Paper Number(s): **E1.4**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2002

EEE/ISE PART I: M.Eng., B.Eng. and ACGI

ANALOGUE ELECTRONICS I

Monday, 27 May 10:00 am

There are FIVE questions on this paper.

Answer THREE questions.

Time allowed: 2:00 hours

Examiners responsible:

First Marker(s): Holmes,A.S.

Second Marker(s): Vickery,J.C.

Corrected Copy

None

1. Figure 1 shows a common-emitter amplifier which is to be manufactured using transistors with a nominal β value of 100.

a) Determine the quiescent output voltage and the collector bias current when $\beta = 100$, taking into account the base current of the transistor. Explain why the bias conditions of this circuit are relatively insensitive to β variations. [6]

b) Draw a small-signal equivalent circuit of the amplifier, and hence show that the voltage gain may be written as:

$$\frac{-\alpha R_C}{r_e + R_E}$$

where α is the common-base current gain of the transistor, and r_e is its emitter resistance. Evaluate this expression, and also determine the small-signal input and output resistances of the amplifier. You may neglect the transistor's small-signal output resistance. [10]

c) By what ratio would the mid-band voltage gain of the amplifier be increased if the emitter resistor R_E were bypassed using a capacitor? [4]

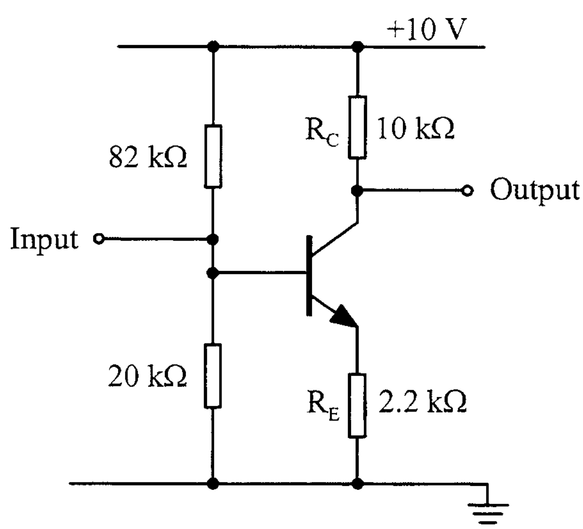

Figure 1

2. The circuit shown in Figure 2a is to be used as a zero-crossing detector i.e. a circuit with a digital output that changes state whenever the input voltage passes through zero. The three transistors are matched and have $\beta = 100$.

a) Explain briefly the operation of the circuit. [4]

b) Choose the value of R_B such that, when V_{IN} is zero, 95 % of the collector current of Q2 flows in R_B. Assuming this value of R_B, choose R_C to give an output voltage of 2.5 V when $V_{IN} = 0$. [6]

c) Using the macromodel shown in Figure 2b, or otherwise, show that the small-signal voltage gain of the overall circuit is given by:

$$Av = -\frac{1}{2} g_{m1} (R_B // r_{be3}) \cdot g_{m3} R_C$$

where g_m and r_{be} are the usual BJT parameters, and the subscripts 1 and 3 refer to Q1 and Q3 respectively. Hence determine the range of input voltages for which the output lies between 4.5 V and 0.5 V. [10]

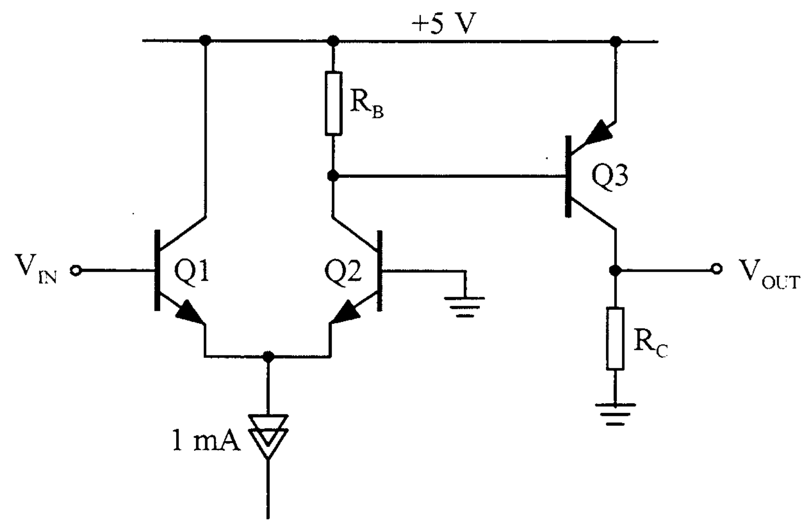

Figure 2a

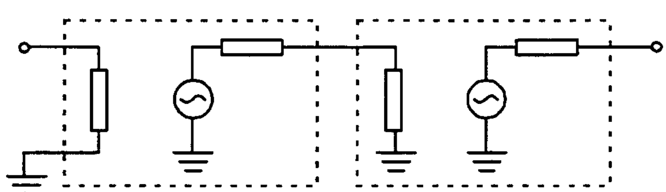

Figure 2b

3. Figure 3 shows a single stage CMOS amplifier in which a p-channel MOSFET provides the active load for an n-channel MOSFET. The active load is biased using an externally applied voltage V_G.

a) Choose the value of V_G to give a quiescent drain current of 0.4 mA. Also determine the quiescent output voltage of the circuit, and verify that both MOSFETs are saturated under quiescent conditions. [8]

b) Draw a small-signal equivalent circuit of the amplifier, and hence determine its small-signal voltage gain. Also calculate the small-signal input resistance of the circuit. You may assume the input capacitor is effectively short-circuit at signal frequencies. [8]

c) Determine the approximate range of output voltages over which both transistors will remain saturated. Show how, by adding one resistor to the circuit, the quiescent output voltage could be moved to the middle of this range. [4]

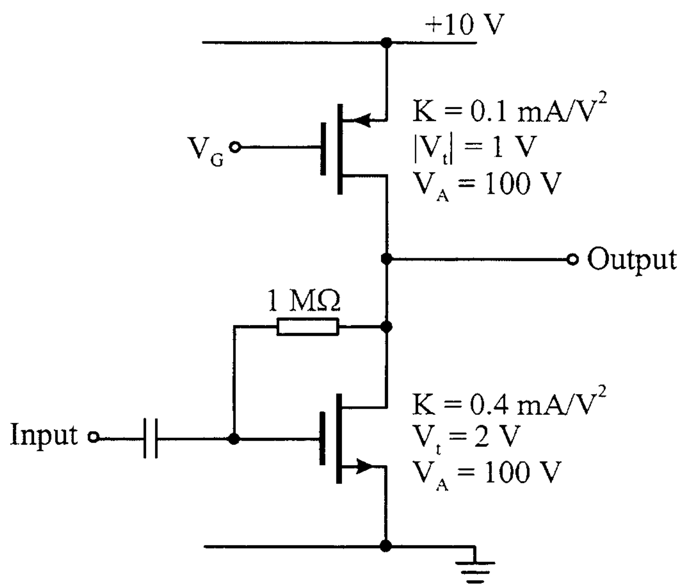

Figure 3

4. a) Figure 4a shows a BJT configured as a so-called *VBE multiplier*, a two-terminal device that can provide a roughly constant terminal voltage over a wide range of currents.

Show that: $$V = \left(\frac{R_1 + R_2}{R_1}\right)V_{BE} + I_B R_2 \quad \text{and} \quad I = I_E + \frac{V_{BE}}{R_1}$$

where V_{BE}, I_B and I_E are respectively the base-emitter voltage, base current and emitter current of the transistor. Which of the terms in the first equation needs to dominate in order for the voltage V to be only weakly dependent on the current I? [8]

b) Figure 4b shows a push-pull output stage in which a VBE multiplier is used to set the quiescent current of the output transistors.

Neglecting the base current of Q2, choose the value of R_1 to give an emitter bias current of 2.5 mA in Q1. You will need to use the relation $V_{BE} = V_T \ln(I_C/I_S)$ where V_T is the thermal voltage. Assume a saturation current of $I_S = 10^{-14}$ A and a current gain of β = 200 for Q1, and use $V_T = 25$ mV.

Assuming your calculated value of R_1, choose the value of R_2 such that the voltage across the VBE multiplier produces a collector bias current of 2 mA in the output transistors. Assume $I_S = 10^{-13}$ A for Q2 and Q3. [8]

c) If Q2 and Q3 have β = 50, calculate the voltage across the VBE multiplier and the emitter current of Q3 when Q2 is delivering 100 mA of output current into a load. [4]

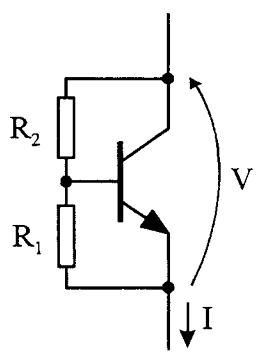

Figure 4a

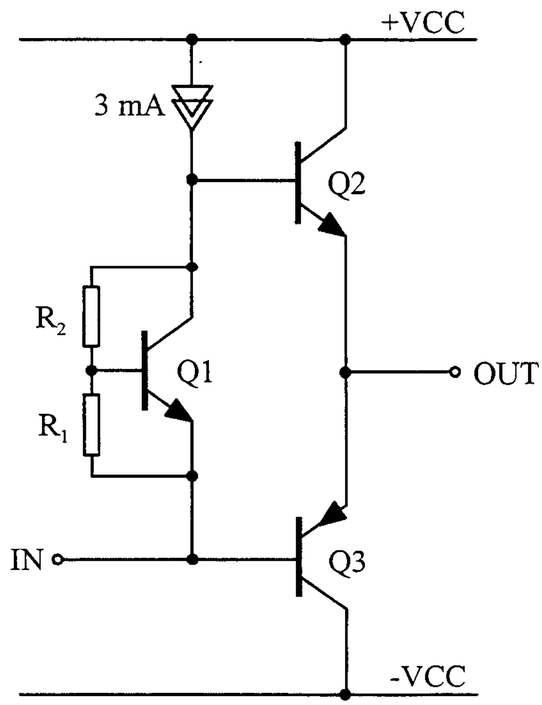

Figure 4b

5. For each of the four circuits in Figure 5 below, determine the operating modes of the transistor(s), and calculate the value of the current I or voltage V. State clearly any assumptions made in your calculations.

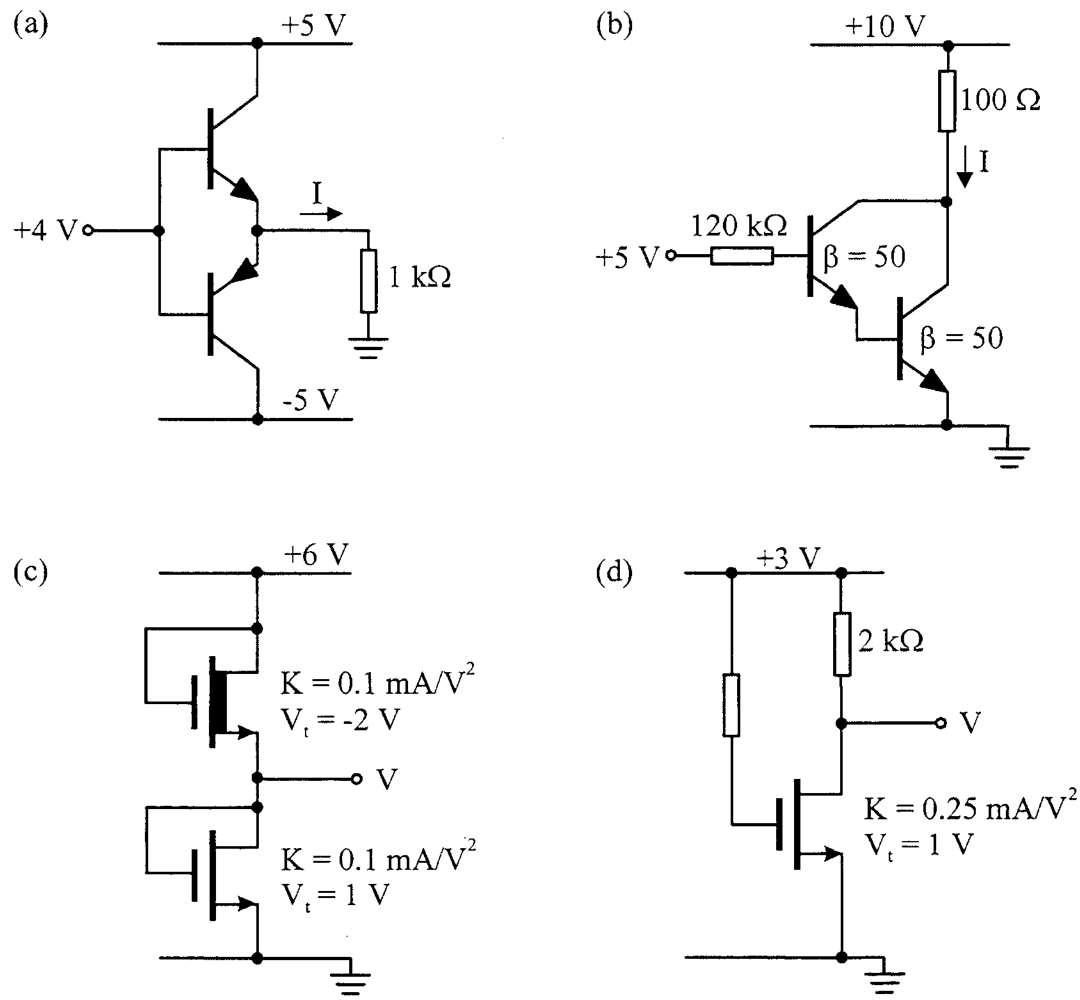

Figure 5

The four parts carry equal marks.

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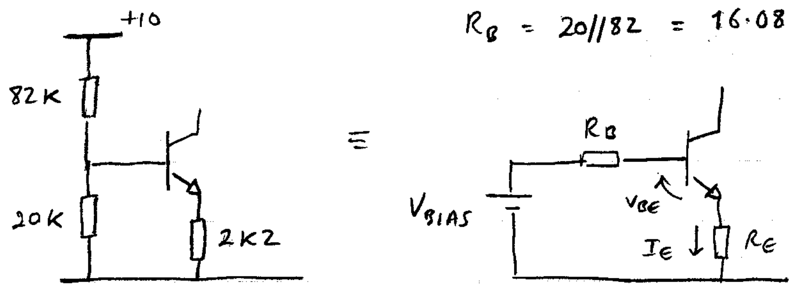
1 a) Bias cct:



$$V_{BIAS} = \frac{20}{102} \times 10 = 1.96V$$

$$R_B = 20 // 82 = 16.08K$$

KVL :-

$$I_E R_E + V_{BE} + I_B R_B = V_{BIAS}$$

$\Rightarrow$

$$I_E = \frac{V_{BIAS} - V_{BE}}{R_E + R_B/_{1+\beta}}$$

$$= \frac{1.96 - 0.7}{2.2k + \frac{16.08K}{101}}$$

$$= 0.534 \text{ mA}$$

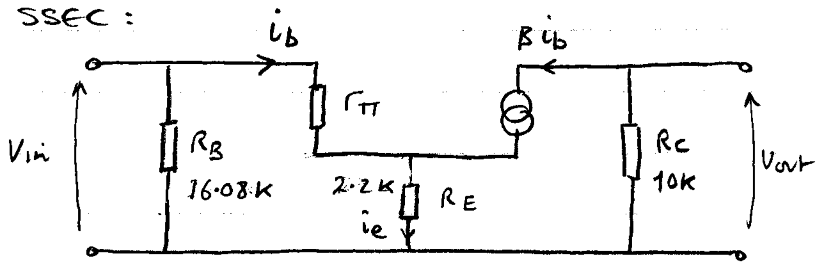
Assuming $V_{BE} = 0.7V$

$$I_c = \alpha I_E = \frac{100}{101} \times 0.534 = \underline{\underline{0.529 \text{ mA}}}$$

$$V_{OUT} = 10 - 10 \times 0.529 = \underline{\underline{4.71 V}}$$

β affects I_E only through $\frac{R_B}{1+\beta}$ term, which is $\ll R_E$
$\Rightarrow$ immunity to β variations 6

b) SSEC:



$$r_\pi = \frac{\beta V_T}{I_C} = \frac{100 \times 25}{0.529} = 4726\ \Omega$$

KVL: $v_{in} = i_b r_\pi + i_e R_E$ ... ①
$= i_b [r_\pi + (1+\beta) R_E]$

$V_{out} = -\beta i_b R_c$ ... ②

②/① $\Rightarrow$

$$V_{out}/_{V_{in}} = \frac{-\beta R_c}{r_\pi + (1+\beta)R_E} = \frac{-\left(\frac{\beta}{1+\beta}\right) R_c}{\left(\frac{r_\pi}{1+\beta}\right) + R_E} = \frac{-\alpha R_c}{r_e + R_E}$$

Putting $r_e = \frac{4726}{101}$, $R_E = 2200$, $R_c = 10000$, $\alpha = \frac{100}{101}$ $\Rightarrow$ $\frac{V_{out}}{V_{in}} = \underline{\underline{-4.41}}$

$$R_i = R_B // [r_\pi + (1+\beta) R_E] = (16.08K // 227K) = \underline{\underline{15\ K\Omega}}, \quad R_o = \underline{\underline{10\ K\Omega}}$$ 10

c) Gain with R_E bypassed is obtained by putting $R_E = 0$ in gain equation

$$A_v \to -\frac{\alpha R_c}{r_e} = -\frac{\beta R_c}{r_\pi} = \frac{-100 \times 10k}{4726\ \Omega} = -212$$

So Ratio $= \frac{212}{4.41} = \underline{\underline{48}}$ 4

2 a) Q1, Q2 form differential pair, amplifying difference between V_{IN} and zero. Q3 provides additional voltage gain. Compound gain is very high, so Q3 moves from full-on to full-off over narrow range of V_{IN} 4

b) When $V_{IN} = 0$, tail current divides equally between Q1 and Q2, giving $I_{C2} = 500\,\mu A$. (neglecting I_B here)

$\Rightarrow$ Require $I_{R2} = 475\,\mu A$, and since $V_{R2} = V_{BE3} \simeq 0.7\,V$

this implies $R_B \sim \frac{0.7V}{475\,\mu A} = \underline{\underline{1.474\,k\Omega}}$

With $I_{C2} = 500\,\mu A$ and $I_{R2} = 475\,\mu A$, $I_{B3} = 25\,\mu A$

$I_{C3} = \beta I_{B3} = 100 \times 25\,\mu A = 2.5\,mA$

$V_{OUT} = I_{C3} R_C$, so for $V_{OUT} = 2.5\,V$ require $R_C = \underline{\underline{1k\Omega}}$

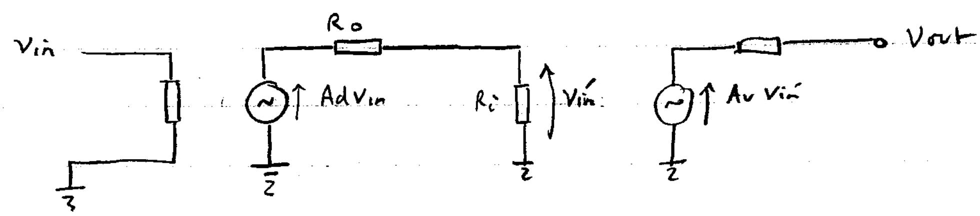
6

c) Using standard results:

Diff Amp has $A_d = \frac{g_{m1} R_B}{2}$, $R_O = R_B$

C-E stage has: $A_v = -g_{m3} R_C$, $R_i = r_{\pi 3}$

$\Rightarrow$ Overall macromodel is:



By inspection, overall gain is

$$\frac{V_{out}}{V_{in}} = +A_d \cdot \frac{R_i}{R_i + R_o} \cdot A_v = \frac{g_{m1}}{2} R_B \cdot \frac{r_{\pi 3}}{R_B + r_{\pi 3}} \cdot \left(-g_{m3} R_C\right)$$

$$= -\underline{\underline{\frac{g_{m1}}{2}\left(R_B \| r_{\pi 3}\right) \cdot g_{m3} R_C}}$$

$I_{C1} = 500\,\mu A \Rightarrow g_{m1} = 0.02\,S$

$I_{C3} = 2.5\,mA \Rightarrow g_{m3} = 0.1\,S$, $r_{\pi 3} = 1k\Omega$

$R_B = 1.474\,k\Omega$, $R_C = 1k\Omega$ $\Rightarrow A_v = -\underline{\underline{596}}$

For o/p swing of $\Delta V_{OUT} = 4V$, corresponding ΔV_{in} is $\frac{\Delta V_{OUT}}{|A_v|} = 6.7\,mV$

$\Rightarrow$ range is -3.35 to $+3.35\,mV$ 10

3 a) For upper FET (assumed active):

$$I_D = K(V_{GS} - V_t)^2 = 0.1\,\text{mA/V}^2 \times (V_G - 10 + 1)^2 = 0.4\,\text{mA}$$

$$\Rightarrow \quad V_G - 9 = \pm 2 \qquad V_G = \cancel{11\,V} \text{ or } \underline{\underline{7\,V}}$$

Lower FET has $V_G = V_D = V_{OUT}$

So $0.4\,\text{mA} = 0.4\,\text{mA/V}^2 \times (V_{OUT} - 2)^2$

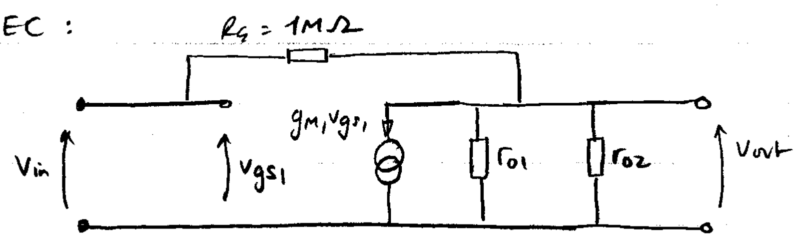
$$\Rightarrow \quad V_{OUT} - 2 = \pm 1 \qquad V_{OUT} = \cancel{1\,V} \text{ or } \underline{\underline{3\,V}}$$

Check for saturation:

Upper FET requires $V_{DS} \le V_{GS} - V_t$ $\quad V_{DS} = 3 - 10 = -7$, $V_{GS} - V_t = 7 - 10 + 1 = -2$ ok

Lower FET requires $V_{DS} \ge V_{GS} - V_t$ $\quad V_{DS} = 3$, $V_{GS} - V_t = 3 - 2 = 1$ ok 8

b) SSEC :



$$r_{o1} = r_{o2} = \frac{V_A}{I_D} = 250\,\text{k}\Omega$$

$$g_{m1} = 2K_1(V_{GS1} - V_{t1}) = 2 \times 0.4 \times 1 = 0.8\,\text{mA/V}$$

KVL @ o/p $\Rightarrow$

$$g_{m1} V_{in} + \frac{V_{out}}{r_{o1}} + \frac{V_{out}}{r_{o2}} + \frac{V_{out} - V_{in}}{R_G} = 0$$

$$\Rightarrow \quad A_V = -\left(g_{m1} - \frac{1}{R_G}\right) \cdot \left(r_{o1} /\!/ r_{o2} /\!/ R_G\right) = \underline{\underline{-88.8}}$$

(neglecting R_G gives -100)

$$R_{in} = \frac{V_{in}}{i_{in}} = \frac{R_G}{1 - A_V} = \underline{\underline{11.1\,\text{k}\Omega}}$$ 8

c) Upper FET active provided $\underbrace{V_{OUT} - 10}_{V_{DS}} \le \overbrace{-2}^{V_{GS} - V_t}$ $\quad V_{OUT} \le 8\,V$

Lower FET active provided $\underbrace{V_{OUT}}_{V_{DS}} \ge \underbrace{3 + V_{in}}_{V_{GS}} - \underbrace{2}_{V_t}$

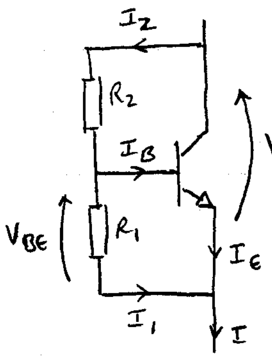
Neglecting small change in V_{GS1} due to i/p signal, latter condition is $V_{OUT} \ge 1$

So range is $1\,V \le V_{OUT} \le 8\,V$

Adding $2\,\text{M}\Omega$ resistor between G & S of lower FET will shift quiescent o/p voltage to $\left(\frac{2+1}{2}\right) \times 3\,V = 4.5\,V$ as required

4

4 a)



KVL: $V = V_{BE} + I_2 R_2$

But $I_2 = I_B + I_1$, and $I_1 = V_{BE}/R_1$

$\Rightarrow V = V_{BE} + \left(I_B + \frac{V_{BE}}{R_1}\right) R_2$

or $V = V_{BE}\left(1 + \frac{R_2}{R_1}\right) + R_2 I_B$

$I = I_E + I_1 = I_E + \frac{V_{BE}}{R_1}$

For stable V, want V_{BE} term to dominate over I_B term. 8

b) Ebers-Moll: $V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right)$

For an I_E of 2.5 mA in Q1, require $V_{BE1} = 25\text{mV} \times \ln\left(\frac{200}{201} \times \frac{2.5\text{ mA}}{10^{-14}\text{A}}\right)$

$= 656\text{ mV}$

R_1 carries current $I_{R_1} = 3\text{mA} - I_{E1} = 0.5\text{mA}$

$\Rightarrow R_1 = \frac{656\text{mV}}{0.5\text{mA}} = \underline{\underline{1.31\text{ k}\Omega}}$

For $I_C = 2\text{mA}$ in Q2, Q3, require $V_{BE2,3} = 25\text{mV} \times \ln\left(\frac{2\text{mA}}{10^{-13}\text{A}}\right)$

$= 593\text{ mV}$

So, total voltage V across multiplier is $V = 2 \times 593\text{mV} = 1.186\text{ V}$

$V = V_{BE}\left(1 + \frac{R_2}{R_1}\right) + I_B R_2$

$\Rightarrow R_2 = \frac{V - V_{BE}}{I_B + V_{BE}/R_1} = \frac{1.186 - 0.656}{\frac{2.5\times10^{-3}}{201} + 0.5\times10^{-3}} = \underline{\underline{1.034\text{ k}\Omega}}$ 8

c) $I_{E2} = 100\text{mA} \Rightarrow I_{B2} = \frac{100}{51} = 1.96\text{ mA}$

So, current I in V_{BE} multiplier is $3 - 1.96 = 1.04\text{ mA}$

Assume current in R_1 hasn't changed, ← NB: a bit tricky (ASH)

Then $I_{E1} = 1.04 - 0.5 = 0.54\text{ mA}$

and $V_{BE1} = 25\text{mV} \times \ln\left(\frac{200}{201} \times \frac{0.54\text{mA}}{10^{-14}\text{A}}\right) = 618\text{ mV}$

$\Rightarrow V = \underline{\underline{1.109\text{ V}}}$

$V_{BE2} = V_T \ln\left(\frac{100\text{mA}}{10^{-13}\text{A}}\right) = 691\text{ mV}$

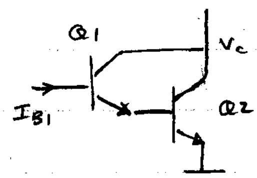
$\Rightarrow V_{BE3} = 1.109 - 0.691 = 418\text{ mV} \Rightarrow I_{E3} = \frac{51}{50} \times 10^{-13} \times \exp\left(\frac{418}{25}\right) = \underline{\underline{1.9\ \mu\text{A}}}$ 4

5 a) Class B o/p stage with resistive load to GND and

$V_{IN} > +V_{BE}$ $\Rightarrow$ Upper Q <u>ACTIVE</u> ; lower Q <u>CUT-OFF</u>

$V_{OUT} = V_{IN} - V_{BE} \simeq 3.3V$ $\Rightarrow$ $I = \frac{3.3V}{1k\Omega} = \underline{\underline{3.3\ mA}}$ 5

assuming $V_{BE} \approx 0.7V$

b) 

$I_{B1} = \frac{5 - 1.4}{120K} = 30\ \mu A$

if both active, then $I = \beta I_{B1} + \beta(1+\beta)I_{B1} = \beta(\beta+2)I_{B1}$

$= \underline{\underline{78\ mA}}$

Check modes : $V_C = 10 - 0.078 \times 100 = 2.2\ V$

$\Rightarrow$ Assumption OK and <u>both Qs ACTIVE</u> 5

c) $V_{DD} = 6V > V_{t_1} + V_{t_2}$ $\Rightarrow$ Both FETs conducting

Both have $V_{GS} = V_{DS}$ $\Rightarrow$ UPPER IS <u>TRIODE</u> ($Vt < 0$)

LOWER IS <u>ACTIVE</u> ($Vt > 0$)

For upper : $V_{GS} = V_{DS} = 6 - V$ and $Vt = -2$

$\Rightarrow$ $I_D = k_u[2(6-V+2)(6-V) - (6-V)^2]$

$= k_u[6-V][10-V]$

For lower : $V_{GS} = V_{DS} = V$ and $Vt = 1$

$\Rightarrow$ $I_D = K_L(V-1)^2$

Solving for V $\Rightarrow$ $V = 59/14 = \underline{\underline{4.214\ V}}$ 5

d) Assume ACTIVE initially

$I_D = K(V_{GS} - Vt)^2 = 0.25 \times (3-1)^2 = 1mA$

But this would imply $V = 3 - 2k \times 1mA = 1V < V_{GS} - Vt$

$\Rightarrow$ FET must be in <u>TRIODE</u> region

Now have $I_D = K[2(V_{GS} - Vt)V_{DS} - V_{DS}^2] = \frac{3-V}{2k\Omega}$

$\Rightarrow$ $\frac{1}{4}[2(3-1)V - V^2] = \frac{3-V}{2}$

$4V - V^2 = 6 - 2V$

$V^2 - 6V + 6 = 0$ $\Rightarrow$ $V = 3 \pm \sqrt{3}$ 5

+ sign not possible (> 3V), so $\underline{\underline{V = (3 - \sqrt{3})\ V}}$