Paper Number(s): **E1.3**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2002

EEE PART I: M.Eng., B.Eng. and ACGI

Modified formula sheet
10.30.

DEVICES AND FIELDS

Monday, 10 June 10:00 am

There are FIVE questions on this paper.

There are two sections. Answer THREE questions including at least ONE question from each section.

Use a separate answer book for each section.

Corrected Copy

Time allowed: 2:00 hours

Examiners responsible:

First Marker(s): Fobelets,K. Popovic,D.

Second Marker(s): Leaver,K.D. Pal,B.C.

SECTION A

Answer at least ONE question. Use a separate answer book for each section.

Formulae and Constants

$$I_{DS} = \frac{WC_{ox}\mu}{L}\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$g_m = \frac{dI_{DS}}{dV_{GS}}$$

$$f_c = \frac{g_m}{2\pi C_{GS}}$$

$$J = \left(\frac{qD_h p_{n_0}}{L_n} + \frac{qD_e n_{p_0}}{L_p}\right)\left(\exp\left(\frac{qV}{kT}\right) - 1\right)$$

$$V_0 = \frac{kT}{q}\ln\frac{N_A N_D}{n_i^2}$$

$$W_n = \sqrt{\frac{2\varepsilon_0\varepsilon_r}{q}(V_0 - V)\frac{N_A}{N_A N_D + N_D^2}}$$

$$W_p = \sqrt{\frac{2\varepsilon_0\varepsilon_r}{q}(V_0 - V)\frac{N_D}{N_A N_D + N_A^2}}$$

$$n = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$p = N_V \exp\left[\frac{(E_V - E_F)}{kT}\right]$$

$$\sigma = q[carrier \quad concentration]\mu$$

$$D = \frac{\mu kT}{q}$$

q	=	$1.602\ 10^{-19}$ C
n_i	=	$1.45\ 10^{10}$ cm^{-3} for Si at 300K
kT/q	=	0.026 V at 300 K
ε_0	=	$8.854\ 10^{-12}$ F/m
N_{CSi}	=	$2.8\ 10^{19}$ cm^{-3}
N_{VSi}	=	$1.04\ 10^{19}$ cm^{-3}

Relative permittivity of SiO_2 = 4

Relative permittivity of Si = 11.7

$\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}

$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}

written on board at 10.30

1 MOSFET

Questions (a) – (e) concern an n-channel enhancement mode MOSFET.

a) What is the majority carrier type in this device? [1]

b) Give a definition for the threshold voltage. [2]

c) Is the threshold voltage of this device positive or negative? [1]

d) Draw the energy band diagram, including E_c,E_v,E_F and E_g of the n-channel enhancement mode MOSFET from gate to substrate (body) when the channel is conducting (inversion). The position of the Fermi level relative to a reference level before the materials are in contact is given in Figure 1. [6]

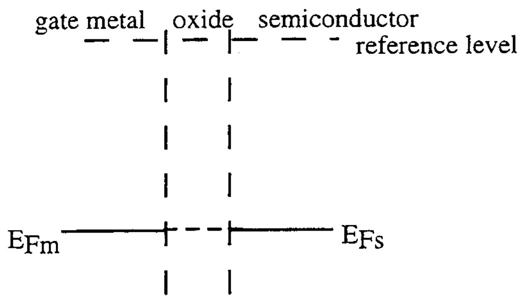


Figure 1: The position of the Fermi level in metal, oxide and semiconductor before contact.

e).Reducing the size of MOS transistors is a way to improve their transconductance g_m.
Derive an expression for the ratio of the transconductances of the enhancement mode MOSFET in *saturation* when both the gate length and the oxide thickness are changed from: $L_1$to L_2 and t_{ox1} to t_{ox2} [10]
If both are halved, what will this ratio be?

2 Junctions

a) Metal-semiconductor contact

Given the energy band diagram of a metal and semiconductor before contact (see Figure 2), sketch the energy band diagram, including E_c,E_v,E_F and E_g of the metal-semiconductor junction in contact. [4]

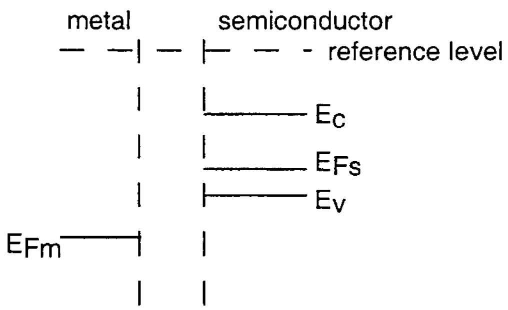


Figure 2: Energy band diagrams of a metal and semiconductor before contact

b) pn junction

Given the energy band diagram of two heavily doped semiconductors before contact (see Figure 3), sketch the energy band diagram of the same semiconductors in contact. Note that although the Fermi levels are lying outside the energy gap all the usual rules are followed. [4]

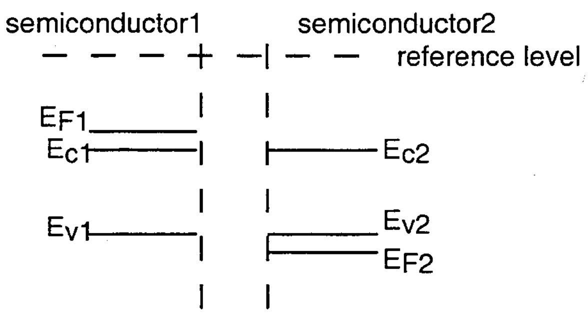


Figure 3: Energy band diagrams of two heavily doped semiconductor before contact

c) Is the following statement correct or wrong?

The hole diffusion current of a pn junction under zero bias is zero.
Give a brief reason for your answer. [4]

d) Calculate the junction capacitance of the following Si pn diode under 1V reverse bias. The diode area is 10 μm x10 μm, the donor concentration is 10^{16} cm^{-3}, the acceptor concentration is 10^{19} cm^{-3}. The temperature T=300K. Justify any approximations you make. [8]

3 Bipolar Junction Transistor

a) State whether the main current flowing from emitter to collector through the base region of a BJT is governed by drift or diffusion. Give a brief reason for your answer. [3]

b) The base current, I_B, of a BJT consists of three components such that $I_B = I_B' + I_B'' + I_{CB0}$ [3]
Give a short description of the three physical processes which cause these three current components. Remember the biasing configuration of the active device given in Figure 4.

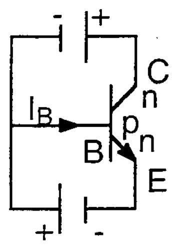


Figure 4: biasing configuration of npn transistor

c) Give the expression for the small signal current gain β of the BJT in terms of the external voltages and currents. [4]
Give two ways in which to improve the current gain.

d) Calculate the current density injected from the emitter into the base of the following Si npn transistor with emitter doping of 10^{19} cm^{-3}, base doping of 10^{16} cm^{-3} and collector doping 10^{16} cm^{-3}. Bias voltages are V_{EB} = 0.7V forward bias and V_{BC} is 3V reverse bias, emitter and collector length : L_n=100 μm, and effective base width L_p= 50 μm. Mobility of emitter electrons is 100 cm^2/Vs, of holes is 200 cm^2/Vs and of collector electrons is 300 cm^2/Vs. The device is operating at room temperature T=300K. [10]

SECTION B

Answer at least ONE question. Use a separate answer book for each section

4. (a) Explain Gauss' Law. Under what circumstances is it useful?

[5marks]

(b) Consider the three closed surfaces, S_1, S_2 and S_3, in the field of four point charges as shown in Figure 1. What is the flux of the electric field intensity vector through each of these surfaces? Assume that all four point charges are situated in a vacuum.

[5marks]

(c) What is the absolute potential at a point P which is 2m from a point charge $Q = +5\mu C$? What is the work required to move a +5nC charge from infinity to point P? Assume a relative permittivity $\epsilon_r = 1$.

[10marks]

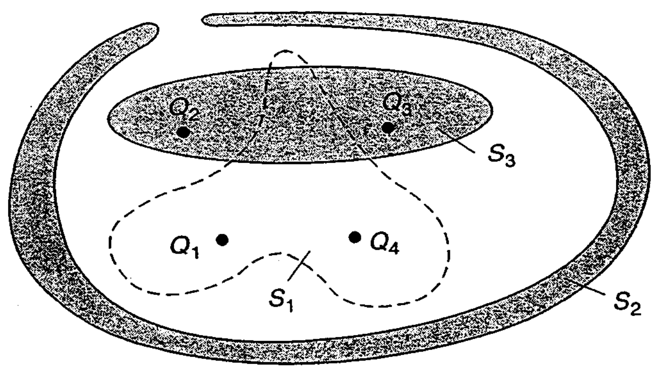


Figure 1

5. (a) What is a fundamental difference between electric field lines and magnetic field lines?

[5marks]

(b) Current I flows in the inner conductor of a long coaxial cable and returns through the outer conductor. What is the magnetic field in the region outside the coaxial cable and why?

[5marks]

(c) The rectangular loop shown in Figure 2 is situated in the $x-y$ plane and moves away from the origin at a velocity $\boldsymbol{u} = \hat{\boldsymbol{y}}\, 5$ (m/s) in a magnetic field given by

$$\boldsymbol{B}(y) = \hat{\boldsymbol{z}}\, 0.2e^{-0.1y} \quad \text{(T)}$$

If $R = 5\Omega$, find the current I at the instant that the loop sides are at $y_1 = 2$m and $y_2 = 2.5$m. The loop resistance may be ignored.

[10marks]

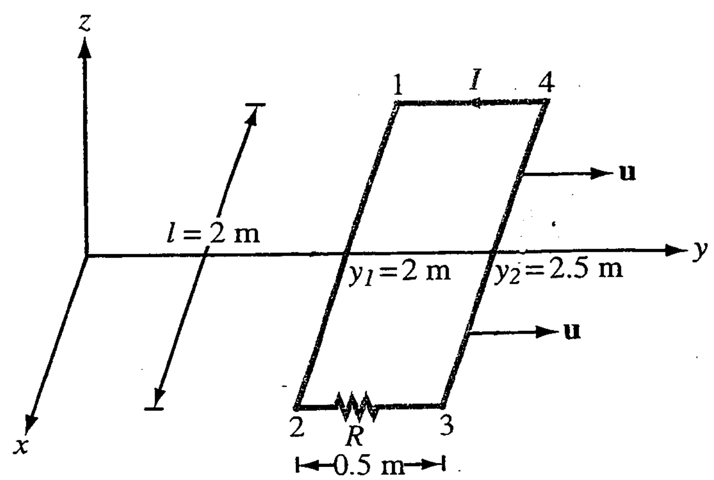


Figure 2

1 MOSFET

a) electrons [1]

b) either of the following answers is acceptable: [2]

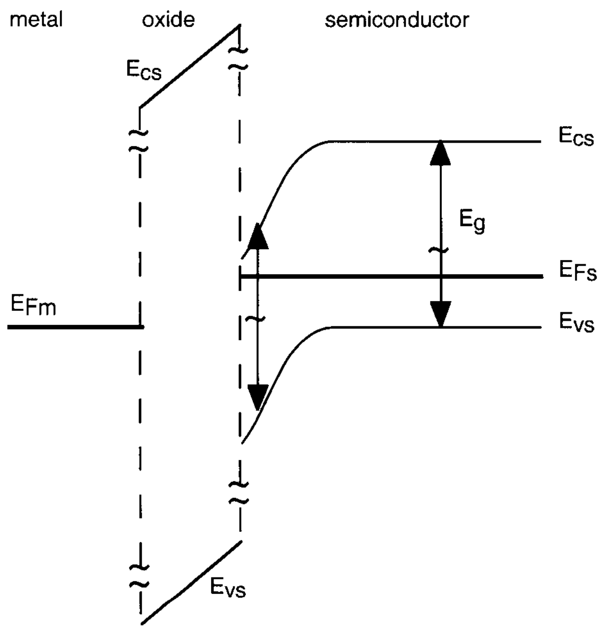
i) the threshold voltage is the *gate – source voltage* at which the *channel* is *created* in an enhancement mode MOSFET or *depleted* in a depletion mode MOSFET

ii) is the *gate – source voltage* at which the *inverted carrier concentration* at the SiO2-Si interface is the same as the majority carrier concentration in the bulk far away from the junction where no band bending occurs

ii) is the *gate – source voltage* at which the MOS capacitance is *minimum*

c) positive [1]

d)



[6]

e)

$$I_{DS} = \frac{WC_{ox}\mu}{L}\left[(V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

in saturation:

$$V_{DS} = (V_{GS}-V_T)$$

$$I_{DS}^{sat} = \frac{WC_{ox}\mu}{2L}(V_{GS}-V_T)^2$$

transconductance:

$$g_m^{sat} = \frac{dI_{DS}^{sat}}{dV_{GS}}$$

$$g_m^{sat} = \frac{WC_{ox}\mu}{L}(V_{GS}-V_T)$$

oxide capacitance:

$$C_{ox} = \frac{\varepsilon_o\varepsilon_{ox}}{t_{ox}}$$

$$g_{m1}^{sat} = \frac{W\varepsilon_o\varepsilon_{ox}\mu}{t_{ox1}L_1}(V_{GS}-V_T) \;\&\; g_{m2}^{sat} = \frac{W\varepsilon_o\varepsilon_{ox}\mu}{t_{ox2}L_2}(V_{GS}-V_T)$$

transconductance ratio:

$$g_{m2}^{sat}\big/g_{m1}^{sat} = \frac{t_{ox1}L_1}{t_{ox2}L_2}$$

Transconductance improvement when:

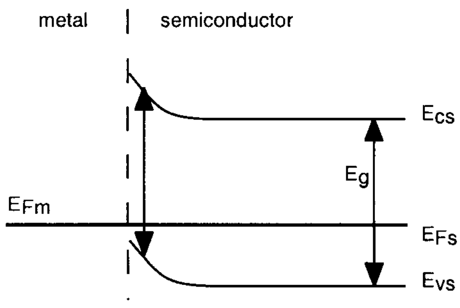
$$t_{ox2} = t_{ox1}/2 \;\&\; L_2 = L_1/2$$

$$g_{m2}^{sat}\big/g_{m1}^{sat} = 4$$

[10]

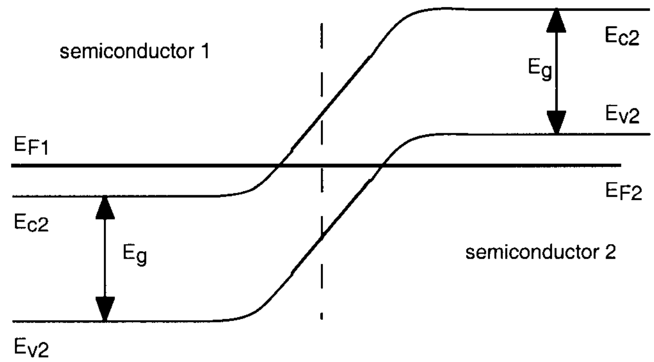
2 Junctions

a)metal-semiconductor contact



[4]

b) pn junction



[4]

c) Correct or wrong

wrong [1]

At zero bias there exists a drift current due to the built-in voltage and therefore a diffusion component has to exist to cancel the drift current and give zero net hole current. [3]

d) pn diode capacitance in reverse bias is due to the depletion width W.

The depletion width extends in the lowest doped region. Since the doping density in the n-type region is 1000 times smaller than in the p-type region, we can neglect the contribution of the depletion width in the highest doped region.

Depletion width extends into the n – doped region :

$$W \approx W_n = \sqrt{\frac{2\varepsilon_0\varepsilon_r}{q}(V_0 - V)\frac{N_A}{N_D(N_A + N_D)}}$$

$$N_A >>> N_D$$

$$W \approx W_n \approx \sqrt{\frac{2\varepsilon_0\varepsilon_r}{q}(V_0 - V)\frac{1}{N_D}}$$

$$\text{built-in voltage}: \; V_0 = \frac{kT}{q}\ln\frac{N_A N_D}{n_i^2}$$

$$V_0 = 0.026V\ln\frac{10^{16}10^{19}cm^{-6}}{2.1025\,10^{20}cm^{-6}} = 0.879V$$

$$W \approx \sqrt{\frac{2\,8.854\,10^{-14}\,F/cm\,11.7}{1.602\,10^{-19}C}(0.879V + 1V)\frac{1}{10^{16}cm^{-3}}}$$

$$W \approx 49.297\,10^{-6}cm$$

$$\text{Depletion capacitance}: C = \frac{\varepsilon_0\varepsilon_r}{W}A$$

$$A = 10\mu m \text{ x } 10\mu m = 10\,10^{-4}cm \text{ x } 10\,10^{-4}cm = 10^{-6}cm^2$$

$$C \approx \frac{8.854\,10^{-14}\,F/cm\,11.7}{49.297\,10^{-6}cm}10^{-6}cm^2$$

$$C \approx 2.1\,10^{-14}F$$

Without approximation :

$$W_n = \sqrt{\frac{2\varepsilon_0\varepsilon_r(V_0 - V)}{qN_D}\frac{(N_A)}{(N_A + N_D)}}$$

$$W_n = \sqrt{\frac{2\,8.854\,10^{-14}\,F/cm\,11.7(0.879V + 1V)}{1.602\,10^{-19}C}\frac{10^{19}cm^{-3}}{(10^{16})(10^{19} + 10^{16})cm^{-6}}}$$

$$W_n = 49.27\,10^{-6}\,cm$$

$$W_p = \sqrt{\frac{2\varepsilon_0\varepsilon_r(V_0 - V)}{qN_A}\frac{(N_D)}{(N_A + N_D)}}$$

$$W_p = \sqrt{\frac{2\,8.854\,10^{-14}\,F/cm\,11.7(0.879V + 1V)}{1.602\,10^{-19}C}\frac{10^{16}cm^{-3}}{(10^{19})(10^{19} + 10^{16})cm^{-6}}}$$

$$W_p = 0.04927\,10^{-6}\,cm$$

$$W = W_n + W_p = 49.32\,10^{-6}cm$$

$$C = \frac{8.854\,10^{-14}\,F/cm\,11.7}{49.32\,10^{-6}cm}10^{-6}cm^2$$

$$C = 2.1\,10^{-14}F$$

[10]

3 Bipolar Junction Transistor

a) diffusion
In the base of the transistor a carrier gradient is generated due to the different signs of the external bias voltages across both base junctions (BE and BC). This concentration gradient in the base results in a diffusion current. [3]

b) $I_B = I_B' + I_B'' + I_{CB0}$
I_B' is the base current injected into the emitter
I_B'' is the base recombination current
I_{CB0} is the reverse bias leakage current into collector [3]

c) The current $\beta = \frac{dI_C}{dI_B}$ gain:

For high gain: small base width and large emitter doping compared to the base doping [4]

d) Current injected into the base is given by the pn diode current for a n^+p (E-B) junction (therefore the electron current is much larger than the hole current which can be neglected). [10]
Current density:

$$J = \left(\frac{qD_h p_{n0}}{L_n} + \frac{qD_e n_{p0}}{L_p} \right) \left(\exp\left(\frac{qV}{kT} \right) - 1 \right)$$

$$J = kT \left(\frac{\mu_h p_{n0}}{L_n} + \frac{\mu_e n_{p0}}{L_p} \right) \left(\exp\left(\frac{qV}{kT} \right) - 1 \right)$$

minority carrier concentration:

$$p_p = N_A \rightarrow n_p = \frac{n_i^2}{p_p} = \frac{n_i^2}{N_A} = \frac{\left(1.45\,10^{10}\ cm^{-3}\right)^2}{10^{16}\ cm^{-3}} = 21025\ cm^{-3}$$

$$n_n = N_D \rightarrow p_n = \frac{n_i^2}{n_n} = \frac{n_i^2}{N_D} = \frac{\left(1.45\,10^{10}\ cm^{-3}\right)^2}{10^{19}\ cm^{-3}} = 21.025\ cm^{-3}$$

$$J = 0.026V\, 1.602\, 10^{-19} C \left(\frac{200\, cm^2/Vs\ 21.025\ cm^{-3}}{100\, 10^{-4} cm} + \frac{100\, cm^2/Vs\ 21025\ cm^{-3}}{50\, 10^{-4} cm} \right) \left(\exp\left(\frac{0.7V}{0.026V} \right) - 1 \right)$$

$$J = 0.864\ C/cm^2 s = 0.864\ A\ cm^{-2}$$

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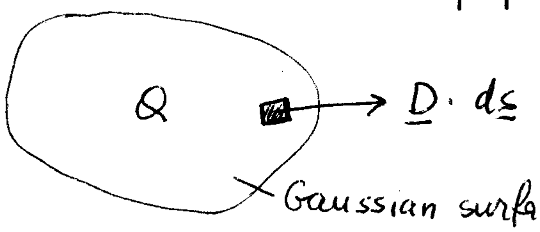
Model Answers and Mark Schemes First Examiner: Popovic

Paper Code: E1.3 - sec II Second Examiner: Pal

Question Number etc. in left margin Mark allocation in right margin

4. a) $\oint_S \underline{D} \cdot d\underline{s} = Q$ The outward flux of $\underline{D}$ through a surface is proportional to the enclosed charge Q.



It is a convenient method for determining $\underline{D}$ and/or $\underline{E}$ when the charge distribution possesses symmetry properties. 5

(Because at every point on the surface the direction of $d\underline{s}$ is the outward normal to the surface, only the normal component of $\underline{D}$ at the surface contributes to the integral above. $\Rightarrow$ the surface S should be chosen such that, from symmetry considerations, the magnitude of $\underline{D}$ is constant and its direction is normal or tangential at every point.)

b) flux through $S_1 = \dfrac{Q_1 + Q_4}{\varepsilon_0}$

flux through $S_2 = 0$ 5

flux through $S_3 = \dfrac{Q_2 + Q_3}{\varepsilon_0}$.

c) $V = -\int_{\infty}^{r} \underline{E} \cdot d\underline{r} = -\dfrac{Q}{4\pi\varepsilon} \int_{\infty}^{r} \dfrac{dr}{r^2} = \dfrac{Q}{4\pi\varepsilon r}$ absolute potential 2

$V = \dfrac{5 \cdot 10^{-6}}{4\pi \cdot 8.85 \cdot 10^{-12} \cdot 2} = 22.5 \text{ kV} \left(= 22.5 \dfrac{kJ}{C}\right)$ 4

$W = (22.5 \cdot 10^3) \cdot (5 \cdot 10^{-9}) = 112.5 \cdot 10^{-6} \text{ J}$ 4

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5. a) Electric field lines originate from positive electric charges and terminate on negative electric charges. Hence, the electric flux through a closed surface surrounding one of the charges is not zero. 5

In contrast, magnetic field lines always form continuous closed loops. ⇒ the net magnetic flux through the closed surface ~~...~~ is always zero. $\oint_S \underline{B}\cdot d\underline{s} = 0$

b) $\underline{H}=0$ outside the cable because the net current enclosed by the Ampèrian contour is zero. 5

c) $\underline{u}\times\underline{B}$ is along $\hat{\underline{x}}$ ⇒ voltages are induced across only the sides oriented along $\hat{\underline{x}}$ (ie sides between points 1 and 2 and the side between 3 and 4)

$$\underline{B}(y_1) = \hat{\underline{z}}\; 0.2\, e^{-0.1 y_1} = \hat{\underline{z}}\; 0.2\, e^{-0.2} \;(T)$$

$y_1 = 2m$

$$V_{12} = \int_2^1 [\underline{u}\times\underline{B}(y_1)]\cdot d\underline{l} = \int_{l/2}^{-l/2} (\hat{\underline{y}}\,5\times\hat{\underline{z}}\;0.2\,e^{-0.2})\cdot\hat{\underline{x}}\,dx$$ 4

$$= -e^{-0.2}\, l = -2\,e^{-0.2} = -1.637 \text{ volts}$$

Similarly:

$$V_{43} = -u B(y_2)\, l = -5\cdot 0.2\, e^{-0.25}\cdot 2 = -1.558 \text{ volts}$$ 4

$$I = \frac{V_{43}-V_{12}}{R} = \frac{0.079}{5} = 15.8 \text{ mA}$$ 2

20