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E1.3

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2006

EEE PART I: MEng, BEng and ACGI

**DEVICES AND FIELDS**

Wednesday, 24 May 10:00 am

Time allowed: 2:00 hours

**There are FIVE questions on this paper.**

**Q1 is compulsory.**
**Answer Q1 and any two of questions 2-5. Answer at least one question from Section B.**

*Q1 carries 40% of the marks. Questions 2 to 5 carry equal marks (30% each)*

**Any special instructions for invigilators and information for candidates are on page 1.**

| | | |
|---|---|---|
| Examiners responsible | First Marker(s) : | K. Fobelets, E.M. Yeatman |
| | Second Marker(s) : | E.M. Yeatman, B.C. Pal |

**Special information for invigilators:** Q1 is Compulsory

**Information for candidates**

| | |
|---|---|
| permittivity of free space: | $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m |
| permeability of free space: | $\mu_0 = 4\pi \times 10^{-7}$ H/m |
| intrinsic carrier concentration in Si: | $n_i = 1.45 \times 10^{10}$ cm$^{-3}$ at $T = 300$K |
| dielectric constant of Si: | $\varepsilon_{Si} = 11$ |
| dielectric constant of $SiO_2$: | $\varepsilon_{ox} = 4$ |
| thermal voltage: | $kT/e = 0.026$V at $T = 300$K |
| charge of an electron: | $e = 1.6 \times 10^{-19}$ C |

Formulae

$$\left.\begin{aligned} J_n(x) &= e\mu_n n(x)E(x) + eD_n \frac{dn(x)}{dx} \\ J_p(x) &= e\mu_p p(x)E(x) - eD_p \frac{dp(x)}{dx} \end{aligned}\right\}$$

Drift and diffusion currents in a semiconductor

$$I_{DS} = \frac{\mu C_{ox} W}{L}\left((V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right)$$

Current in a MOSFET

$$\left.\begin{aligned} J_n &= \frac{eD_n n_p}{L_n}\left(e^{\frac{eV}{kT}} - 1\right) \\ J_p &= \frac{eD_p p_n}{L_p}\left(e^{\frac{eV}{kT}} - 1\right) \end{aligned}\right\}$$

Diffusion currents in a pn-junction

$$V_0 = \frac{kT}{e}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Built-in voltage

$$c = c_0 \exp\left(\frac{eV}{kT}\right) \text{ with } \begin{cases} c = p_n \text{ or } n_p \\ c_0 \text{ bulk minority carrier concentration} \end{cases}$$

Minority carrier injection under bias $V$

$$L = \sqrt{D\tau}$$

Diffusion length

$$D = \frac{kT}{e}\mu$$

Einstein relation

SECTION A: SEMICONDUCTOR DEVICES

1. **Compulsory!**

a) Give the doping type of Material 1 and 2 with energy band diagrams given in Figure 1. [4]

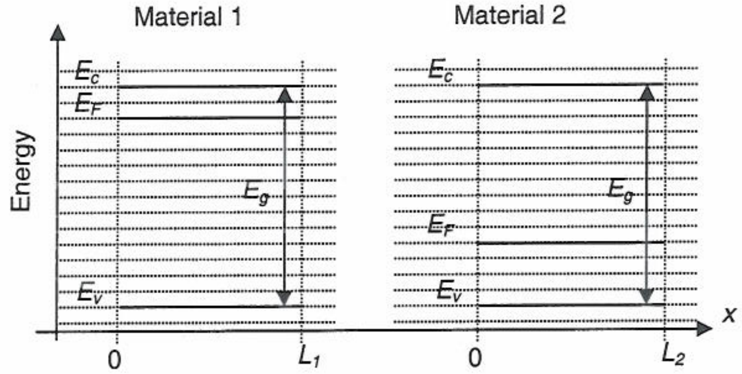


*Figure 1: The energy band diagrams of two materials.*

b) Give the relationship (<, > or =) between the density of the free majority carriers in material 1 and material 2 shown in Figure 1. Explain your answer briefly. [4]

c) Draw the energy band diagram for each of Material 1 and 2 when a positive voltage is applied to the right edge of each material (in $x=L_1$ respectively $L_2$). Add the direction of the electric field and direction of majority carrier flux. [4]

d) Draw the energy band diagram of Material 1 and 2 of Figure 1 in contact (no bias applied). Make sure the relative widths of the depletion region across the junction are correct [4]

e) Given a doped Si substrate with a donor concentration of $N_D = 2.1 \times 10^{17}\ cm^{-3}$, calculate the density of free electrons and holes in this material. [4]

f) Give the relationship (<, > or =) between the majority carrier current through a p-type and n-type Si substrate of the same geometry and doping density, under the same bias condition. Explain your answer briefly. [4]

g) Explain briefly the reason for the time delay that occurs when switching a pn junction from on to off (zero bias). [4]

h) A certain one-dimensional system has an electric field:

$$E(x) = [4x - 2]\ \text{V/m}$$

and the potential at $x = 0$ is $V(0) = 0V$. Find the maximum positive potential $V_{max}$ in the system [4]

i) A circular open-circuit coil of 10 turns is placed in a spatially uniform magnetic field with a flux density which is increasing linearly at a rate 0.1 T/sec. If the coil area is 10 cm$^2$ and the coil lies perpendicularly to the field, find the voltage induced in it. [4]

j) Two long, straight, parallel wires, 10 cm apart, are each carrying a current of 10 A, in opposite directions. Calculate the magnetic flux density B at a point mid-way between the two wires. [4]

2.

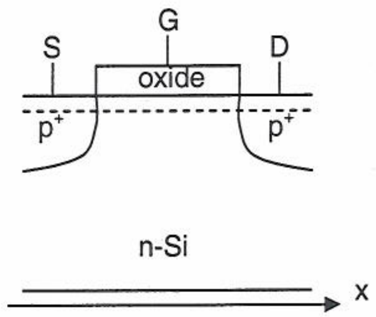


*Figure 2.1: A Si MOSFET on an n-type substrate. (The dashed line relates to part c only)*

a) Define the type of the MOSFET of Figure 2.1 (enhancement-, depletion-mode, n- or pMOS). [4]

b) Is the threshold voltage $V_{th}$ of the MOSFET in Figure 2.1 >, < or = to 0? Explain your answer briefly. [4]

c) Plot the energy band diagram ($E_c$, $E_v$, $E_F$, $E_g$) from source S to drain D through the channel along the dashed line for the MOSFET of Figure 2.1 when:

i) $V_{GS} = V_{DS} = 0$V with bulk and source contact grounded. [6]
ii) $|V_{GS}| > |V_{th}|$, $V_{DS} < 0$V and bulk and source contact grounded. [6]

d) In Figure 2.2 the transfer characteristic $\sqrt{I_{DS}}$ versus $V_{GS}$ of an ideal n-channel MOSFET with a gate length of 1 μm is given. The maximum capacitance measured between gate and bulk contact of the MOSFET is $C_{max} = 1.45 \times 10^{-13}$ F. Determine the mobility $\mu$ and the threshold voltage $V_{th}$ of the MOSFET. [10]

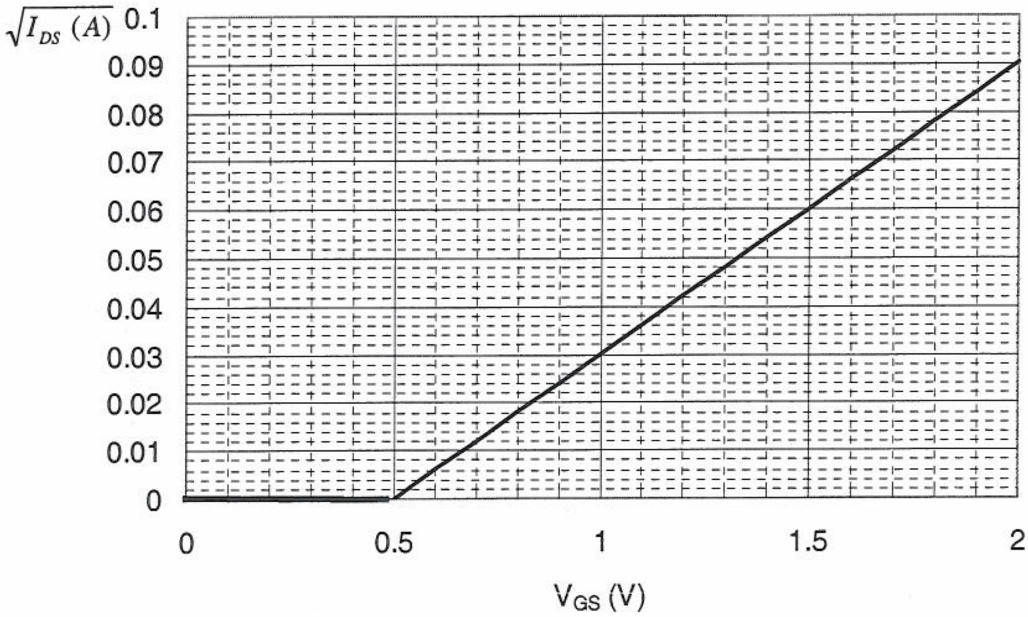


*Figure 2.2: The transfer characteristics: $\sqrt{I_{DS}}$ versus $V_{GS}$ of an ideal n-channel MOSFET*

3.

a) Sketch the energy band diagram ($E_c$, $E_v$, $E_F$, $E_g$) through an unbiased $p^+ n n^+ p$ Si junction (note: there are 4 different layers in the structure. The layers are sufficiently wide to allow bulk conditions to be re-established far from the junctions). Ensure that the relative distances between the energy levels are consistent. [6]

b) Explain why the reverse bias current in a pn junction remains constant for increasing reverse bias voltage. [4]

c) A **short** Si pn junction has the properties given in Table 3. The length of the p-type region is the electron diffusion length $L_n$, and the length of the n-type region is the hole diffusion length $L_p$. The cross-sectional area of the diode is $10^{-4}$ cm$^2$. Ignore the depletion region. The contacts are ideal.

| parameter | p-side | n-side | units |
|---|---|---|---|
| Doping density | $10^{17}$ | $10^{15}$ | cm$^{-3}$ |
| Lifetime of minority carriers ($\tau$) | 0.1 | 10 | μs |
| Mobility of majority carriers ($\mu$) | 200 | 1300 | cm$^2$/(Vs) |
| Mobility of minority carriers ($\mu$) | 700 | 450 | cm$^2$/(Vs) |
| Length of un-depleted region (L) | $L_n$ | $L_p$ | cm |

*Table 3: Parameters of the pn junction*

i) Calculate the diffusion lengths, $L_n$ and $L_p$ of the minority carriers in both the n and p regions of the diode. [6]

ii) Draw the minority carrier concentration variation in the n and p regions of the diode given in Table 3 under a forward bias of 0.5V. Give the magnitude of the minority carrier concentration at each edge of the depletion region and at each contact.
Tip: Add the lengths of the un-depleted p and n regions on the x-axis. [8]

iii) Calculate the value of the forward bias current. [6]

SECTION B: FIELDS

4. An iron C-core as shown in Fig. 4.1 has a gap length of $g = 0.5\ mm$, a total path length in the iron of $\ell_i = 40\ cm$, and the coil has 200 turns

a) Assuming the iron has $\mu_r = 8000$, and the coil current $I = 0.5\ A$, calculate:

i. the magnetic flux density in the gap $B_g$.
ii. the magnetic flux density in the iron $B_i$.
iii. the magnetic field in the gap $H_g$.
iv. the magnetic field in the iron $H_i$

State any assumptions or approximations made. [16]

b) Assume now that the core has a *B-H* characteristic as shown in Fig. 4.2, i.e. linear up to $H = 100\ A/m$, and saturated at $B = 1\ T$ for $H > 100\ A/m$.
Calculate:

i. the coil current $I$ at which the flux density in the core will reach saturation. [7]
ii. the magnetic field in the iron $H_i$ for $I = 2.5\ A$. [7]

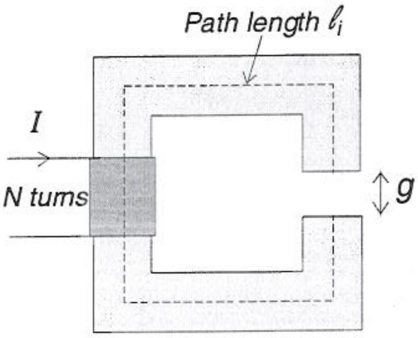


*Figure 4.1*

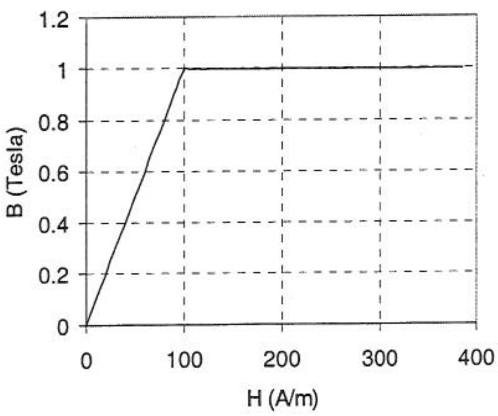


*Figure 4.2*

5. A coaxial cable cross-section is shown in Figure 5.1 below. The radii of the inner and outer conductors are $a = 4\ mm$ and $b = 8\ mm$ respectively, and the relative permittivity of the dielectric between them $\varepsilon_r = 6$.

   a) Using Gauss' Law, derive an expression for the electric field in the dielectric, $E(r)$, where $r$ is the radial distance from the cable axis, when the charge per unit length on the inner conductor is $\rho$. [7]

   b) Hence, determine the value of $\rho$, and the position and magnitude of the maximum electric field, if the applied voltage is $V = 10\ V$. [7]

   c) For an applied voltage of $V = 10\ V$ as in (b), find the voltage at radius $r = 6\ mm$ with respect to ground. [6]

   d) The uniform dielectric material is now replaced by one whose relative permittivity is a function of the distance from the axis, i.e. $\varepsilon_r = \varepsilon_r(r)$. Find the function $\varepsilon_r(r)$ for which the electric field in the dielectric is uniform, and the capacitance per unit length of the cable is 200 pF/m. [10]

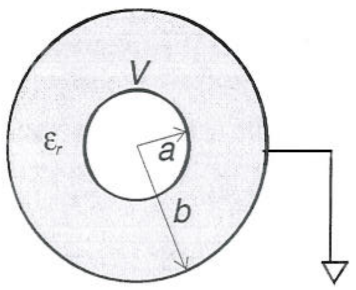


*Figure 5.1*

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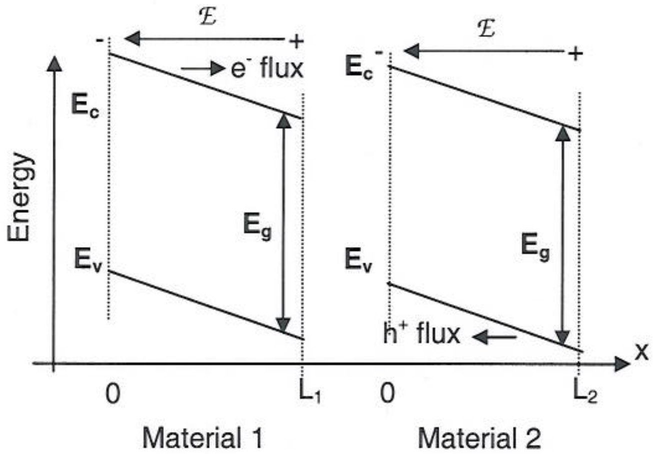
1. **Compulsory!**

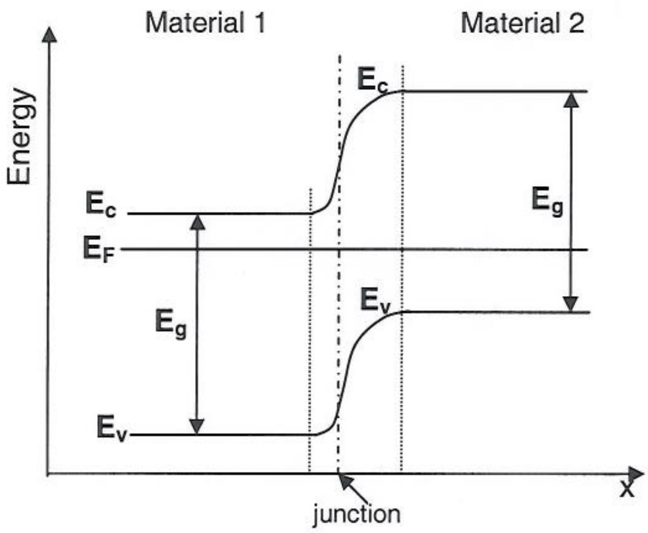
a) Material 1: n-type
Material 2: p-type [2]

b) n>p [2]

c) [2]




d) [2]




e)

$$n = N_D = 2.1 10^{17} cm^{-3}$$

$$p = \frac{n_i^2}{N_D} = \frac{(1.45\times10^{10})^2}{2.1\times10^{17}} = 10^3 cm^{-3}$$ [2]

f) $I_p < I_n$ [2]

g) The delay occurs because the excess minority carrier charges that are built-up in the p and n region of the diode under forward bias need to change from the large amount in forward bias to none at zero bias via recombination. [2]

h) $V = -\int E(x)dx = -4x^2/2 + 2x$. $V_{max}$ is at E=0, i.e. x = 0.5, so $V_{max}$ = 0.5V [2]

i) V = -d(NBA)/dt = $-10 \times 10^{-3}$ $m^2 \times 0.1$ T/s = 1 mV [2]

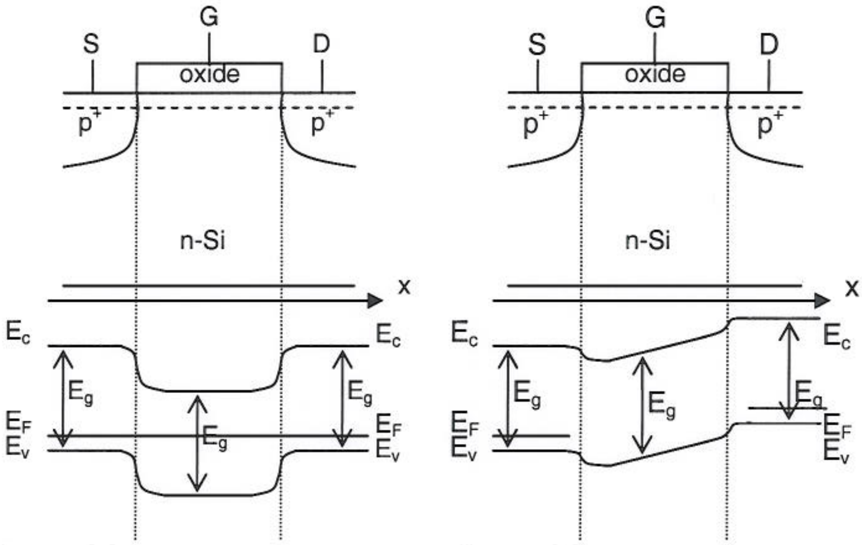
j) At a distance r from a long straight wire $B = \mu_o I/(2\pi r)$, and mid-way between the two wires each contributes an equal B in the same direction, perpendicular to the wires and to the line connecting them. Thus
$B = 2 \times 4\pi \times 10^{-7} \times 10 / (2\pi \times 0.05) = 0.08$ mT [2]

2.

a) enhancement-mode pMOS. [2]

b) $V_{th} < 0$ [2]

c)




i) [4] ii) [4]

d) $V_{th}$ = 0.5V from graph (where current starts to be non-zero)

Current of MOSFET in saturation for $V_{DS}=V_{GS}-V_{th}$

$$I_{DS} = \frac{\mu C_{ox} W}{L}\left( (V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2} \right)$$

$$I_{DS}^{sat} = \frac{\mu C_{ox} W}{2L}(V_{GS} - V_{th})^2 \Rightarrow \sqrt{I_{DS}^{sat}} = \sqrt{\frac{\mu C_{ox} W}{2L}}(V_{GS} - V_{th})$$

Maximum capacitance

$$C_{max} = C_{ox} \times W \times L \Rightarrow C_{ox} \times W = \frac{C_{max}}{L}$$

Thus: $\sqrt{I_{DS}^{sat}} = \sqrt{\frac{\mu C_{max}}{2L^2}}(V_{GS} - V_{th})$

The gradient of the transfer characteristics, *grad* in Fig. 2.2 is $grad = \sqrt{\frac{\mu C_{max}}{2L^2}}$

At $V_{GS}$= 1V, $\sqrt{I_{DS}}$ =0.03$\sqrt{A}$ and at $V_{GS}$=2V, $\sqrt{I_{DS}}$ =0.09$\sqrt{A}$

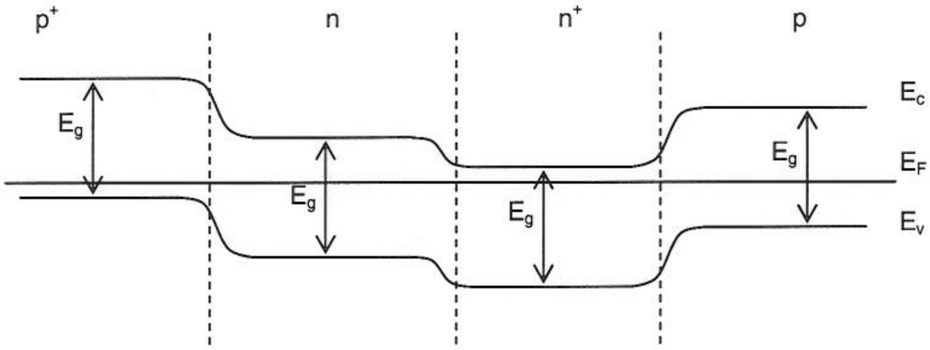
$$grad = \frac{0.03 - 0.09}{1 - 2} = 0.06\frac{\sqrt{A}}{V}$$

$$\mu = grad^2 \frac{2L^2}{C_{max}} = 0.06^2 \frac{A}{V} \frac{2 \times 10^{-8} cm^2}{1.45\, 10^{-13} F} = 496.6 \frac{cm^2}{Vs}$$

[8]

3.

a) [4]




b) The reverse bias current in a pn diode is determined by the amount of minority carriers that are available in each side of the junction for injection across the depletion region. The amount is small and does not increase with increasing field. [4]

c)

i) Note: $L_p$ is the diffusion length of the holes in the n-type region, $L_n$ is the diffusion length of the electrons in the p-type region.

$$L_p = \sqrt{D_p \tau_p} = \sqrt{\frac{kT}{e}\mu_p \tau_p} = \sqrt{0.026 \times 450 \times 10\,10^{-6}} = 1.08\,10^{-2}\,cm$$

[2]

$$L_n = \sqrt{D_n \tau_n} = \sqrt{\frac{kT}{e}\mu_n \tau_n} = \sqrt{0.026 \times 700 \times 0.1\,10^{-6}} = 1.35\,10^{-3}\,cm$$

ii) Minority carrier concentration: [6]

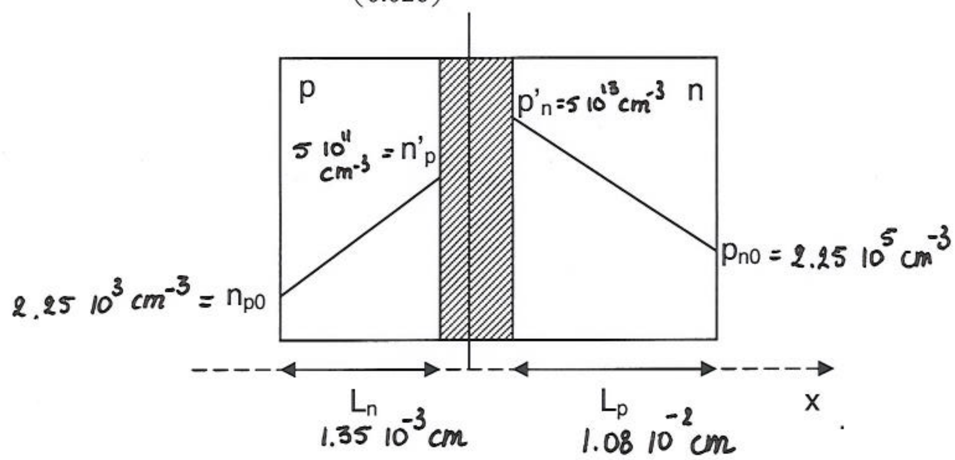
$$n_{p_0} = \frac{n_i^2}{N_A} = \frac{(1.45\,10^{10})^2}{10^{17}} = 2.25 \times 10^3\,cm^{-3}$$

$$p_{n_0} = \frac{n_i^2}{N_D} = \frac{(1.45\,10^{10})^2}{10^{15}} = 2.25 \times 10^5\,cm^{-3}$$

Injected carrier concentration:

$$p'_n = p_{n0} \exp\left(\frac{0.5}{0.026}\right) = 2.25 \times 10^5 \times 2.25 \times 10^8 = 5 \times 10^{13}\,cm^{-3}$$

$$n'_p = n_{p0} \exp\left(\frac{0.5}{0.026}\right) = 2.25 \times 10^3 \times 2.25 \times 10^8 = 5 \times 10^{11}\,cm^{-3}$$



Values for minority carrier density see calculation results above. Values of thickness of n and p layer see table and results in i)

iii) [4]

$$I_{tot} = \left(\frac{eD_p p_n}{L_p} + \frac{eD_n n_p}{L_n}\right)\left(e^{\frac{eV}{kT}} - 1\right)A$$

$$I_{tot} = 1.6\,10^{-19} \times 0.0001\left(\frac{11.66}{0.0108} \times 2.25\,10^{5} + \frac{18.13}{0.00135} \times 2.25\,10^{3}\right)\left(\exp\left(\frac{0.5}{0.026}\right) - 1\right)$$

$$I_{tot} = 4.37\,10^{-15} \times 2.25\,10^{8}\ A = 0.98\,10^{-6}\,A$$

if chosen to approximate (ignoring the electron current as $N_A >> N_D$):

$$I_{tot} = \left(\frac{eD_p p_n}{L_p}\right)\left(e^{\frac{eV}{kT}} - 1\right)A$$

$$I_{tot} = 1.6\,10^{-19} \times 0.0001\left(\frac{11.66}{0.0108} \times 2.25\,10^{5}\right)\left(\exp\left(\frac{0.5}{0.026}\right) - 1\right)$$

$$I_{tot} = 3.89\,10^{-15} \times 2.25\,10^{8}\ A = 0.87\ 10^{-6}\,A$$

(4) a)

$$\oint \underline{H} \cdot d\underline{l} = NI = H_i l_i + H_g g$$

$$H = \frac{B}{\mu} \quad \therefore \quad H_i = \frac{B_i}{\mu_0 \mu_r} \qquad H_g = \frac{B_g}{\mu_0}$$

But continuity of flux means $B_i = B_g = B$

$$\frac{B}{\mu_0 \mu_r} l_i + \frac{B g}{\mu_0} = NI$$

$$B\left(\frac{0.4}{8000} + 0.5\times10^{-3}\right) = 4\pi\times10^{-7} \times 200 \times 0.5 = 1.26\times10^{-4}$$

i) $B_g = B = 0.228\ T$

ii) $B_i = B = 0.228\ T$

iii) $H_g = B/\mu_0 = 1.81\times10^{5}\ A/m$

iv) $H_i = B/(\mu_0\mu_r) = 22.7\ A/m$

b) Now $B = \mu H$ no longer holds in iron,
but $B_g = \mu_0 H_g$ still holds, & $B_i = B_g = B$

$$H_i l_i + \frac{B g}{\mu_0} = NI$$

At the inflection point $H_i = 100\ A/m$, $B = 1T$

$$100(0.4) + \frac{1(0.5\times10^{-3})}{4\pi\times10^{-7}} = 200 I$$

i) $I = \underline{\underline{2.19\ A}}$

ii) For $I > 2.19A$, $B = 1\ T$

$$H_i(0.4) + \frac{1(0.5\times10^{-3})}{4\pi\times10^{-7}} = 200(2.5)$$

$$H_i = \underline{\underline{255\ A/m}}$$

(5)

a) $\oint \underline{D} \cdot d\underline{s} = Q$

For cable of length $L$, $Q = \rho L$

By symmetry $D$ is a function only of $r$, and $\underline{D} \parallel d\underline{s}$ on surface of a cylinder (but $\underline{D} \perp d\underline{s}$ on ends of cylinder)
so taking a cylinder of radius $a < r < b$:

$$D(2\pi r L) = \rho L \quad \therefore D(r) = \frac{\rho}{2\pi r}$$

and $$E = \frac{D}{\varepsilon} = \frac{\rho}{2\pi \varepsilon_0 \varepsilon_r r}$$

b) $$\Delta V = -\int_a^b E(r)\,dr = \frac{\rho}{2\pi\varepsilon_0\varepsilon_r} \ln\left(\frac{b}{a}\right)$$

$$\rho = \frac{2\pi \varepsilon_0 \varepsilon_r V}{\ln(b/a)} = \frac{2\pi(8.85\times10^{-12})(6)(10)}{\ln(2)}$$

$$= \underline{\underline{4.81\times10^{-9}\ C/m}}$$

$$E_{max} = E(r=a) = \frac{\rho}{2\pi\varepsilon_0\varepsilon_r a} = \underline{\underline{3607\ V/m}}$$

Location of max field: surface of inner conductor.

c) Since outer conductor is earthed and inner conductor is at $V$,

$$V(r) = 10 - \int_a^r E(r)\,dr = 10 - \frac{\rho}{2\pi\varepsilon_0\varepsilon_r}\ln\left(\frac{r}{a}\right)$$

For $r = 6$, $V = \underline{\underline{4.15\ V}}$

d) Now $$E(r) = \frac{D(r)}{\varepsilon_0\varepsilon_r(r)} = E_0$$

$$E_0 = \frac{\rho}{2\pi\varepsilon_0\varepsilon_r(r) r} \quad \therefore \varepsilon_r(r) = \frac{\rho}{E_0 2\pi\varepsilon_0 r}$$

since in this case $V = E_0(b-a)$, $\varepsilon_r(r) = \frac{\rho(b-a)}{V 2\pi\varepsilon_0 r}$

Capacitance per unit length $\frac{C}{L} = \frac{Q}{LV} = \frac{\rho}{V}$

$$\therefore \varepsilon_r(r) = \frac{(C/L)(b-a)}{2\pi\varepsilon_0 r} = \frac{200\times10^{-12}\times4\times10^{-3}}{2\pi\times8.85\times10^{-12} r} = \frac{14.4\times10^{-3}}{r}$$

$$\boxed{\varepsilon_r(r) = 14.4\text{mm}/r}$$