Master – June 07

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2007

EEE PART I: MEng, BEng and ACGI

DEVICES AND FIELDS

Tuesday, 22 May 10:00 am

Time allowed: 2:00 hours

There are SIX questions on this paper.

Question ONE and Question FOUR are compulsory. Answer Question One and Question Four, plus one additional question from Section A and one additional question from Section B.

Questions One and Four each carry 20% of the marks; remaining questions each carry 30% of the marks.

Use a separate answer book for each section.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	K. Fobelets, E.M. Yeatman
	Second Marker(s) :	E.M. Yeatman, B.C. Pal

Special instructions for students

permittivity of free space:	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
permeability of free space:	$\mu_o = 4\pi \times 10^{-7}$ H/m
intrinsic carrier concentration in Si:	$n_i = 1.45 \times 10^{10}$ cm^{-3} at $T = 300$K
dielectric constant of Si:	$\varepsilon_{Si} = 11$
dielectric constant of SiO_2:	$\varepsilon_{ox} = 4$
thermal voltage:	$kT/e = 0.026$V at $T = 300$K
charge of an electron:	$e = 1.6 \times 10^{-19}$ C

Formulae

$$\left.\begin{aligned} J_n(x) &= e\mu_n n(x)E(x) + eD_n\frac{dn(x)}{dx} \\ J_p(x) &= e\mu_p p(x)E(x) - eD_p\frac{dp(x)}{dx} \end{aligned}\right\}$$

Drift and diffusion currents in a semiconductor

$$I_{DS} = \frac{\mu C_{ox}W}{L}\left((V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2}\right)$$

Current in a MOSFET

$$\left.\begin{aligned} J_n &= \frac{eD_n n_p}{L_n}\left(e^{\frac{eV}{kT}} - 1\right) \\ J_p &= \frac{eD_p p_n}{L_p}\left(e^{\frac{eV}{kT}} - 1\right) \end{aligned}\right\}$$

Diffusion currents in a pn-junction

$$V_0 = \frac{kT}{e}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Built-in voltage

$$c = c_0 \exp\left(\frac{eV}{kT}\right) \text{ with } \begin{cases} c = p_n \text{ or } n_p \\ c_0 \text{ bulk minority carrier concentration} \end{cases}$$

Minority carrier injection under bias V

$$D = \frac{kT}{e}\mu$$

Einstein relation

SECTION A: SEMICONDUCTOR DEVICES

1. This question is **obligatory**.

a) Give the definition of the Fermi level. [2]

b) Given a slab of n-type Si with an electric field as given in Fig.1.1, what is the direction of the electron flow? (Answer L or R) [2]

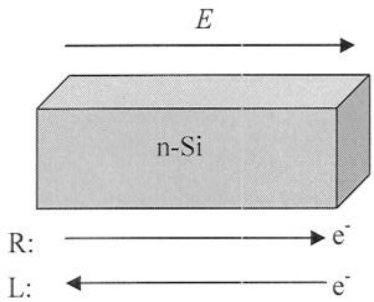


Figure 1.1: The electron flow in a slab of n-Si under an electric field E.

c) Give the physical reason for the constant but very low reverse bias (saturation) current in a pn diode. [4]

d) Sketch the material cross section, including doping and contacts, of an un-biased enhancement mode MOSFET that under inversion will have holes in the channel. Give the doping type in each necessary region and the correct annotations to the contacts. [6]

e) Sketch the energy band diagram (E_c, E_v, E_F) of an un-biased p^+np BJT and explain briefly the function of both the emitter-base and the base-collector junction in the BJT in active mode. [6]

2.

a) Draw the energy band diagram (E_c, E_v, E_F) of an n^+p diode under zero bias. The doping in the n-region is 10^3 times larger than in the p-region. Indicate the depletion region and potential barrier on your diagram. [5]

b) Give the charge neutrality equation across the depletion region and derive the relation between the extend of the depletion width in the n and the p type regions. Define all parameters you use. [5]

c) Determine the doping density in each region for the Si diode given in a) such that at an applied forward bias of V=0.5V, a current of I=10μA is flowing. The parameters of the diode are the following:
The diffusion length of the minority carriers: $L_p=10^{-2}$ cm.
$L_n=10^{-3}$ cm.
The diffusion constant of the minority carriers: $D_p=10$ cm^2/s.
$D_n=20$ cm^2/s.
The area of the diode is: $A=10^{-2}$ cm^2.
The temperature is: T=300K. [10]
Indicate clearly any approximations you make.

d) Derive the expression for current gain β in a short n^+pn bipolar transistor as a function of material and geometrical parameters. Ignore I_{CB0}. Define all the parameters you use. [8]

e) Based on your result in d), explain briefly why the emitter is doped higher than the base. [2]

3.

a) Give the definition of threshold voltage (in words) and explain how an increase in doping in the substrate of an enhancement mode n-channel MOSFET will influence the threshold voltage? [6]

b) In Fig.3.1 the electron drift velocity-electric field curves are given for two different semiconductors, GaAs and Si. Which of the two has the highest mobility? Why? [4]

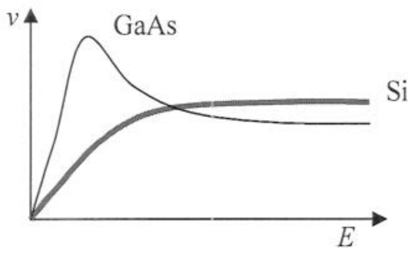


Figure 3.1: Velocity (v) versus electric field (E) for two semiconductors GaAs and Si.

c) If two identical MOSFETs (same geometry, gate and doping densities) are fabricated but one is in GaAs and one in Si, which of the two would carry the highest current for the same gate voltage overdrive V_{GS}-V_{th}? Why? [4]

d) A Si MOSFET is measured and the characteristics given in Fig.3.2.
   i) Give the value of the threshold voltage and explain how you derive it. [4]
   ii) Determine in which region the MOSFET is measured (triode or saturation region.) [2]
   iii) Estimate the value of the mobility knowing that the workfunction of the gate contact is equal to the workfunction of the Si substrate, and that the gate length and width are respectively W_g = 100 μm and L_g = 10 μm. [10]

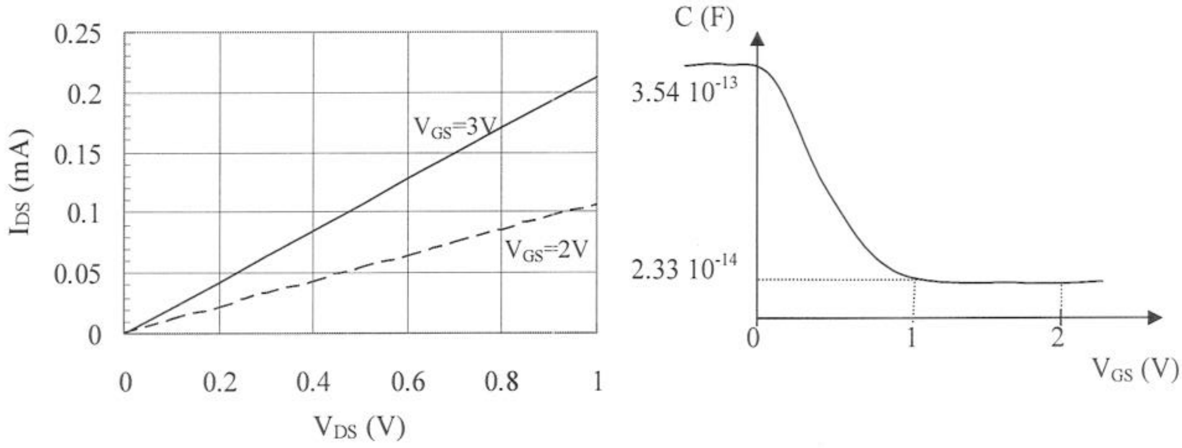


Figure 3.2: Left: output characteristics I_{DS}-V_{DS} of the MOSFET for 2 gate voltages as shown. Right: MOS capacitance C as a function of gate voltage. Note: C takes into account the gate area.

SECTION B: FIELDS

4. This question is **obligatory**.

a) When an electric field line meets a conductive surface, what is its direction relative to that surface? [4]

b) For a single point charge of +0.1 C in free space, what will be the magnitude of the electric flux density at a distance 2 cm from this charge? [4]

c) For a toroidal inductor, what will be the effect on its inductance value of doubling the number of turns N, assuming all other parameters are unchanged? [4]

d) An ideal transformer has a turns ratio $N_1:N_2 = 2:1$, and a load of 100 Ω is connected to the secondary coil terminals. If a constant DC current of 100 mA flows in the primary coil, what current will flow in the secondary coil? [4]

e) A linear ferromagnetic material contains a magnetic flux density B = 0.4 T when the magnetic field strength H = 40 A/m. Calculate the relative permeability μ_r of this material. [4]

5. A circular ring of thin wire in air, as shown in Fig. 5.1, has radius R and carries a net charge of $+Q$, uniformly distributed around the ring.

a) Derive an expression for the magnitude of the electric field E, at a point P on the z axis at $z = h$, due to a small segment of the wire of length dl. You may approximate the wire segment as a point charge. [8]

b) Derive an expression for E_z , the z component of the field in part (a). [6]

c) Hence, derive an expression for the electric field E_z in the z direction, at point P, due to the whole ring. Will the field at P have x or y components? [8]

d) Using your solution to (c), find the voltage V at the centre of the ring ($x = y = z = 0$), taking $z = \infty$ as the voltage reference ($V(\infty) = 0$). [8]

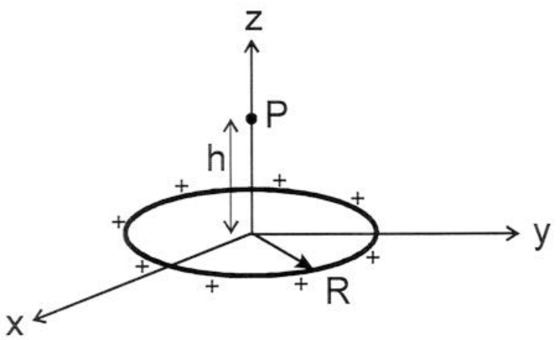


Figure 5.1 A circular thin wire with net charge +Q and radius R.

6. a) An inductor is made by winding N turns of wire around a cylindrical iron core of relative permeability μ_r, length s and cross-sectional area A, as shown in Fig. 6.1. When current flows in the winding in the direction shown, what will be the direction of the magnetic field H in the core? Note that the core axis is in the y direction. [6]

b) State Ampere's Law for the magnetic field strength H. Using this law, taken along the dotted path as shown, find an expression for H_i in the core for a winding current I. You may approximate the field strength outside the core as zero. Hence, calculate the magnetic flux density in the core, B_i, for the following values: $N = 10$, $A = 1\ \text{cm}^2$, $s = 8$ cm, $\mu_r = 8000$, $I = 0.5$ A. State any approximations or assumptions made. [9]

c) Give an expression for the total magnetic flux Φ flowing in the core. Hence, remembering that inductance is given by flux linkage per unit current, find an expression for the inductance L. Calculate L for the parameter values given in (b) above. [9]

d) Why is it reasonable to make the approximation given in (b) above, that H is negligible outside the core? Does this approximation result in an overly high, or overly low estimate of the inductance L? [6]

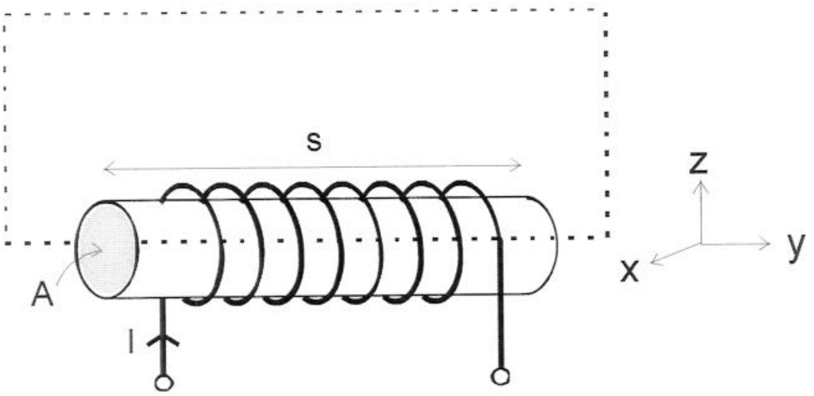


Figure 6.1 A cylindrical iron core having N windings.

E1.3 Devices + Fields 2007

SECTION A: SEMICONDUCTOR DEVICES - **ANSWERS**

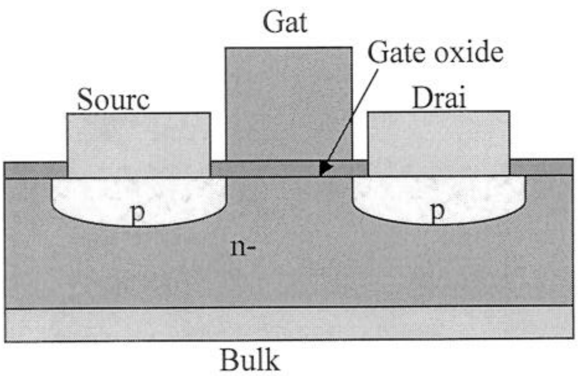
1. This question is **obligatory**.

a) The Fermi level defines the position in energy where the probability of finding an electron is equal to 1/2. [2]

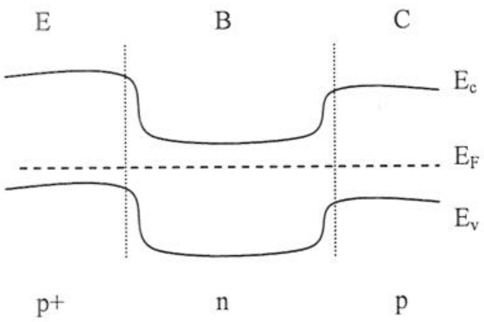
b) L [2]

c) The reverse bias current in a pn diode is due to the concentration of minority carriers available at each side of the junction that can be injected across the junction in reverse bias. Since this number is small (minority carriers) this limits the current that can flow rather than the applied voltage. As the number is small the current is small. [4]

d) pMOS. [6]



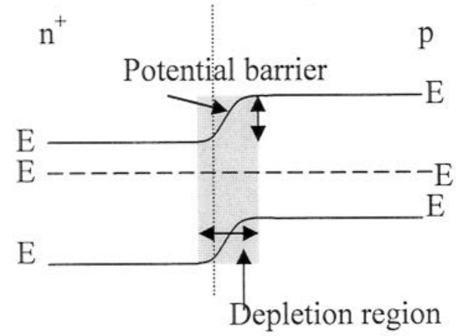
e)



In active mode the emitter-base junction is forward biased and injects carriers (mainly holes in pnp BJT) into the base. The base-collector junction is reverse biased and thus collects the minority carriers (holes) that are injected in to the base. [6]

2.

a) . [5]



b) Charge neutrality: $eN_D w_n A = eN_A w_p A$ [5]

with: e charge of 1 electron

A cross sectional area of the diode

N_D: donor doping concentration

N_A: acceptor doping concentration

w_n: depletion region extending in the n-doped region

w_p: depletion region extending in the p-doped region

since $N_D >> N_A$, $w_n << w_p$

c) From formulae sheet: [10]

$$J_n = \frac{eD_n n_p}{L_n}\left(e^{\frac{eV}{kT}} - 1\right)$$
$$J_p = \frac{eD_p p_n}{L_p}\left(e^{\frac{eV}{kT}} - 1\right)$$
thus $I = eA\left(\frac{D_n n_p}{L_n} + \frac{D_p p_n}{L_p}\right)\left(e^{\frac{eV}{kT}} - 1\right)$

Approximations that one can make are:

1) since $N_D >> N_A$ the diode current will be approximately equal to the electron current I_n thus $I \approx I_n = eA\frac{D_n n_p}{L_n}\left(e^{\frac{eV}{kT}} - 1\right)$

2) since forward bias the "-1" term with the exponential is negligible thus

$$I \approx I_n = eA\frac{D_n n_p}{L_n} e^{\frac{eV}{kT}}$$

Replacing the minority carrier concentration by its expression as a function of doping density: $n_p = \frac{n_i^2}{N_A}$ gives $I \approx eA\frac{n_i^2 D_n}{N_A L_n} e^{\frac{eV}{kT}}$. Thus the p-side doping concentration can be extracted via: $N_A \approx eA\frac{n_i^2 D_n}{IL_n} e^{\frac{eV}{kT}}$

$$N_A \approx 1.6\,10^{-19} C \times 10^{-2}\,cm^{-2}\,\frac{\left(1.45\,10^{10}\,cm^{-3}\right)^2 20\,cm^2/s}{10\,10^{-6}\,A\,10^{-3}\,cm} e^{\frac{0.5}{0.026}} = 1.5\,10^{17}\,cm^{-3}$$

$N_D = 10^3 \times N_A = 1.5\,10^{20}\,cm^{-3}$.

d) Current gain $\beta = \frac{I_C}{I_B}$ by definition, with I_C collector current and I_B base current.

In an n$^+$pn BJT the electrons are the carriers collected by the collector. Thus I_C is the electron current of the emitter-base forward biased junction. I_B is equal to the hole current that is escaping from the base into the emitter. Thus using the pn junction diffusion current definition in the formulae sheet:

$$I_C = I_n = \frac{eD_n n_p}{L_n}\left(e^{\frac{eV}{kT}} - 1\right)A$$

$$I_B = I_p = \frac{eD_p p_n}{L_p}\left(e^{\frac{eV}{kT}} - 1\right)A$$

$$\beta = \frac{I_C}{I_B} = \frac{I_n}{I_p} = \frac{\frac{D_n n_p}{L_n}}{\frac{D_p p_n}{L_p}} = \frac{D_n n_p L_p}{D_p p_n L_n} = \frac{D_n N_D L_p}{D_p N_A L_n}$$ [8]

If the lengths of the emitter and the base are different from the diffusion lengths, then L_n (= diffusion length of the electrons in the p-type region = base) becomes W_b (base width) and L_p (=diffusion length of the holes in the n-type region = emitter) becomes x_e and thus the gain formulae should be rephrased as:

$$\beta = \frac{D_n N_D x_e}{D_p N_A W_B}$$

e) For a higher current gain. [2]

3.

a) The threshold voltage is the voltage that needs to be applied on the gate in order to invert the channel. The definition of inversion is that the number of free carriers in the inverted channel must be the same as the number of free carriers in the bulk. If the number of free carriers in the bulk is increased by increasing the doping concentration of the substrate then a larger amount of free carriers must be generated in the channel. Thus a higher gate voltage will be to be applied to do this, thus the threshold voltage increases. (Note: threshold voltage of an n-type enhancement mode MOSFET is positive) [6]

b) GaAs, because the *v-E* slope at low electric fields is steeper than for Si. The slope of the *v-E* curve at low *E* is the mobility following: $v=\mu E$. [4]

c) The GaAs MOSFET due to the higher mobility and the current is directly proportional to the mobility of the carriers. [4]

d) i) The threshold voltage can be derived from the C-V characteristic when depletion is reached. Full depletion is reached when the capacitance is minimum. Thus V_{th}=1V. [4]

ii) The measurement is done in the triode region (low V_{DS}, current linear). [2]

iii) From the capacitance-voltage characteristics we can derive the oxide capacitance:

$$C_{ox} = \frac{C(V_{GS}=0V)}{A} = \frac{C(V_{GS}=0V)}{L_g W_g} = \frac{3.54\,10^{-13}\,F}{1000\times 10^{-12}\,m^2} = 3.54\,10^{-4}\,F/m^2$$ [5]

The current I_{DS} in the triode region is given by:

$$I_{DS} = \frac{\mu C_{ox} W}{L}(V_{GS}-V_{th})V_{DS}$$

At V_{DS}=0.5V and V_{GS}=2V we estimate the current $I_{DS}\approx 5\ 10^{-5}$ A [5]

$$\mu = \frac{I_{DS} L_g}{C_{ox} W_g (V_{GS}-V_{th}) V_{DS}} = \frac{5\,10^{-5}\,A \times 10\,\mu m}{3.54\,10^{-4}\,F/m^2 \times 100\,\mu m \times 1V \times 0.5V}$$

$$\mu = 0.0283\frac{m^2}{Vs} \approx 0.03\frac{m^2}{Vs} = 300\frac{cm^2}{Vs}$$

Fields and Devices Part B: SOLUTIONS

4\. a) When an electric field line meets a conductive surface, what is its direction relative to that surface? *It is perpendicular to the surface.*

b) For a single point charge of +0.1 C in free space, what will be the magnitude of the electric flux density at a distance 2 cm from this charge?
$D = Q/(4\pi r^2) = 19.9\ C/m^2$.

c) For a toroidal inductor, what will be the effect on its inductance value of doubling the number of turns N, assuming all other parameters are unchanged?
The inductance is proportional to N^2, so doubling N will increase L by a factor of 4.

d) An ideal transformer has a turns ratio $N_1:N_2 = 2:1$, and a load of 100 W is connected to the secondary coil terminals. If a constant DC current of 100 mA flows in the primary coil, what current will flow in the secondary coil?
There is no coupling of direct current across a transformer so the current in the secondary coil will be zero.

e) A linear ferromagnetic material contains a magnetic flux density B = 0.4 T when the magnetic field strength H = 40 A/m. Calculate the relative permeability ε_r of this material.
$B = \mu H = \mu_r \mu_o H$, so $\mu_r = B/(\mu_o H) = (0.4/(4\pi \times 10^{-7} \times 40) = 7958$.

5\. A circular ring of thin wire, as shown in Fig. 5.1, has radius R and carries a net charge of +Q, uniformly distributed around the ring.

a) Derive an expression for the magnitude of the electric field, at a point P on the z axis at z = h, due to a small segment of the wire of length $d\ell$. You may approximate the wire segment as a point charge. [8]

This is just the field of a point charge, $q/(4\pi\varepsilon_o r^2)$, with $q = Q\,d\ell/(2\pi R)$, and $r^2 = R^2 + h^2$, giving $E = Q\,d\ell/[8\pi^2\varepsilon_o R(R^2 + h^2)]$

b) Derive an expression for the z component of the field in part (a). [6]

Field direction is parallel to the line from the segment to P, so:
$E_z/E = h/(R^2 + h^2)^{1/2}$ *giving* $E_z = Qhd\ell/[8\pi^2\varepsilon_o R(R^2 + h^2)^{3/2}]$

c) Hence, derive an expression for the electric field E_z in the z direction, at point P, due to the whole ring. Will the field at P have x or y components? [8]

The z component is equal for every segment, so the total E_z is just the value from (b) times $2\pi R/d\ell$, giving: $E_z = Qh/[4\pi\varepsilon_o(R^2 + h^2)^{3/2}]$
By symmetry we can see that there is no x or y field.

d) Using your solution to (c), find the voltage at the centre of the ring (x = y = z = 0), taking z = ∞ as the voltage reference (V(∞) = 0). [8]

$V = -\int E_z dz = [Q/(4\pi\varepsilon_o)] \int z(R^2 + z^2)^{-3/2} dz$ evaluated from z= ∞ to 0.

It is straightforward to determine that $\int x(a^2 + x^2)^{-3/2} dx = -(a^2 + x^2)^{-1/2}$

So $V = [Q/(4\pi\varepsilon_o)][(R^2 + z^2)^{-1/2}]$

This gives $V(\infty) = 0$ directly, and $V(0) = Q/(4\pi\varepsilon_o R)$.

6\. a) An inductor is made by winding N turns of wire around a cylindrical iron core of relative permeability μ_r, length L and cross-sectional area A, as shown in Fig. 6.1. When current flows in the winding in the direction shown, what will be the direction of the magnetic field H in the core? Note that the core axis is in the *y* direction. [6]

The magnetic field will be along the direction of the core axis, and by right hand rule in this case we can find that H is in the +y direction.

b) State Ampere's Law for the magnetic field strength H. Using this law, taken along the dotted path as shown, find an expression for H_i in the core for a winding current I. Sate any approximations or assumptions made. You may approximate the field strength outside the core as zero. Hence, calculate the magnetic flux density in the core, B_i, for the following values: N = 10, A = 1 cm^2, s = 8 cm, μ_r = 8000 I = 0.5 A. [9]

Ampere's law: $\int H \cdot dl = NI$, so ignoring H outside core, and assuming H_i is parallel to dl, this gives $H_i = NI/s$. Then:
$B_i = \mu_r \mu_o H_i = 8000 \times 4\pi \times 10^{-7} \times 10 \times 0.5/0.08 = 0.628$ T

c) Give an expression for the total magnetic flux Φ flowing in the core. Hence, remembering that inductance is given by flux linkage per unit current, find an expression for the inductance L. Calculate L for the parameter values given in (b) above. [9]

$\Phi = BA$ (assuming uniform B in cross section). Need to recall that flux linkage equals ΦN, then $L = \Phi N/I = BAN/I = 0.628 \times 10^{-4} \times 10/0.5 = 1.26$ mH.

d) Why is it reasonable to make the approximation given in (b) above, that H is negligible outside the core? Does this approximation result in an overly high, or overly low estimate of the inductance L? [6]

Once outside the core, the flux spreads over a very wide area, so the flux density B along the return path outside the core is hugely reduced, and thus H is also small. In reality $H \cdot dl$ is not zero outside the core, so this reduces H_i for a given NI, and thus the actual B is less, and consequently the approximation over-estimates L.