IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

EEE PART II: M.Eng., B.Eng. and ACGI

POWER, FIELDS AND DEVICES

Day, Date, Time

Time Allowed: 3:00 hours

There are NINE questions on this paper.

There are three sections. Answer FIVE questions including at least ONE question from each of sections A, B and C.

Use a separate answer book for each section.

SAMPLE QUESTIONS FOR SECTION C - DEVICES

Section C
Use a separate answer book for this section

7. The wave equation for an undamped, vibrating beam is:

$$EI\ \partial^4 y/\partial x^4 = -\rho A\ \partial^2 y/\partial t^2$$

Here A and I are the cross-sectional area and second moment of area of the beam, respectively. E and ρ are the Young's modulus and density of the beam material.

(a) Show that time-harmonic solutions to the wave equation are associated with space variations X(x) having the general form:

$$X(x) = A \sin(kx) + B \cos(kx) + C \sinh(kx) + D \cosh(kx)$$

Find an expression for the constant k. [4]

(b) A cantilever beam of length L is built in at one end and free at the other. What boundary conditions must be satisfied in solving the wave equation? Show that the possible angular frequencies of vibration are given by:

$$\omega_0 = (\beta/L)^2 \sqrt{\{EI/\rho A\}}$$

where β satisfies the eigenvalue equation cos(β) cosh(β) = -1. [10]

(c) A built in silicon cantilever beam of length 1 mm has depth 2 μm and breadth 100 μm. Calculate the frequency of the lowest order bending mode. [6]

(You may assume that $E = 1.08 \times 10^{11}$ N/m^2 and $\rho = 2330$ kg/m^3 for Si)

8 (a) Derive scaling laws for *gravitational*, *elastic* and *surface tension* forces. Explain the consequence of the differences between each possible pair of laws. [9]

(b) A Gaussian optical beam travelling in the z-direction has an electric field distribution in the (x, y) plane given by:

$$E(r, z) = A_0 \exp\{-r^2/w^2\} \exp\{-jk_0r^2/2R\}$$

Here, w, R and z_0 are given by:

$$(w/w_0)^2 = \{1 + (z/z_0)^2\} \; ; \; R = z\,\{1 + (z_0/z)^2\} \; ; \; z_0 = k_0w_0^2/2, \text{ where } k_0 = 2\pi/\lambda$$

Sketch the variations of the beam radius and phase-front with z. What is the significance of the parameter z_0? Find an expression for the waist radius w_0 that gives the minimum beam radius w after travelling a fixed distance z. What scaling law does this correspond to? [11]

9. (a) What advantages do MEMS switches have over semiconductor switches at RF frequencies? What are the main limitations of MEMS switches? [6]

(b) Sketch the structure of a typical MEMS capacitive shunt switch based on a simple metal bridge. Explain why such a device exhibits *snap-down* behaviour, and sketch the variation of the bridge height with applied voltage. [10]

(c) Assuming the device in part (b) is designed to be resonant in the down state, show that the down-state isolation at the resonant frequency is given approximately by:

$$|S_{21}|^2 \approx \frac{4R_s^2}{Z_0^2}$$

where R_s is the series resistance of the shunt, and Z_0 is the characteristic impedance of the transmission line. [4]