IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2007

EEE PART I: MEng, BEng and ACGI

ENGINEERING MATERIALS

Friday, 1 June 10:00 am

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible   First Marker(s) :   W.T. Pike, W.T. Pike

Second Marker(s) :   T.J. Tate, T.J. Tate

Special instructions for students

Fundamental constants

Permittivity of free space, ε_0 = 8.85 x 10^{-12} F/m
Permeability of free space, μ_0 = 4π x 10^{-7} H/m
Planck's constant, h = 6.6 x 10^{-34} Js
Boltzmann's constant, k = 1.38 x 10^{-23} J/K
Electron charge, e = 1.6 x 10^{-19} C
Electron mass, m = 9.1 x 10^{-31} kg
Speed of light, c = 3.0 x 10^8 ms^{-1}

Schrödinger's equation

General form:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

In one dimension:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

In spherical coordinates:

$$\nabla^2 = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}$$

Free-electron theory

Density of states (3D):

$$g(E) = \frac{1}{\pi^2\hbar^3}\left(m\right)^{3/2}\sqrt{2E}$$

Fermi energy

$$E_f = \frac{\hbar^2\pi^2}{2m}\left(\frac{3n}{\pi}\right)^{2/3}$$

Fermi distribution

$$f(E) = \frac{1}{1+\exp\left(\dfrac{E-E_f}{kT}\right)}$$

Electrons in semiconductors

Effective mass:

$$m_e^* = \frac{\hbar^2}{d^2E(k)/dk^2}$$

Concentration of electrons in a semiconductor of bandgap E_g:

$$n = \frac{1}{\sqrt{2}\hbar^3}\left(\frac{m_e^* kT}{\pi}\right)^{3/2} e^{-\frac{(E_g - E_f)}{kT}}$$

$$= N_c e^{-\frac{(E_g - E_f)}{kT}}$$

Concentration of holes

$$p = \frac{1}{\sqrt{2}\hbar^3}\left(\frac{m_h^* kT}{\pi}\right)^{3/2} e^{-\frac{E_f}{kT}}$$

$$= N_v e^{-\frac{E_f}{kT}}$$

Polarization

Lorentz correction for local field:

$$\mathbf{E}_{loc} = \mathbf{E} + \frac{\mathbf{P}}{3\varepsilon_0}$$

Electronic polarization:

$$P_0 = \frac{\varepsilon_0 \omega_p^2 E_0}{\omega_m^2 - \omega^2 + j\omega\gamma}$$

where

$$\gamma = \frac{r}{m},$$

$$\omega_m^2 = \omega_0^2 - \frac{\omega_p^2}{3},$$

$$\omega_0^2 = k/m,$$

$$\omega_p^2 = \frac{ne^2}{m\varepsilon_0}.$$

Orientational Polarization:

Static:

$$P = n\mu\, L(\mu E/kT) \text{ where } L(x) = \coth(x) - 1/x$$

Dynamic:

$$P_0 = \frac{P_s}{1 + j\omega\tau},$$

Magnetism

Magnet dipole due to electron angular momentum:

$$\mu_m = -\frac{e\mathbf{L}}{2m}$$

Magnet dipole due to electron spin:

$$\mu_m = -\frac{e\mathbf{S}}{m}$$

Paramagnetism:

$$M = n\mu_m\, L\left(\frac{\mu_m\mu_0 H}{kT}\right)$$

The Questions

1.

a) Show that $\psi(x) = A \exp(-jkx)$ is a solution of the 1-D Schrödinger equation for a particle of mass m when the potential $V(x)$ is zero, and derive an expression for the total energy, E, of the particle. [4]

b) Show that the occupancy of electron states of energy E much higher than the Fermi level (that is where $E - E_f \gg kT$) is given by $\exp\text{-}(E - E_f)/kT$. [4]

c) What is the angle between the (110) and the (111) planes of silicon? [4]

d) Plot a typical variation of bond potential with bond distance, labelling the bond energy and the equilibrium bond length. [4]

e) Where are donor and acceptor states located in an energy diagram of an n- or p-doped semiconductor? Explain the occupancy of these states in the extrinsic regime for both cases. [4]

g) Plot the variation of orientational polarisation with electric field for a dielectric. labelling the linear and saturation regimes. Describe the orientation of the individual dipoles in each of these regimes with respect to the direction of the applied electric field. [8]

h) Explain, using the Bohr model of electron orbitals, why diamagnetic materials have a relative permeability slightly less than 1. [4]

i) Explain, with reference to labelled B-H curves, the difference between a hard and soft magnet, and for each type give an example of an application. [8]

2.

a) The expression for the vetical deflection v of a horizontal cantilever of length l at a distance x from the support due to a vetical force P applied at the cantilever's tip is given by

$$\frac{d^2v}{dx^2} = \frac{P(l-x)}{EI}$$

where E is Young's modulus for the cantilever material and I is the second moment of the cross-sectional area. Show that the deflection of the tip is given by $Pl^3/(3EI)$. For a cantilever of cross sectional dimensions a by b, show that $I = ab^3/12$, where b is the vertical dimension. [12]

b) A laser beam is used to measure the deflection of the cantilever. The laser is aligned in the deflection direction to hit the cantilever at the end of the tip. The laser has an output power of 30 mW. Assuming the photons are perfectly reflected by the cantilever, from the definition of force as the rate of change of momentum, show that the force at the end of the cantilever will be 0.2 nN. [10]

c) Hence calculate the deflection of the end of a cantilever of length 500 μm, and dimensions $a \times b$ of 100 μm x 10 μm with a Young's modulus of 1.3×10^{11} Pa when the laser is turned on. [8]

3.

a) Fig. 3 shows an unlabelled plot which can be used to determine the concentration of carriers in an intrinsic semiconductor.
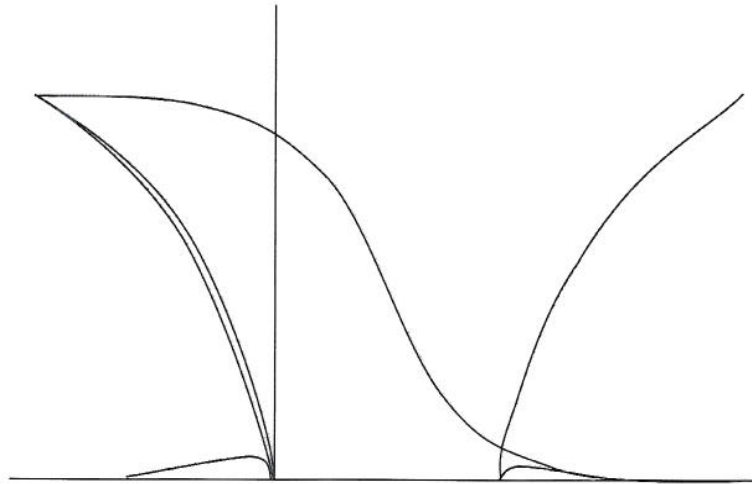
[6]

Reproduce this plot and fully label it.

Write out, but do not evaluate, expressions for the integrals required to determine the concentration of carriers from the variables you have labelled.   [14]

b) From the expressions provided for these carrier concentrations, determine the position of the Fermi level for an intrinsic semiconductor, with equal electron and hole effective masses.   [8]

c) Using the same expressions, determine the product of the electron and hole concentrations under any doping conditions, and show that this product is independent of doping and depends only on the semiconductor properties. Hence determine for Si, which has an intrinsic carrier concentration of 10^{10}/cm^3, the electron carrier concentration for extrinsic p-type material with doping of 10^{15}/cm^3.   [8]

4.

a) Describe graphically how the complex dielectric constant of a material varies with frequency as a result of electronic, molecular and orientational polarisation mechanisms. [10]

b) Dielectric losses in a parallel-plate capacitor can be modelled by using the complex dielectric constant, $\varepsilon_r = \varepsilon_r' - j \varepsilon_r''$. Show that the introduction of the dielectric causes the capacitor to behave as a capacitor and resistor in parallel, and show that the respective values are $\varepsilon_r' C_0$ and $1/(\omega \varepsilon_r'' C_0)$ where C_0 is the capacitance of the capacitor without any dielectric and ω is the angular frequency of the signal across the capacitor. [10]

c) A 100 MHz signal is applied across a capacitor which incorporates a dielectric with $\varepsilon_r = 5 - 0.3 j$. What is the phase difference between the voltage and current? [10]

The Answers

1.

a)   Substituting $\psi(x) = A \exp(-jkx)$ into the 1-D Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \text{ for } V(x) \text{ is zero}$$

gives $\dfrac{\hbar^2}{2m}k^2\psi(x) = E\psi(x)$ which shows that $\psi(x)$ is a solution iff
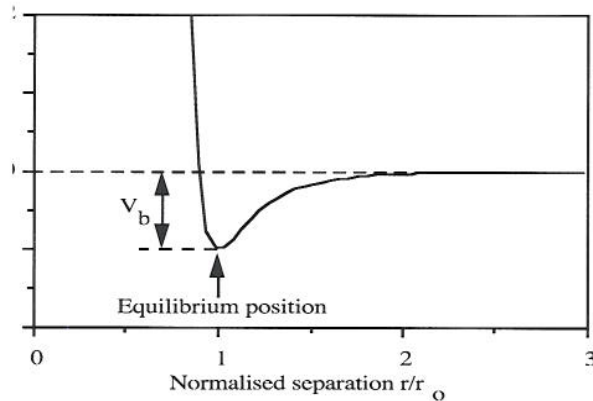
$$E = \frac{\hbar^2 k^2}{2m}.$$ [4]

b)   From $f(E) = \dfrac{1}{1+\exp\left(\dfrac{E-E_f}{kT}\right)}$ if $E - E_f \gg kT$, then $\exp\left(\dfrac{E-E_f}{kT}\right) \gg 1$,

and $f(E) = \exp{-\left(\dfrac{E-E_f}{kT}\right)}$ [4]

c)   The angle between the planes, θ, is the angle between the surface normals for a cubic system. Hence by the dot product:
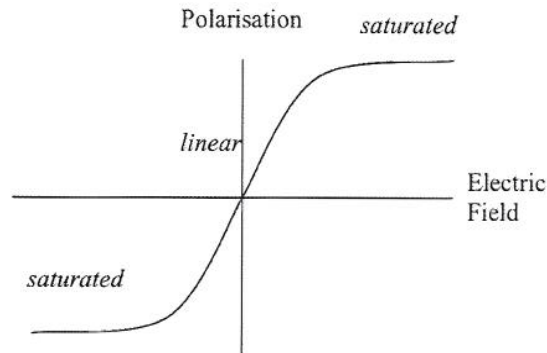
$$\cos\theta = \frac{(111).(110)}{|111||110|}$$

$$= \frac{2}{\sqrt{3}\sqrt{2}}$$ [4]

$$= \sqrt{\frac{2}{3}}, \quad \Rightarrow \theta = 35.2^0$$

d) $V(r)$.   [4]

e)   The donor states are just below the bottom of the conduction band, the acceptor states just above the top of the valence band. In the extrinsic regime, the donor states are unoccupied and the acceptor states are occupied.   [4]

g)

Polarisation — *saturated* — *linear* — *saturated* — Electric Field
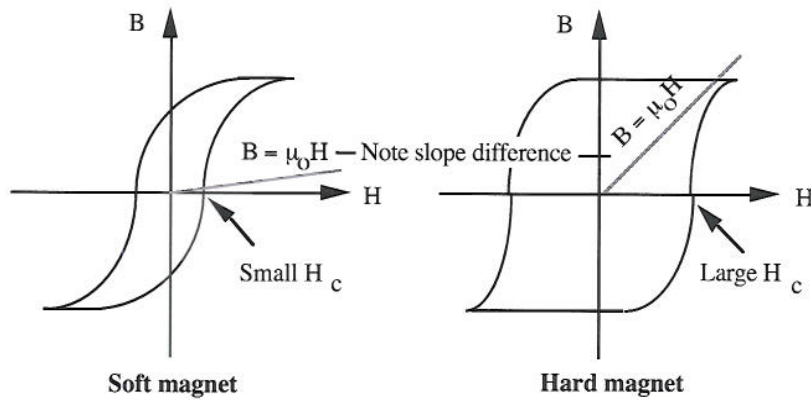
[4]

The dipoles are all aligned parallel to the electric field in the saturation regimes, but are randomised in the linear regime, with a level of alignment parallel and proportional to the electric field.

[4]

h)   By Lenz's Law, the effective currents due to the orbiting electrons will change to oppose the external field H, reducing the flux below that without the diamagnet and so producing a permeability a little less than 1.

[4]

i)

$B = \mu_0 H$ — Note slope difference — Small H_c — **Soft magnet**

$B = \mu_0 H$ — Note slope difference — Large H_c — **Hard magnet**

[6]

Soft magnets: transformers; hard magnets: data storage.

[2]

2.

a) From

$$\frac{d^2v}{dx^2} = \frac{P(l-x)}{EI}$$

Integrating once:

$$\frac{dv}{dx} = \frac{P(lx - x^2/2)}{EI} + c_1$$

with boundary conditions, $dv/dx = 0$ at $x = 0$: $c_1 = 0$

Integrating again:

$$v = \frac{P(lx^2/2 - x^3/6)}{EI} + c_2$$

with boundary conditions, $v = 0$ at $x = 0$: $c_2 = 0$

Hence at $x = l$, $v = Pl^3/(3EI)$.   [8]

From

$$I = \int_{-b/2}^{b/2} ay^2 dy = \frac{ab^3}{12}$$

[4]

b) The momentum of each photon is given by $p = h/\lambda$. The rate of photons striking the cantilever is given by $r = W/hf = W\lambda/(hc)$.   [6]

Assuming the photons are perfectly reflected by the cantilever, the force is given by $2rp = 2W/c = 2 \times 30 \times 10^{-3}/(3 \times 10^8) = 0.2$ nN.   [4]
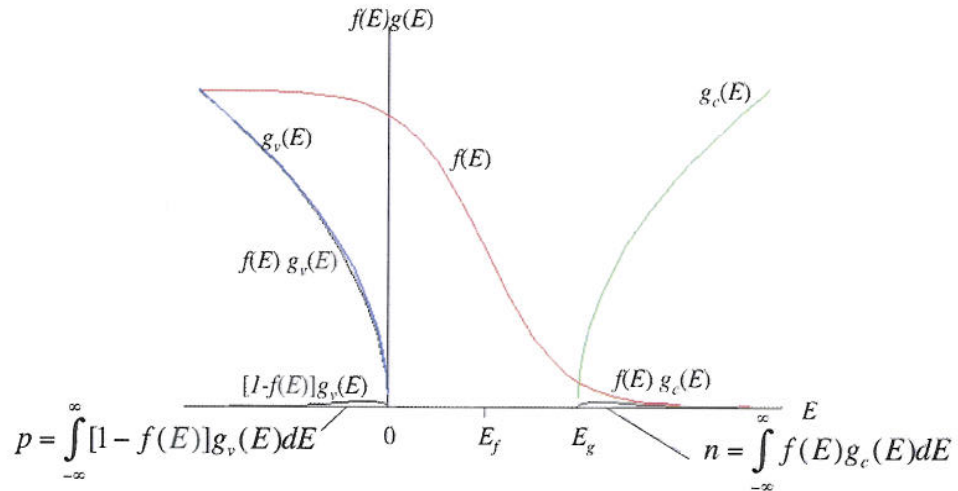
c) Combining the equations in part a, $v = 4Pl^3/(Eab^3)$. Therefore:

$v = 4 \times .2 \times 10^{-9} \times (0.5 \times 10^{-3})^3/(1.3 \times 10^{11} \times 100 \times 10^{-6} \times (10 \times 10^{-6})^3)$
$= 7.7 \times 10^{-12}$ m   [8]

3.

a)

$$p = \int_{-\infty}^{\infty} [1 - f(E)] g_v(E) dE$$

$$n = \int_{-\infty}^{\infty} f(E) g_c(E) dE$$

where $f(E)$ is the Fermi distribution, $g_c(E)$ is the density of states in the conduction band, $g_v(E)$ is the density of states in the valence band and n and p are the concentration of electrons and holes. [14]

b) From

$$n = \frac{1}{\sqrt{2}\hbar^3} \left(\frac{m^* kT}{\pi} \right)^{3/2} e^{-\frac{(E_g - E_f)}{kT}}$$

and

$$p = \frac{1}{\sqrt{2}\hbar^3} \left(\frac{m^* kT}{\pi} \right)^{3/2} e^{-\frac{E_f}{kT}}$$

equating the two:

$$e^{-\frac{(E_g - E_f)}{kT}} = e^{-\frac{E_f}{kT}}$$

and so $E_f = E_g/2$ and the Fermi level is in the middle of the band gap [8]

c) The product of the two expressions:

$$np = \frac{1}{2\hbar^6} \left(\frac{m^* kT}{\pi} \right)^3 e^{-\frac{E_g}{kT}}$$

[4]

which is independent of the position of the Fermi level and so independent of doping.

Hence for p-type extrinsic material, $n = n_i^2/\mathrm{NA} = 10^5$ /cc [4]

4.

a) Polarisation, in general, will increase the real part of the dielectric constant, while any absorption will cause a increase in the imaginary part. Electronic, molecular and orientational polarisation is due to the formation or orientation of electric dipoles at decreasing frequencies – above the characteristic frequency of each mechanism it does not contribute to the dielectric constant. Hence, as each mechanism becomes effective wirth decreasing frequency there is a step in the real part and a peak in the imaginary part of the dielectric constant. Graphically:
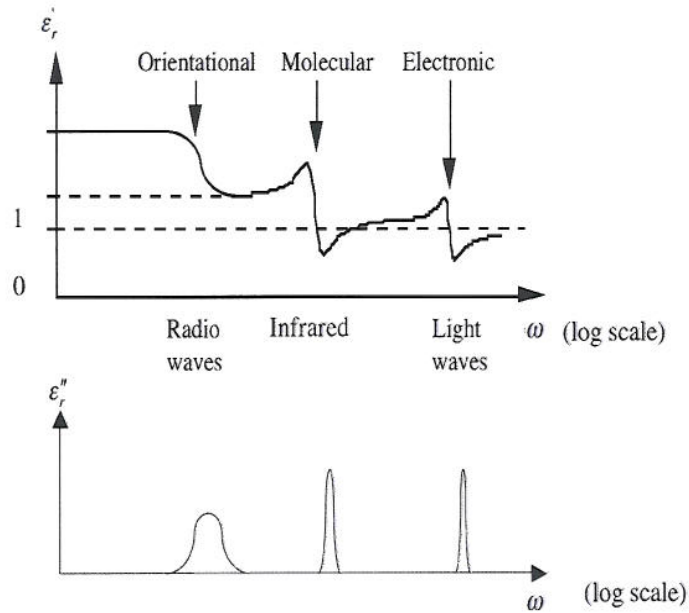
[10]

b) The capacitance is given by:

$$C_d = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{\varepsilon_0 (\varepsilon_r' - j\varepsilon_r'') A}{d}$$

and so the admittance is

$$Y = j\omega C_d$$
$$= \frac{j\omega \varepsilon_0 (\varepsilon_r' - j\varepsilon_r'') A}{d}$$

which can be written as:

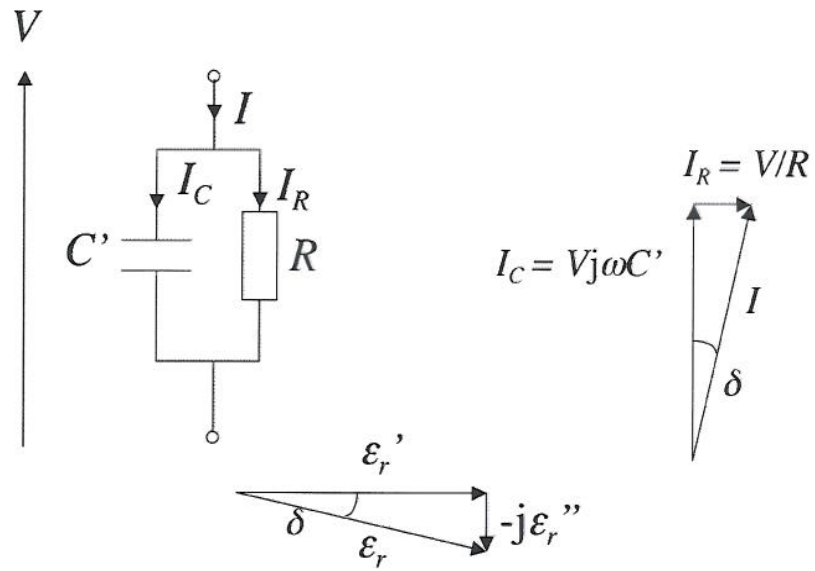$$Y = j\omega C' + \frac{1}{R}$$

where

$$C' = \frac{\varepsilon_0 \varepsilon_r' A}{d} = \varepsilon_r' C_0$$

and

$$R = \frac{d}{\omega \varepsilon_0 \varepsilon_r'' A} = \frac{1}{\omega \varepsilon_r'' C_0}$$

[10]

c)

From considering the currents in the effective capacitor and resistor, the overall angle will be $90 - \delta = 90 - \tan^{-1}(0.3/5) = 86.6°$

[10]