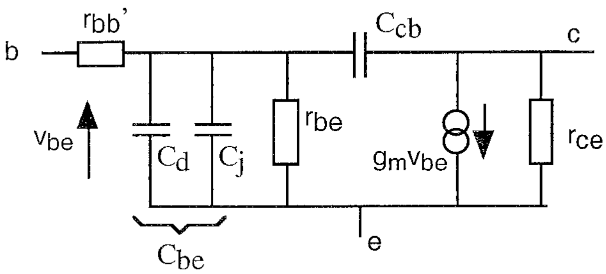
$\varepsilon_r = 2.7$ so $\eta = \dfrac{377}{\sqrt{2.7}} \approx 229\ \Omega$, and given $d = 100\ \mu m$

For $Z = 50\ \Omega$ require $W = \dfrac{\eta d}{2Z} = \dfrac{229 \times 100\ \mu m}{2 \times 50} \approx \underline{\underline{0.23\ mm}}$ 3

Answers - key points only

7. (i)

**Small-signal equivalent circuit**




$TF = 1/\omega_T = 1/120\pi\ \mu s = 2.5$ ns.

$r_{be} = \beta/g_m = 40/(I_c/V_T) = 40/(20/25) = 50\ \Omega$

$r_{be}C_{be} = \beta TF = 100$ ns

Thus $C_{be} = 100/50$ nF = 2 nF.

(ii) Assume $V_{be}(on) = 0.7$ V

(a) When transistor is in active region, $Q_B = I_C TF = 20\text{x}10^{-3}\text{x}2.5\text{ns} = 5\text{x}10^{-11}$ C

(b) In saturation, $Q_B = I_B\tau_B = I_B\beta TF = [(3.5-0.7)/50]\text{x}100\text{ns} = 5.6\text{x}10^{-9}$ C

(iii) Since $V_{ce}$ is constant in saturation, there is no gain to give feedback thro' $C_{cb}$.

The delay is due to the time taken to remove the majority base charge ~~thro' the base contact~~ by recombination.

Until the transistor enters the active region,

$$\frac{dQ_B}{dt} = -\frac{Q_B}{\tau_B} - I_B \quad \text{where } I_B \text{ is approximately constant at } 0.7\text{V}/50\Omega.$$

Hence (since $\tau_B$ is a constant), $\frac{d(Q_B + I_B\tau_B)}{dt} = -\frac{Q_B + I_B\tau_B}{\tau_B}$

Solution $Q_B(t) = [Q_B(0)+I_B\tau_B]\exp(-t/\tau_B) - I_B\tau_B$

$$= [5.6\text{x}10^{-9} + (0.7/50)\text{x}10^{-5}]\exp(-t/10^{-5}) - (0.7/50)\text{x}10^{-5}$$

and when $Q_B(t) = 5\text{x}10^{-11}$ C, i.e the transistor just leaves saturation,

$$-t/t_B = \ln\frac{[5\text{x}10^{-11} + (0.7/50)\text{x}10^{-5}]}{[5.6\text{x}10^{-9} + (0.7/50)\text{x}10^{-5}]} = -0.0392$$

i.e. t = 3.92 ns.

8. (i) The gate-substrate capacitance must be charged as the gate voltage is raised, until the channel is formed.

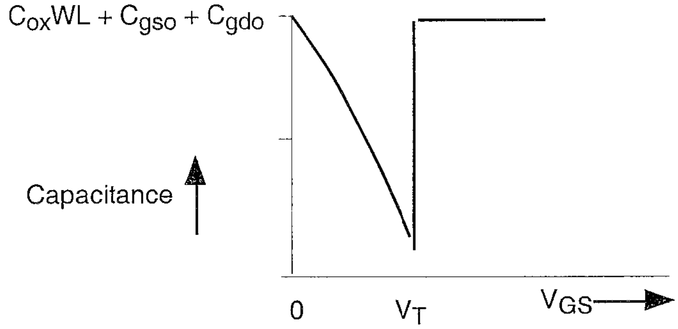
Counter-charge lies in bulk/substrate, in a depletion layer directly below the gate. Dimensions of region are WxLxd =2μm x 2μm x d.

Charge $3x10^{-15}$ C is stored at uniform density $eN_A$ in this volume. Hence at threshold

$(2x2xd)x10^{-18} = 3x10^{-15}/3x10^{22}x1.602x10^{-19}$ Hence d = 0.156 μm.

Transistor T2 has WxL =1/4 of that for T1, so charge needed is $7.5x10^{-16}$ C.

(ii)




Below threshold, depletion layer capacitance ($\propto$ variable depth d) is in series with oxide capacitance $C_{ox}WL$. As $V_{GS}$ rises, so d increases from zero, causing series combination to fall. At threshold, channel forms from source-drain and connects gate-source across oxide capacitance alone, so capacitance rises abruptly to $C_{ox}WL$ ($=C_{GS}+C_{GD}$). Good students will remember to add overlap capacitances.

(iii) Assume (a) $Q_G = Q_B + (V_G - V_{th})C_{GS}$ when $V \geq V_{th}$ (b) $Q_{B1}$ (bulk charge of T1) does not change during charge sharing, because $V_G>1.8V$ (since T2 turns on).

(c) Since T2 is said to turn fully on ($V_{DS}$ small), so $C_{GS2} = 0.5C_{ox}WL$

Then, if final voltage is V:

$$Q_{B1}+(5 - 1.8)C_{ox}x2x2x10^{-12} = [Q_{B1}+(V - 1.8)C_{ox}x2x2x10^{-12}] + [0.25Q_{B1}+(V - 1.8)x0.5C_{ox}x1x10^{-12}]$$

$$\text{Thus } (V - 1.8) = \frac{(5 - 1.8)x2x2x10^{-12} - 0.25Q_{B1}/C_{ox}}{4.5x10^{-12}}$$

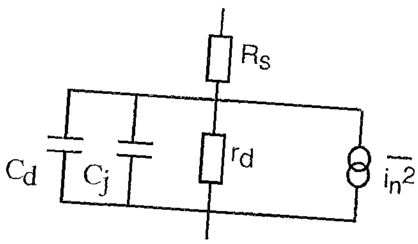
Now $C_{ox} = 4.0x8.85x10^{-12}/0.05x10^{-6}$, so that

$$V = 1.8 + [3.2x4x10^{-12} - 7.5x10^{-16}/(4.0x8.85x10^{-12}/0.05x10^{-6})]/4.5x10^{-12}$$
$$= 1.8 + [12.8x10^{-12} -1.06x10^{-12}]/4.5x10^{-12} \qquad = 4.41 \text{ V}$$

9. (i) $I_d = I_s[\exp(V/V_T) - 1] + I_{RG}[\exp(V/2V_T) - 1]$ where $V_T = kT/e$

The first term is normally dominant in forward bias, because it increases faster with bias voltage. At small forward currents and in reverse bias the second term, due to thermal generation and recombination of carriers within the depletion region, becomes important.

(ii)




Noise equations $\overline{i_n^2} = 2e(I+2I_s)\Delta f$

GMIN is used because the value of $r_d$ may become so large that SPICE cannot converge unless all other impedances are correspondingly raised. Good students may attribute this to the numerical resolution neede to model the resulting current flow.

Reasons for SPICE to fail:

(a)Too large a ratio of impedances present.

(b) errors in the drawn circuit or netlist.

(c) 'floating' nodes

(d) An over-large number of nonlinear circuit elements.

(iii) The triode equation inappropriately includes the factor $(1 + \Lambda V_{DS})$ in order that the two equations have no discontinuity at the onset of saturation. In the physical model this factor models the decrease in effective channel length which occurs only in the saturation region.