Paper Number(s): **E1.6**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2002

EEE/ISE PART I: M.Eng., B.Eng. and ACGI

COMMUNICATIONS I

Wednesday, 22 May 10:00 am

There are FIVE questions on this paper.

Answer THREE questions.

Corrected Copy

Time allowed: 2:00 hours

Examiners responsible:

First Marker(s): Gurcan,M.K.

Second Marker(s): Yeatman,E.M.

Useful equations

If the signal $g(t)$ is periodic with period $T_0 = \frac{2\pi}{\omega_0}$. The trigonometric Fourier series is

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \qquad \text{where } a_0 = \frac{1}{T_0}\int_0^{T_0} g(t)\, dt \text{, and}$$

$$a_n = \frac{2}{T_0}\int_0^{T_0} g(t)\cos(n\omega_0 t)\, dt \text{ for } n \geq 1 \qquad b_n = \frac{2}{T_0}\int_0^{T_0} g(t)\sin(n\omega_0 t)\, dt \text{ for } n \geq 1$$

The compact Fourier series is given by

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n),\ C_n = \sqrt{a_n^2 + b_n^2}\ , \qquad C_0 = a_0 \text{ and } \theta_n = \tan^{-1}\frac{-b_n}{a_n}$$

The exponential Fourier series is given by

$$g(t) = \sum_{n=-\infty}^{\infty} D_n \exp(jn\omega_0 t) \text{ where}$$

$$D_n = \frac{1}{T_0}\int_{T_0} g(t)\exp(-jn\omega_0 t)\, dt \quad \text{or} \quad D_n = \frac{C_n}{2}\exp(j\theta_n) \text{ and } D_{-n} = \frac{C_n}{2}\exp(-j\theta_n).$$

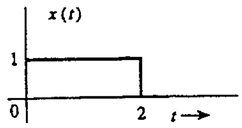
Useful Fourier Transformations

$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)\exp(j\omega t)\, d\omega$	$\Leftrightarrow$	$G(\omega) = \int_{-\infty}^{\infty} g(t)\exp(-j\omega t)\, dt$
$g(-t)$	$\Leftrightarrow$	$G(-\omega)$
$\exp(-at)\, u(t)$	$\Leftrightarrow$	$\frac{1}{a + j\omega}$
$\cos\omega_0 t$	$\Leftrightarrow$	$\pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$
$\sin\omega_0 t$	$\Leftrightarrow$	$j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$
$\text{rect}\left(\frac{t}{\tau}\right)$	$\Leftrightarrow$	$\tau\ \text{sinc}\left(\frac{\omega\tau}{2}\right)$
$\frac{W}{\pi}\text{sinc}(W\, t)$	$\Leftrightarrow$	$\text{rect}\left(\frac{\omega}{2W}\right)$
$g(t) = \Delta\left(\frac{t}{\tau}\right)$	$\Leftrightarrow$	$\frac{\tau}{2}\text{sinc}^2\left(\frac{\omega\tau}{4}\right)$
$g(t \pm T)$	$\Leftrightarrow$	$G(\omega)\exp(\pm j\omega T)$
$\frac{1}{2j}[g(t+T) - g(t-T)]$	$\Leftrightarrow$	$G(\omega)\sin(\omega T)$
$g(t)\cos\omega_c t$	$\Leftrightarrow$	$\frac{1}{2}[G(\omega-\omega_c) + G(\omega+\omega_c)]$

Two useful integrals : $\int \frac{1}{x^2 + a^2}\, dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$ and

$$\int \frac{x^2}{x^2 + a^2}\, dx = x - a\tan^{-1}\frac{x}{a}$$

1 a) Find E_x and E_y, the energies of the signals $x(t)$ and $y(t)$ shown in Fig. 1.1. Sketch the signals $x(t)$ + $y(t)$ and $x(t)$ -$y(t)$ and calculate the energies of either of these two signals. [5]



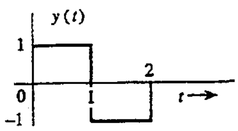


Figure 1.1.

b) A periodic signal $g(t)$ is expressed by the following Fourier series:

$$g(t) = 3\cos(t) + \sin\left(5t - \frac{\pi}{6}\right) - 2\cos\left(8t - \frac{\pi}{3}\right).$$

i) Sketch the amplitude and phase spectra for the compact Fourier series. [2]

ii) By inspection of spectrum in part (i), write the exponential Fourier series for $g(t)$. [2]

iii) By inspection of spectra in part (ii), sketch the double-sided magnitude spectrum for $g(t)$. [2]

iv) By inspection of coefficients in part (iii), plot the double-sided power spectral density for $g(t)$. Using the power spectral density coefficients calculate the signal power. [1]

c) Consider the time waveform, $g(t) = e^{-at}\,u(t)$ where $a>0$, given in Figure 1.2.

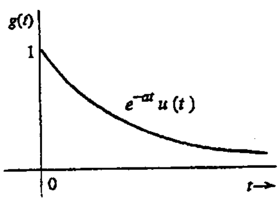


Figure 1.2.

i) Find the Fourier transform of $g(t)$. [2]

ii) Using $g(-t) \Leftrightarrow G(-\omega)$ and the result of part (i), find the Fourier transform of $e^{at}\,u(-t)$ and $e^{-|at|}$. [2]

iii) Verify Parseval's theorem for the signal $g(t) = e^{-at}\,u(t)$. [2]

iv) Estimate the essential bandwidth W rad/s of the signal $g(t) = e^{-at}\,u(t)$ if the essential band is required to contain 95% of the signal energy. [2]

2 a) The signal in Figure 2.1 is modulated with carrier $\cos(10t)$. Find the Fourier transform of this signal using the appropriate properties of the Fourier transform given in the Fourier transformation table on page 1. Sketch the magnitude spectrum for the signal given in Figure 2.1. *Hint:* This function can be expressed in the form $g(t)\cos\omega_0 t$. [6]

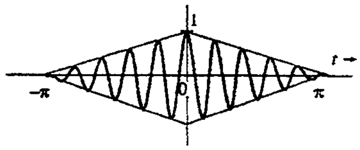


Figure 2.1.

b) A certain channel has ideal amplitude, but non-ideal phase response (Figure 2.2) given by [7]

$|H(\omega)| = 1$

$\theta_h(\omega) = -\omega t_0 - k \sin \omega T \qquad k << 1$

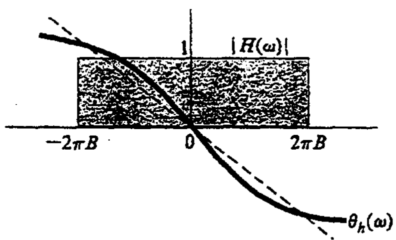


Figure 2.2.

Show that *y(t)*, the channel response to an input pulse *g(t)* band-limited to *B* Hz, is

$$y(t) = g(t - t_0) + \frac{k}{2}\left[g(t - t_0 - T) - g(t - t_0 + T)\right]$$

Hint: use $\exp(-jk\sin\omega T) \approx 1 - jk\sin\omega T$

c) Find the mean square value (or power) of the output voltage *y(t)* of the system shown in Figure 2.3 if the input voltage power spectral density is $S_x(\omega) = \mathrm{rect}\left(\frac{\omega}{2}\right)$. [7]

Calculate the power (mean square value) of the input signal *x(t)*.

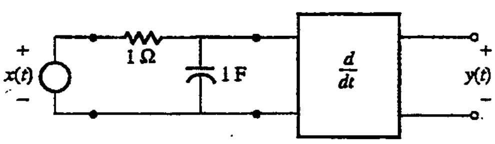


Figure 2.3.

3 a) Two signals $m_1(t)$ and $m_2(t)$, both band-limited to 5000 rad/s, are to be transmitted simultaneously over a channel by the multiplexing scheme shown in Figure 3.1. The signal at point b is the multiplexed signal, which now modulates a carrier of frequency 20,000 rad/s. The modulated signal at point c is transmitted over a channel.

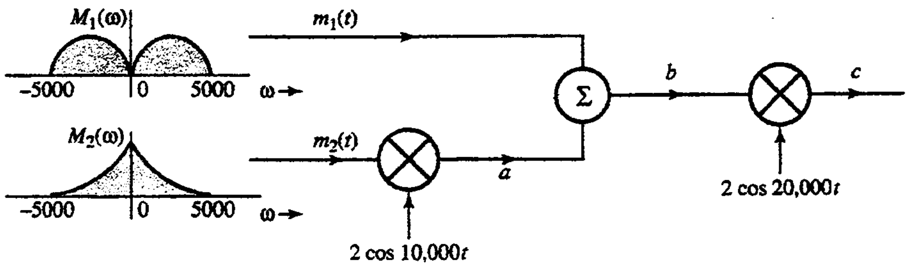


Figure 3.1.

i) Sketch the amplitude spectra at points a, b, and c. [3]

ii) Design a receiver to recover signals $m_1(t)$ and $m_2(t)$ from the modulated signal at point c. [4]

b) For each of the following baseband signals: (I) $m_1(t) = \cos 1000t$; (II) $m_2(t) = 2\cos 1000t - \cos 2000t$; (III) $m_3(t) = \cos(1000t)\ \cos(3000t)$:

i) sketch the amplitude spectra of $m_i(t)$, $i = 1,2,3$. [2]

ii) sketch the amplitude spectrum of the DSB-SC signals $m_i(t)\cos(10000t)$, $i = 1,2,3$. [3]

iii) identify the upper sideband (USB) and the lower sideband (LSB) of the spectra. [2]

iv) identify the frequencies in the baseband, and the corresponding frequencies in the DSB-SC USB, and LSB spectra. [1]

c) Figure 3.2 shows a scheme for coherent (synchronous) demodulation. Show that this scheme can demodulate the full AM signal $[A + m(t)]\cos \omega_c t$ regardless of the value of A. Show that a scheme that can demodulate DSB-SC can also demodulate full AM. Is the converse true? Explain. [5]

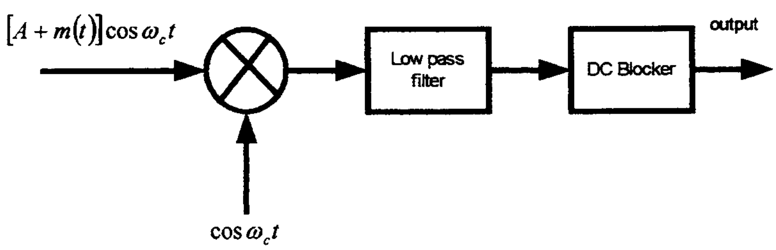


Figure 3.2.

4 a) A frequency-modulated signal with carrier frequency $\omega_c = 2\,\pi\,10^5$ rad/s is described by the equation $S_{FM} = 10\cos(\omega_c\, t + 5\sin 3000\, t + 10\sin 2000\pi\, t)$

i) Find the power of the modulated signal. [1]

ii) Find the frequency deviation $\Delta\, f$ and the modulation index β. [2]

iii) Estimate the bandwidth of S_{FM}. [2]

b) Consider the signal *m(t)*, given in Figure 4.1. A modulating signal is generated by passing the signal *m(t)* through a filter, which lets through the first 3 harmonics of the fundamental frequency and maintains the maximum and minimum signal amplitudes. Estimate the bandwidth for a frequency modulated signal when the filtered modulating signal is used to modulate a carrier of $f_c = 100$ MHz. Assume that the angular frequency deviation constant is $k_f = 2\pi \times 10^5$ rad/Vs. [5]

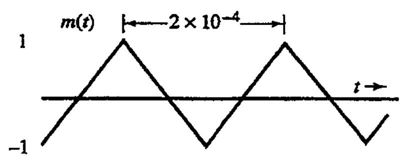


Figure 4.1.

c) A signal *g(t)* band-limited to *B* Hz is sampled by a periodic pulse train $P_{T_s}(t)$ made up of a unity amplitude rectangular pulse of width *1/8B* seconds (centered at the origin) repeating at the Nyquist rate *(2B* pulses per second). Show that the sampled signal $\bar{g}(t)$ is given by [5]

$$\bar{g}(t) = \frac{1}{4} g(t) + \sum_{n=1}^{\infty} \frac{2}{n\,\pi} \sin\left(\frac{n\pi}{4}\right) g(t)\cos(n\omega_s t), \qquad \omega_s = 4\,\pi\, B.$$

d) A compact disc (CD) records each audio channel signals digitally by using PCM. Assume the audio signal bandwidth to be 15 kHz.

i) What is the Nyquist rate? [1]

ii) If the Nyquist samples are quantised into L = 65,536 levels and then binary coded, determine the number of binary digits required to encode a sample. [2]

iii) Determine the number of binary digits per second (bit/s) required to encode the audio signal. [1]

iv) For practical reasons, signals are sampled at a rate well above the Nyquist rate. Practical CDs use 44,100 samples per second. If L = 65,536, determine the number of bits per second required to encode the signal, and the minimum bandwidth required to transmit the encoded signal. [1]

5 a) In Figure 5.1, the input signal is $\phi(t) = m(t)$ and the amplitude satisfies the relationship $A >> |\phi(t)|$. The two diodes are identical with a resistance r ohms in the conducting mode and infinite resistance in the cut-off mode. Figure 5.2 shows the diode current versus voltage characteristics.

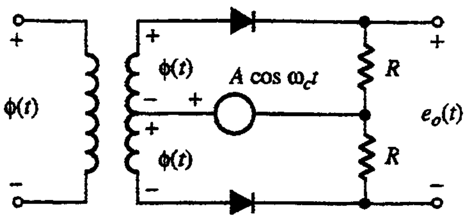


Figure 5.1

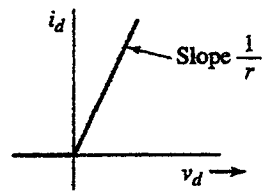


Figure 5.2

i) Show that the output $e_o(t)$ is given by [4]

$$e_o(t) = \frac{2R}{R+r} w(t)\, m(t)$$

where $w(t)$ is the switching periodic signal shown in Figure 5.3 with amplitude spectrum given in Figure 5.4.

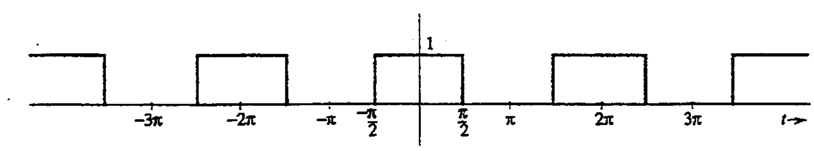


Figure 5.3

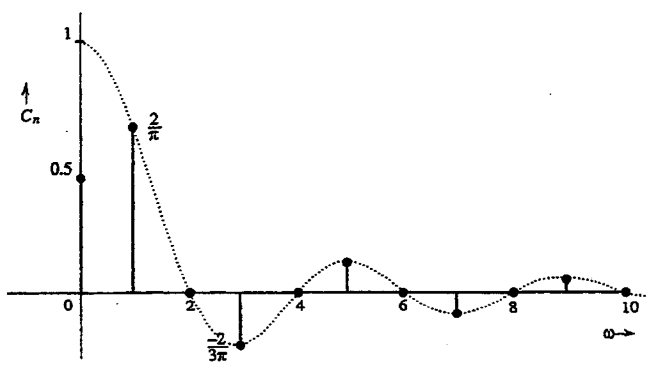


Figure 5.4.

ii) Show that this circuit can be used as a DSB-SC modulator. [3]

iii) How would you use this circuit as a synchronous demodulator for DSB-SC signals. [2]

b) A frequency-modulated signal is given as $u(t) = 100 \cos[2000\,\pi\, t + \phi(t)]$ where $\phi(t) = 5 \sin(20\,\pi\, t)$.

Question continued over

Using the values of $J_n(5)$ for $n=0,1,.....5$ given in the following table,

n	0	1	2	3	4	5
$J_n(5)$	-0.178	-0.328	0.047	0.365	0.391	0.261

i) determine and sketch the amplitude spectrum [4]

ii) determine the percentage of the modulated signal power carried by these harmonics [1]

iii) From Carlson's rule, determine the approximate bandwidth for the FM signal. [2]

c) An analogue signal, whose amplitude is uniformly distributed between –A and +A V, is to be quantized using a uniform quantizer, which has the transfer function shown in Figure 5.5. [5]

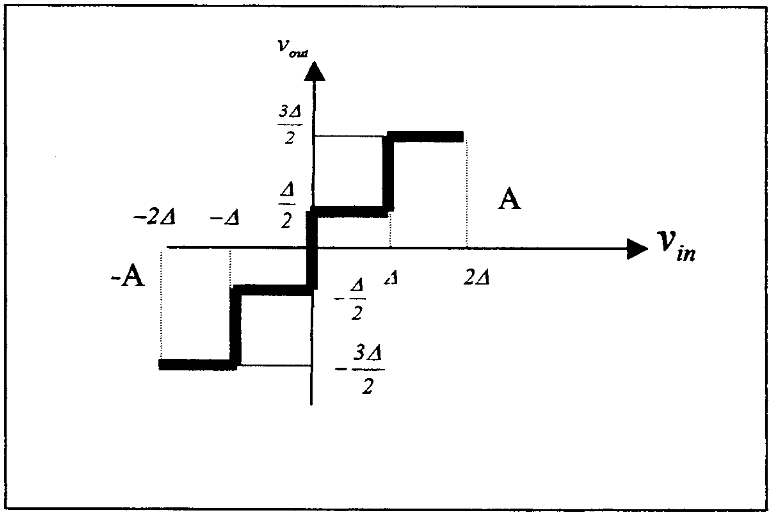


Figure 5.5

Find the quantization noise power in terms of the maximum signal amplitude.

Examinations : EEE/ISE Part I, Communications I, 2002

MODEL ANSWER and MARKING SCHEME

First Examiner: Dr. Mustafa Gurcan	**Paper Code: E1.6**
Second Examiner Dr. E. M. Yeatman	**Question 1-a /b Page 1 *out of 15***
Question labels in left margin	**Marks allocations in right margin**

1-a $E_x = \int_0^2 (1)^2 dt = 2$ $\quad E_{x+y} = \int_0^1 (2)^2 dt = 4$

$E_y = \int_0^1 (1)^2 dt + \int_1^2 (-1)^2 dt = 2$ $\quad E_{x-y} = \int_1^2 (2)^2 dt = 4$

$E_{x+y} = E_{x-y} = E_x + E_y$

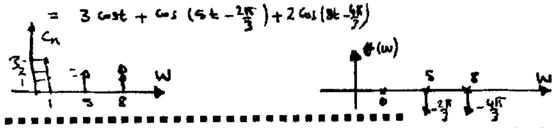
1-b $g(t) = 3\cos t + \sin\left(5t - \frac{\pi}{6}\right) - 2\cos\left(8t - \frac{\pi}{3}\right)$

For compact trigonometric form all terms must have cosine form and amplitudes must be positive.

We re-write $g(t)$ as

i) $g(t) = 3\cos t + \cos\left(5t - \frac{\pi}{6} - \frac{\pi}{2}\right) + 2\cos\left(8t - \frac{\pi}{3} - \pi\right)$

$= 3\cos t + \cos\left(5t - \frac{2\pi}{3}\right) + 2\cos\left(8t - \frac{4\pi}{3}\right)$

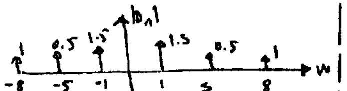


ii) $D_0 = D_2 = D_{-2} = D_3 = D_{-3} = D_4 = D_{-4} = 0$, $D_1 = D_{-1} = 3/2$, $D_5 = \frac{1}{2}\exp\left(-j\frac{2\pi}{3}\right)$

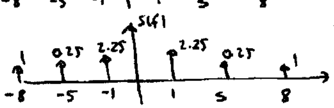
$D_{-5} = \frac{1}{2}\exp\left(j\frac{2\pi}{3}\right)$, $D_8 = \exp\left(-j\frac{4\pi}{3}\right)$, $D_{-8} = \exp\left(j\frac{4\pi}{3}\right)$

$g(t) = \sum_{k=-\infty}^{\infty} D_k \exp(jkt)$

iii)



iv)



P: $2(2.25 + 0.25 + 1) = 2 \times 3.5 = 7$

Examinations : EEE/ISE Part I, Communications I, 2002

MODEL ANSWER and MARKING SCHEME

First Examiner: Dr. Mustafa Gurcan	**Paper Code: E1.6**
Second Examiner Dr. E. M. Yeatman	**Question 1-c Page 2 *out of 15***
Question labels in left margin	**Marks allocations in right margin**

1-c

i) $\exp(-at) \cdot u(t) \Longleftrightarrow \frac{1}{a + j\omega}$

ii) also

$g(-t) \Longleftrightarrow G(-\omega)$

let

$\exp(at)\, u(-t) \Longleftrightarrow \frac{1}{a - j\omega}$

$\exp(-a|t|) = \exp(-at)\, u(t) + \exp(at)\, u(-t)$

iii) $g(t) = \exp(-at)\, u(t) \qquad (a > 0)$

$E_g = \int_{-\infty}^{\infty} g^2(t)\, dt = \int_0^{\infty} \exp(-2at)\, dt = \frac{1}{2a}$

$G(\omega) = \frac{1}{a + j\omega}$

$E_g = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{\omega^2 + a^2} d\omega$

$= \frac{1}{2\pi a} \tan^{-1} \frac{\omega}{a} \Big|_{-\infty}^{\infty} = \frac{1}{2a}.$

$\square$

iv) $\frac{0.95}{2a} = \frac{1}{2\pi} \int_{-W}^{W} \frac{d\omega}{\omega^2 + a^2} = \frac{1}{2\pi a} \tan^{-1} \frac{\omega}{a} \Big|_{-W}^{W}$

$= \frac{1}{\pi a} \tan^{-1} \frac{W}{a}$

$\frac{0.95\pi}{2} = \tan^{-1} \frac{W}{a} \Rightarrow W = 12.706a$ rad/s

Examinations : EEE/ISE Part I, Communications I, 2002

MODEL ANSWER and MARKING SCHEME

First Examiner: Dr. Mustafa Gurcan	Paper Code: E1.6		
Second Examiner Dr. E. M. Yeatman	Question2-a	Page 3	*out of 15*
Question labels in left margin	Marks allocations in right margin		

2-a The signal $g(t)$ is a triangular pulse $\Delta\left(\frac{t}{2\pi}\right)$

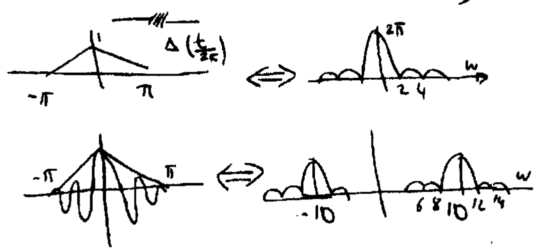
$$g(t) = \Delta\left(\frac{t}{2\pi}\right)\cos(10t)$$

From Fourier transformations table

$$\Delta\left(\frac{t}{2\pi}\right) \Longleftrightarrow \pi\,\mathrm{sinc}^2\left(\frac{\pi w}{2}\right)$$

From the modulation property

$$g(t) = \Delta\left(\frac{t}{2\pi}\right)\cos(10t) \Longleftrightarrow \frac{\pi}{2}\left\{\mathrm{sinc}^2\left[\frac{\pi(w-10)}{2}\right] + \mathrm{sinc}^2\left[\frac{\pi(w+10)}{2}\right]\right\}$$



Examinations : EEE/ISE Part I, Communications I, 2002

MODEL ANSWER and MARKING SCHEME

First Examiner: Dr. Mustafa Gurcan	Paper Code: E1.6		
Second Examiner Dr. E. M. Yeatman	Question 2-b	Page 4	*out of 15*
Question labels in left margin	Marks allocations in right margin		

2-b

$$Y(w) = G(w)\,\mathrm{rect}\left(\frac{w}{4\pi B}\right)\exp\left(-j(wt_0 + k\sin wT)\right)$$

$$\simeq G(w).\mathrm{rect}\left(\frac{w}{4\pi B}\right)\left[1 - jk\sin wT\right]\exp(-jwt_0)$$

This follows from the fact that

$\exp(x) \simeq 1 + x$ when $x \ll 1$.

More over, $G(w)\,\mathrm{rect}\left(\frac{w}{4\pi B}\right) = G(w)$

Because $G(w)$ is band limited to B Hz.

Hence

$$Y(w) = G(w)\exp(-jwt_0) - jkG(w)\sin wT\exp(-jwt_0)$$

from Fourier transform table

$$\frac{1}{2j}\left[g(t+T) - g(t-T)\right] \Longleftrightarrow G(w)\sin wT$$

Hence

$$y(t) = g(t-t_0) + \frac{k}{2}\left[g(t-t_0-T) - g(t-t_0+T)\right]$$

Examinations : EEE/ISE Part I, Communications I, 2002

MODEL ANSWER and MARKING SCHEME

First Examiner: Dr. Mustafa Gurcan	**Paper Code: E1.6**
Second Examiner Dr. E. M. Yeatman	**Question 2-c Page 5 *out of 15***
Question labels in left margin	**Marks allocations in right margin**

2-c The ideal differentiator transfer function is jw. Hence the transfer function of the entire system is

$$H(w) = \frac{1}{1+jw} \cdot jw = \frac{jw}{1+jw} \quad \text{and}$$

$$|H(w)|^2 = \frac{w^2}{w^2+1}$$

$$\langle x^2(t) \rangle = \frac{1}{\pi}\int_0^{\infty} \text{rect}\left(\frac{w}{2}\right) dw = \frac{1}{\pi}\int_0^1 dw = \frac{1}{\pi}$$

$$\langle y^2(t) \rangle = \frac{1}{\pi}\int_0^{\infty} \text{rect}\left(\frac{w}{2}\right)\frac{w^2}{w^2+1}\, dw$$

$$= \frac{1}{\pi}\int_0^1 \frac{w^2}{w^2+1}\, dw = \frac{1}{\pi}\left(1-\frac{\pi}{4}\right)$$

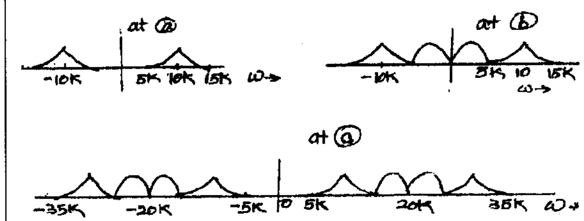
$$= 0.06831$$

Examinations : EEE/ISE Part I, Communications I, 2002

MODEL ANSWER and MARKING SCHEME

First Examiner: Dr. Mustafa Gurcan	**Paper Code: E1.6**
Second Examiner Dr. E. M. Yeatman	**Question 3-a Page 6 *out of 15***
Question labels in left margin	**Marks allocations in right margin**

3-a



The above figure shows the signals at points a, b and c.

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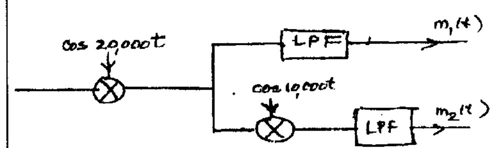


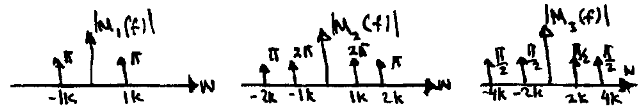
The receiver to recover $m_1(t)$ and $m_2(t)$ from the received signal is shown above.

Examinations : EEE/ISE Part I, Communications I, 2002

MODEL ANSWER and MARKING SCHEME

First Examiner: Dr. Mustafa Gurcan	**Paper Code: E1.6**
Second Examiner Dr. E. M. Yeatman	**Question 3-b Page 7 *out of 15***
Question labels in left margin	**Marks allocations in right margin**

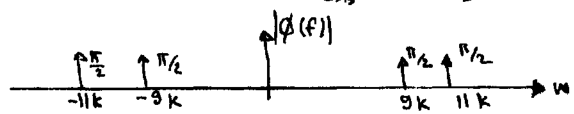
3-b



For $m_1(t) = \cos 1000t$

$\phi(t) = m_1(t) \cdot \cos 10{,}000t$

$= \cos 1000t \ \cos 10{,}000t = \frac{1}{2}\left[\underbrace{\cos 9000t}_{LSB} + \underbrace{\cos 11{,}000t}_{USB}\right]$

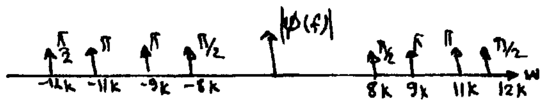


For $m_2(t) = 2\cos 1000t + \cos 2000t$

$\phi(t) = m_2(t) \cdot \cos 10{,}000t$

$= 2\left[\cos 1000t + \cos 2000t\right]\cos 10{,}000t$

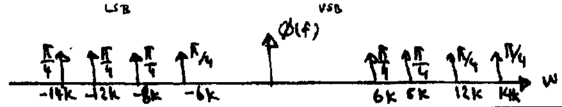
$= \cos 9000t + \cos 11{,}000t + \frac{1}{2}\left[\cos(8000t) + \cos 12{,}000t\right]$



For $m_3(t) = \cos 1000t \cdot \cos 3000t$

$\phi(t) = m_3(t)\cos 10{,}000t = \frac{1}{2}\left[\cos 2000t + \cos 4000t\right]\cos 10{,}000t$

$= \underbrace{\frac{1}{2}\left[\cos 8000t + \cos 6000t\right]}_{LSB} + \underbrace{\frac{1}{2}\left[\cos(12{,}000t) + \cos 14{,}000t\right]}_{USB}$



Examinations : EEE/ISE Part I, Communications I, 2002

MODEL ANSWER and MARKING SCHEME

First Examiner: Dr. Mustafa Gurcan	**Paper Code: E1.6**
Second Examiner Dr. E. M. Yeatman	**Question 3-c Page 8 *out of 15***
Question labels in left margin	**Marks allocations in right margin**

3-c $\phi_a(t) = [A + m(t)]\cos w_c t$. Hence,

The signal at the output of multiplier is

$g_a(t) = [A + m(t)]\cos^2 w_c t$

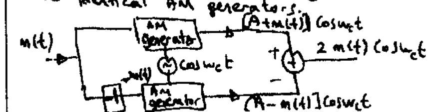
$= \frac{1}{2}[A + m(t)] + \frac{1}{2}[A + m(t)]\cos 2w_c t$

The first term is a baseband signal because its spectrum is centered at $w = 0$. The low-pass filter allows this term to pass, but supresses the second term, whose spectrum is centered at $\pm 2w_c$. Hence the output of the low pass filter is

$y(t) = A + m(t)$

When this signal is passed through a dc blocker, the dc term A is suppressed yielding the output $m(t)$. This shows that the system can demodulate AM signal regardless of the value of A. This is a synchronous or coherent demodulator.

When an input to a DSB-SC generator is $m(t)$, the corresponding output is $m(t)\cos w_c t$. Clearly if the input is $A + m(t)$, the corresponding output will be $[A + m(t)]\cos w_c t$. This is precisely the AM signal. Thus by adding a dc of value A to the baseband signal $m(t)$, we can generate AM signal using a DSB-SC generator. The converse is generally not true. However we can generate DSB-SC using AM generators if we use two identical AM generators.



4-a The signal bandwidth is the highest frequency $m(t)$. $B = \frac{2000\pi}{2\pi} = 1000$ Hz

The carrier amplitude is 10, and the power is

$P = \frac{10^2}{2} = 50$

To find the frequency deviation Δf, we find the instantaneous frequency w_i given by

$w_i = \frac{d}{dt}\theta(t) = w_c + 15{,}000 \cos 3000t + 20{,}000\pi \cos 2000\pi t$

The carrier deviation is $15{,}000 \cos 3000t + 20{,}000\pi \cos 2000\pi t$. The two sinusoids will add phase at some point, and the maximum value of this expression is $15{,}000 + 20{,}000\pi$. This is the maximum carrier deviatio Δw. Hence,

$\Delta f = \frac{\Delta w}{2\pi} = 12{,}387.32$ Hz

$\beta = \frac{\Delta f}{B} = \frac{12{,}387.32}{1000} = 12.387$

Bandwidth $= 2(\Delta f + B) = 26{,}774.65$ Hz

....

4-b $w_i = w_c + k_f m(t)$

Dividing throughout by 2π, we have the equation in terms of the variable f.

The instantaneous frequency is

$f_i = f_c + \frac{k_f}{2\pi} m(t)$

$= 10^8 + 10^5 m(t)$

$(f_i)_{min} = 10^8 + 10^5 [m(t)]_{min} = 99.9$ MHz

$(f_i)_{max} = 10^8 + 10^5 (m(t))_{max} = 100.1$ MHz

$\Delta f = 10^5$ kHz

$T_0 = 2 \cdot 10^{-4}$ s, $f_0 = \frac{1}{T_0} = 5 \cdot 10^3$

Signal bandwidth $= B = 3 \cdot f_0 = 15 \cdot 10^3$ Hz

Modulated signal bandwidth $= 2(\Delta f + B) = 2(10^5 + 15 \cdot 10^3)$

$= 2(100 + 15)10^3 = 230 \times 10^3$ Hz

$= 230$ kHz.

4-c ....

The pulse train is a periodic signal with fundamental frequency $2B$ Hz. Hence $w_s = 2\pi(2B) = 4\pi B$. The period is $T_0 = 1/2B$. It is an even function of t. Hence the Fourier series for the pulse train can be expressed as

$P_{T_s}(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos n w_s t$

Using

$a_0 = \frac{1}{T_0}\int_{T_0} g(t)\,dt$

$a_n = \frac{2}{T_0}\int_{T_0} g(t) \cdot \cos n w_0 t \, dt \quad n = 1, 2, \ldots$

and

$b_n = \frac{2}{T_0}\int_{T_0} g(t) \cdot \sin n w_0 t \, dt \quad n = 1, 2 \ldots$

we get

$a_0 = C_0 = \frac{1}{T_0}\int_{-1/16B}^{1/16B} dt = \frac{1}{4}$

$a_n = C_n = \frac{2}{T_0}\int_{-1/16B}^{1/16B} \cos n w_s t \, dt = \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right)$

4-d

The bandwidth is 15 kHz. The Nyquist rate is 30 kHz.

$65536 = 2^{16}$, so that 16 binary digits are needed to encode each sample.

$\cancel{30000} \times 16 = 480000$ bits/s

$44100 \times 16 = 705600$ bits/s

$R = 2B$

$B = \frac{R}{2} = \frac{705600}{2} = 352800$ Hz

5-a

The resistance of each diode is r ohms while conducting, and ∞ when off. When the carrier $A \cos w_c t$ is positive, the diodes conduct (during the entire positive half cycle), and when the carrier is negative the diodes are open (during the entire negative half cycle). Thus during during the positive half cycle, the voltage $\frac{R}{R+r}\,\phi(t)$ appears accross each of the resistors R. During the negative half cycle, the output voltage is zero. Therefore the diodes act as a gate in the circuit that is basically a voltage divider with a gain $\frac{2R}{R+r}$. The output is therefore

$e_o(t) = \frac{2R}{R+r}\, w(t).m(t)$

The period of $w(t)$ is $T_o = \frac{2\pi}{w}$. Using the Fourier series

$w(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos w_c t - \frac{1}{3}\cos 3w_c t + \frac{1}{5}\cos 5w_c t + \cdots\right]$

Fourier series

$w(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos w_c t - \frac{1}{3}\cos 3w_c t + \frac{1}{5}\cos 5w_c t + \cdots\right]$

The output $e_o(t)$ is

$e_o(t) = \frac{2R}{R+r}\, w(t).m(t)$

$= \frac{2R}{R+r}\, m(t)\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos w_c t - \frac{1}{3}\cos 3w_c t + \frac{1}{5}\cos 5w_c t + \cdots\right]\right.$

Examinations : EEE/ISE Part I, Communications I, 2002

MODEL ANSWER and MARKING SCHEME

First Examiner: Dr. Mustafa Gurcan — Paper Code: E1.6

Second Examiner Dr. E. M. Yeatman — Question 5 a — Page 13 — out of 15

Question labels in left margin — Marks allocations in right margin

5-a If we pass the output $e_o(t)$ through a bandpass filter centered at w_c, the filter suppresses the signal $m(t)$ and $m(t)\cdot\cos nw_c t$ for all $n \neq 1$, leaving only the modulated term $\frac{4R}{\pi(R+r)} m(t)\cdot\cos w_c t$ intact. Hence the system acts as a modulator.

The same circuit can be used as a demodulator if we use a ~~bandpass~~ low pass filter at the output. In this case, the input is $x(t) = m(t)\cos w_c t$ and the output is $\frac{2R}{\pi(R+r)} m(t)$

$g_a(t) = [A + m(t)]\cos w_c t$. Hence,

The signal at the output of multiplier is

$$g(t) = [A+m(t)]\cos^2 w_c t$$
$$= \frac{1}{2}[A+m(t)] + \frac{1}{2}[A+m(t)]\cos 2w_c t.$$

The first term is a baseband signal because its spectrum is centered at $w=0$. The low-pass filter allows this term to pass, but supresses the second term, whose spectrum is centered at $\pm 2w_c$. Hence the output of the low pass filter is

$$y(t) = A + m(t)$$

When this signal is passed through a dc blocker, the dc term A is suppressed yielding the output $m(t)$. This shows that the system can demodulate AM signal regardless of the value of A. This is a synchronous or coherent demodulator.

Examinations : EEE/ISE Part I, Communications I, 2002

MODEL ANSWER and MARKING SCHEME

First Examiner: Dr. Mustafa Gurcan — Paper Code: E1.6

Second Examiner Dr. E. M. Yeatman — Question 5 b — Page 14 — out of 15

Question labels in left margin — Marks allocations in right margin

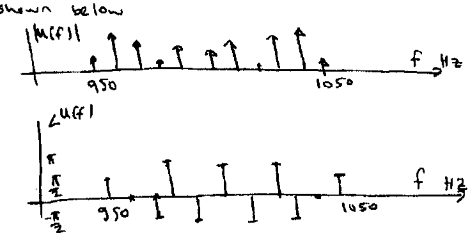
5-b The Fourier series expansion of $\exp(j\beta \sin(2\pi f_m t))$ is

$$c_n = f_m \int_{\frac{1}{4f_m}}^{\frac{5}{4f_m}} \exp(j\beta \sin(2\pi f_m t))\exp(-j2\pi n f_m t)\,dt$$
$$= \frac{1}{2\pi}\int_0^{2\pi} \exp(j\beta\cos u - jnu)\exp\left(j\frac{n\pi}{2}\right)du$$
$$= \exp\left(j\frac{2\pi}{2}\right)J_n(\beta)$$

Hence

$$u(t) = A_c \,\mathrm{Re}\left[\sum_{n=-\infty}^{\infty} c_n \cdot \exp(j2\pi f_c t)\exp(j2\pi n f_m t)\right]$$
$$= A_c \,\mathrm{Re}\left[\sum_{n=-\infty}^{\infty} \exp\left(j2\pi(f_c+nf_m)t + \frac{n\pi}{2}\right)\right]$$

The magnitude and phase spectra of $u(t)$ for $\beta=5$ and frequencies in the interval $[950, 1000]$ are shown below



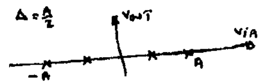
Examinations : EEE/ISE Part I, Communications I, 2002

MODEL ANSWER and MARKING SCHEME

First Examiner: Dr. Mustafa Gurcan	**Paper Code: E1.6**
Second Examiner Dr. E. M. Yeatman	**Question 5 b Page 15 *out of 15***
Question labels in left margin	**Marks allocations in right margin**

5-c

$\Delta = \frac{A}{2}$



Noise over one step

$$\int_{-\frac{\Delta}{2}}^{\Delta/2} \left(\frac{\Delta}{2}-x\right)^2 \frac{1}{2A}\,dx = \frac{\Delta^3}{2A\cdot 12}$$

When we have four levels

$$\sigma^2 = \text{Total noise} = 4\cdot\frac{\Delta^3}{2A\cdot 12} \quad \text{where } \Delta = \frac{A}{2}$$

$$\sigma^2 = \frac{A^3}{8\cdot 6\,A} = \frac{A^2}{48} \quad \text{for } A=3 \quad \boxed{\sigma^2 = \frac{9}{48}}$$

—//—