IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2005

EEE/ISE PART I: MEng, BEng and ACGI

Corrected Copy

ANALYSIS OF CIRCUITS

Monday, 6 June 10:00 am

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

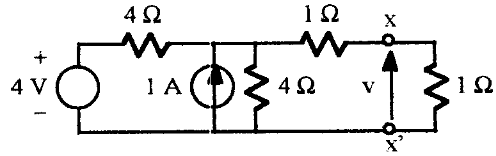
Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D.G. Haigh, D.G. Haigh

Second Marker(s) : G. Weiss,

a) Use source transformations to derive a simplified equivalent circuit for the part of the circuit in Figure 1(a) to the left of the terminals x, x':

Figure 1(a)

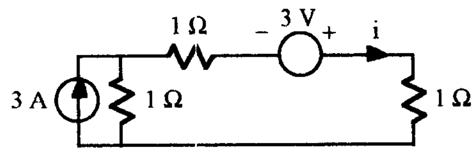


Hence determine the voltage v.

[4]

b) Use the principle of superposition to find the current i in the circuit in Figure 1(b):

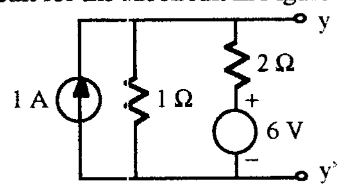
Figure 1(b)



[4]

c) Find the Thevenin equivalent circuit for the subcircuit in Figure 1(c).

Figure 1(c)

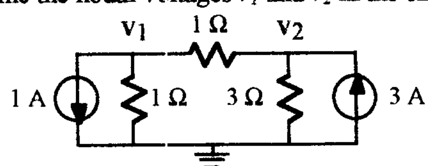


Hence, determine what current flows in a 2 Ω resistor connected between the terminals y, y'.

[4]

d) State Kirchhoff's current law.
Use nodal analysis to determine the nodal voltages v_1 and v_2 in the circuit of Figure 1(d):

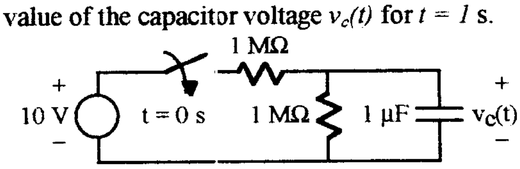
Figure 1(d)



[4]

e) In the circuit of Figure 1(e), the switch remains in the open state a long time before closing at time $t = 0$ s. Find the value of the capacitor voltage $v_c(t)$ for $t = 1$ s.

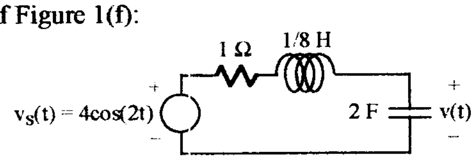
Figure 1(e)



[4]

f) Write the voltage $v(t) = 4\cos(2t)$ in phasor form. Write the impedances at a frequency of 2 rad/s of a 1/8 H inductor and a 2 F capacitor. Hence use the phasor method to determine the voltage $v(t)$ in the circuit of Figure 1(f):

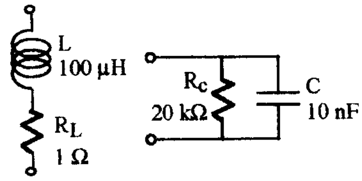
Figure 1(f)



[4]

g) The diagram on the left in Figure 1(g) shows an inductor with equivalent series resistance. Determine the Q-factor of the inductor at a frequency of 10^6 rad/s. Convert the resistance in series with the inductor to a resistance in parallel with the inductor which is approximately equivalent to it at that frequency. The inductor is now connected in parallel with a capacitor to form a tuned circuit; a model for the capacitor is shown on the right in figure 1(g). Calculate the resonant frequency of the tuned-circuit and the Q-factor at the resonant frequency.

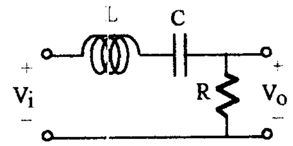
Figure 1(g)



[4]

h) For the filter circuit shown in Figure 1(h), determine the frequency response function $H(j\omega) = V_o/V_i$. By considering the behaviour of $H(j\omega)$ at the resonant frequency, at zero frequency and for frequency $\omega \to \infty$, show that the filter is a band-pass filter.

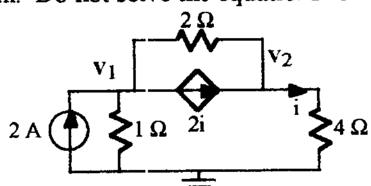
Figure 1(h)



[4]

i) The circuit shown in Figure 1(i) contains a current-controlled current source. By taping the source, perform by-inspection nodal analysis of the circuit. Un-tape the source and write the nodal equations in standard form. Do not solve the equations for v_1 and v_2.

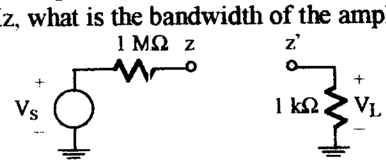
Figure 1(i)



[4]

j) It is desired to connect the voltage source with a 1 MΩ internal resistance in Figure 1(j) to the load resistor shown in such a way that $V_L = (+10) \times V_s$. Suggest a suitable circuit to connect between nodes z and z' using an op-amp and two resistors. If the gain-bandwidth product of the op-amp is 1 MHz, what is the bandwidth of the amplifier system?

Figure 1(j)



[4]

2

(a) Figure 2 shows a circuit which has 4 nodes and for which the reference node has not yet been chosen:

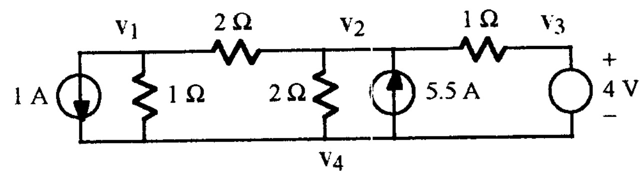


Figure 2

(i) Classify the nodes as either ordinary nodes or generalised nodes.
(ii) Given that node 4 is chosen to be the reference node, classify the remaining nodes as either essential, non-essential or super nodes.
(iii) Identify suitable unknown voltages for nodal analysis and write the Kirchhoff's Current Law equations.
(iv) Organise the equations by grouping like terms.
(v) Solve the equations for the unknown nodal voltages.

[18]

(b) Nodal analysis of a circuit containing resistors and independent current sources leads to the following set of nodal equations, which are un-scaled:

$$\begin{aligned} 5v_1 - 2v_2 + 0v_3 &= 1 \\ -2v_1 + 5v_2 - 2v_3 &= -2 \\ 0v_1 - 2v_2 + 5v_3 &= 3 \end{aligned}$$

From these equations, derive the structure of the original circuit and all of its element values.
Check your synthesised circuit by circuit analysis.

[8]

(c) Explain briefly the role which DC circuit analysis might play in the design of an analogue circuit, such as a transistor amplifier.

[4]

(a) Analysis of a filter circuit can lead to a frequency response function $V_{out}/V_{in} = H(j\omega)$. Write expressions for the filter amplitude response function $A(\omega)$ and the phase response function $\phi(\omega)$ in terms of $H(j\omega)$.

[2]

(b) Figure 3 shows a filter circuit which has one input terminal and two output terminals.

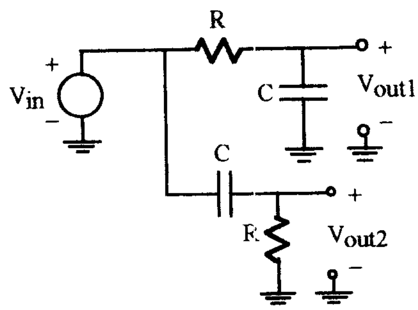


Figure 3

Use the voltage divider rule to determine the two separate frequency response functions $H_1(j\omega)$ and $H_2(j\omega)$, where:

$$H_1(j\omega) = \frac{V_{out1}}{V_{in}} \quad \text{and} \quad H_2(j\omega) = \frac{V_{out2}}{V_{in}}$$

State the types of filter, eg bandpass, that $H_1(j\omega)$ and $H_2(j\omega)$ describe.

[10]

(c) Determine the frequency response function formed by taking the difference of the output voltages of the circuit in Figure 3:

$$H(j\omega) = \frac{V_{out1} - V_{out2}}{V_{in}}$$

Determine the amplitude and phase response functions for $H(j\omega)$.
State what type of filter $H(j\omega)$ describes.

[8]

(d) Draw a circuit using op-amps and resistors which takes V_{out1} and V_{out2} in Figure 3 as input signals and produces a single output signal given by:

$$V_{out} = V_{out1} - V_{out2}$$

taking into account the no-load conditions i = 0 at the output terminals in Figure 3.

[6]

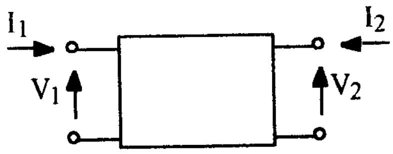
(e) Explain briefly using a sketch the key role played by a filter in a radio receiver.

[4]

4

(a) Give two reasons why we use 2-port descriptions for circuits and for some elements. [4]

(b) The 2-port circuit in Figure 4(b) can be described by the given general 2-port equations containing admittance parameters.



$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

Figure 4(b)

The admittance parameters y_{11}, y_{12}, y_{21} and y_{22} may be determined for a circuit by applying a short-circuit to one port with a known voltage source at the other port and measuring a current. For example, $y_{11} = I_1/V_1$ with $V_2 = 0$; hence we measure I_1 for known V_1 with short-circuit at port 2 and form I_1/V_1.
Now derive the remaining admittance parameters in this way. [5]

(c) Consider the circuit in Figure 4(c).

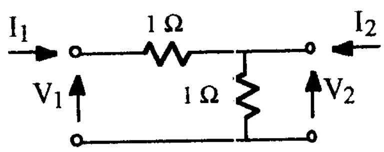


Figure 4(c)

Illustrate the determination of all four admittance parameters using the method in part (b) by applying it to this circuit. Make sure that the signs of the currents are correct. [8]

(d) Use the general 2-port equations given above in Figure 4(b) to show that the voltage gain (V_2/V_1) for a 2-port circuit when it is terminated at port 2 in an open-circuit ($I_2 = 0$) is given by:

$$\left.\frac{V_2}{V_1}\right|_{I_2=0} = -\frac{y_{21}}{y_{22}}$$

[3]

(e) A simple amplifier consisting of a FET and a resistor is shown in Figure 4(e). Figure 4(e) also shows a small-signal model for the amplifier circuit, where g_m and r_0 are the transconductance and output resistance of the small-signal FET model.

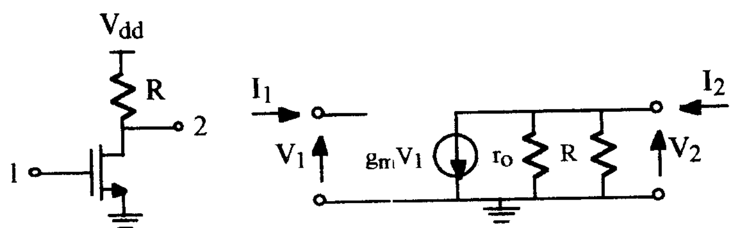


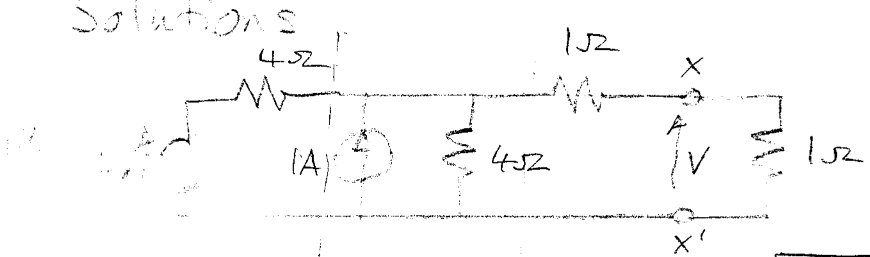
Figure 4(e)

Use the method of part(b) to determine the 2-port parameters y_{21} and y_{22} for the small-signal amplifier circuit.
Hence, derive an expression for the open-circuit voltage gain of the amplifier circuit. [10]

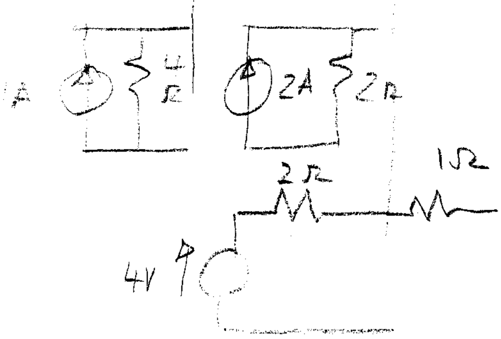
Analysis of Circuits 2005

Solutions

1(a)

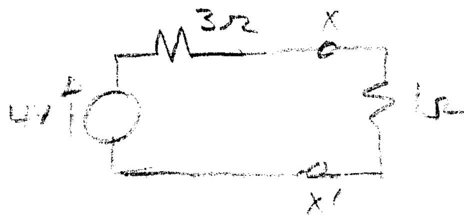


[A]



Key

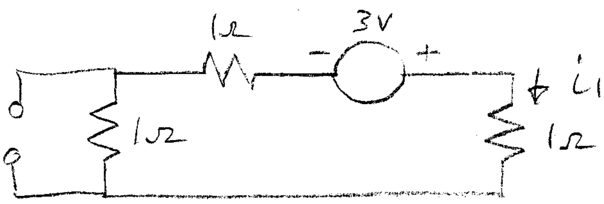
- [A] Application
- [B] Bookwork
- [T] Theory



$V = 1V$.

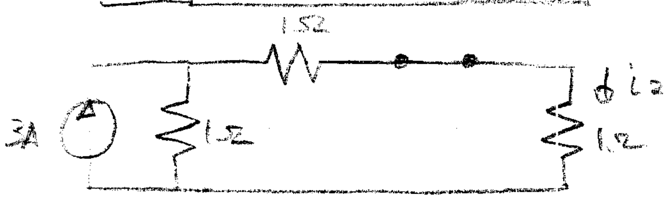
(4)

1(b)



$i_1 = \frac{3V}{3\Omega} = 1A$

[A]

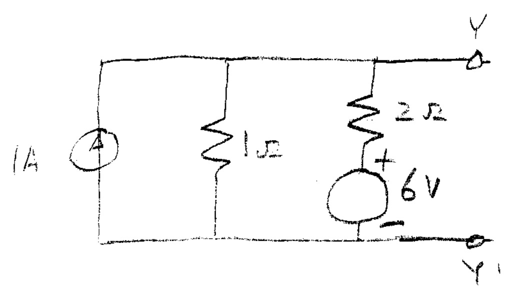


$i_2 = 3A \times \frac{1}{1+2} = 1A$

(4)

$i = i_1 + i_2 = 2A$

1(c)

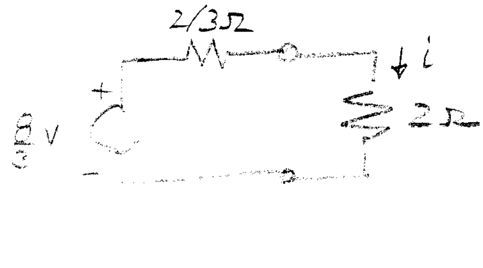


[A]

Using Superposition: $V_{oc} = 1 \times \frac{1 \times 2}{1+2} + 6 \times \frac{1}{1+2}$

$= \frac{2}{3} + 2 = \frac{8}{3} V$

$R_{eq} = \frac{1 \times 2}{1+2} = \frac{2}{3} \Omega$

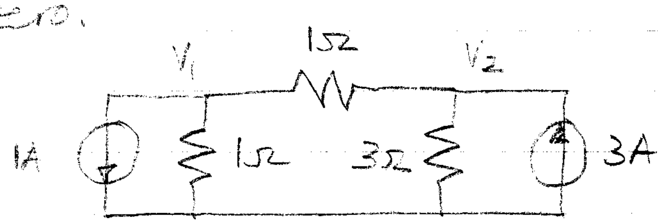


$i = \frac{8/3}{2 + 2/3} = 1A$

(4)

1 (a) The sum of currents entering (or leaving) a node is zero.

A



V_1) $1 \cdot V_1 + 1\,(V_1 - V_2) = -1$

V_2) $1 \cdot (V_2 - V_1) + \frac{1}{3} \cdot V_2 = 3$

$2V_1 - V_2 = -1$

$-V_1 + \frac{4}{3} V_2 = 3$

$-2V_1 + \frac{8}{3} V_2 = 6$

4

$\frac{5}{3} V_2 = 5 \qquad V_2 = 3V, \quad V_1 = \frac{1}{2}(V_2 - 1) = 1V$

1 (b) $x(t) = x_\infty - (x_\infty - x_0)\, e^{-t/\tau}$

A

$V_{c0} = 0V$

$V_{c\infty} = 5V$

$\tau = 0.5 \times 10^6 \times 1 \times 10^{-6} = 0.5\,s$

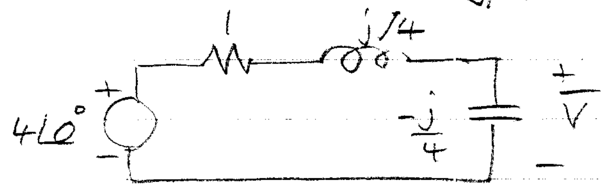
$v_c(t) = 5 - 5e^{-t/\tau} = 5(1 - e^{-t/\tau})$

$= 5 \times 0.865 = 4.325\,V$

4

1 (c) $\bar{V} = 4\angle 0^\circ \qquad Z_L = j/4\,\Omega \qquad Z_C = -j/4\,\Omega$

A

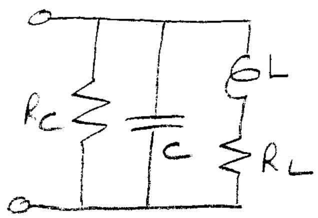


$\bar{V} = \dfrac{-j/4}{1 + j/4 - j/4} \cdot 4\angle 0^\circ = -j/4 \cdot 4\angle 0^\circ = 1\angle -90^\circ$

$v(t) = \cos\left(2t - \frac{\pi}{2}\right)$

4

1 (a)



$L = 10^{-4}\,H \quad C = 10^{-8}\,F$
$R_L = 1\,\Omega \quad R_C = 20\,k\Omega$

A

$$\omega_0 \approx \frac{1}{\sqrt{LC}} = 10^6\ r/s \qquad Q_L = \frac{\omega_0 L}{R_L} = \frac{10^6 \times 10^{-4}}{1} = 100$$

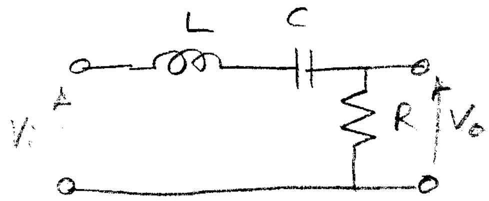
4

Parallel resistance $R_L' \approx Q_L^2 R_L = 10^4\,\Omega$

$$R_{eff} = R_C // R_L' = \frac{10 \times 20}{10+20} = 6.667\ k\Omega$$

$$Q_{eff} = \frac{R_{eff}}{\omega_0 L} = 66.67$$

1 (h)



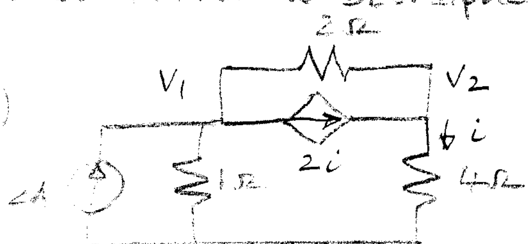
A

$$H(j\omega) = \frac{V_o}{V_i} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{1 + \frac{j}{R}\left(\omega L - \frac{1}{\omega C}\right)}$$

For $\omega = 0$, $H(j\omega) \to 0$; For $\omega \to \infty$, $H(j\omega) \to 0$, For $\omega = 1/\sqrt{LC}$, $H(j\omega) \to 1$
Hence filter is bandpass

4

(i)



A

$$\begin{bmatrix} 1+\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2}+\frac{1}{4} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 - i_c \\ i_c \end{bmatrix} = \begin{bmatrix} 2 - V_2/2 \\ V_2/2 \end{bmatrix}$$

since $i_c = 2i = 2\frac{V_2}{4} = \frac{V_2}{2}$

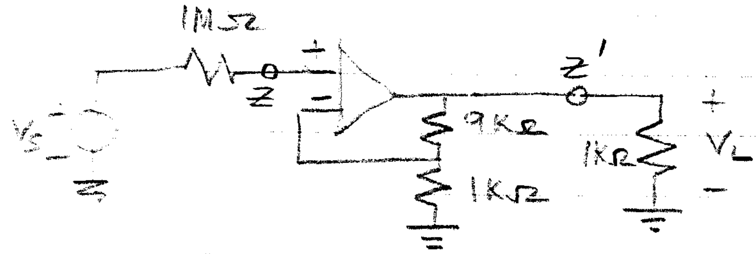
$$\begin{bmatrix} \frac{3}{2} & 0 \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

4

EE
1.1

1 (j)

A



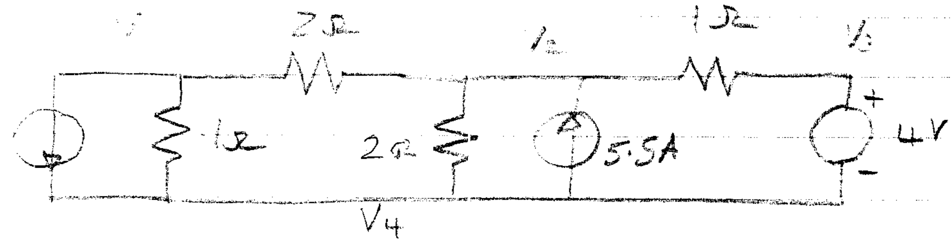
For GBW (op-amp) = 1 MHz and closed-loop gain = 10, bandwidth = 100 kHz.

(4)

40

2 (a)

A



(i) V_1, V_2 ordinary nodes; V_3 and V_4, generalised node

(ii) V_1, V_2 essential nodes; V_3, non-essential node.

(iii) V_1) $1(V_1) + \frac{1}{2}(V_1 - V_2) = -1$

V_2) $\frac{1}{2}(V_2 - V_1) + \frac{1}{2} V_2 + 1(V_2 - 4) = 5.5$

(iv) $\frac{3}{2} V_1 - \frac{1}{2} V_2 = -1$

$-\frac{1}{2} V_1 + 2V_2 = 9.5$

(v) $-\frac{3}{2} V_1 + 6V_2 = 28.5$

18

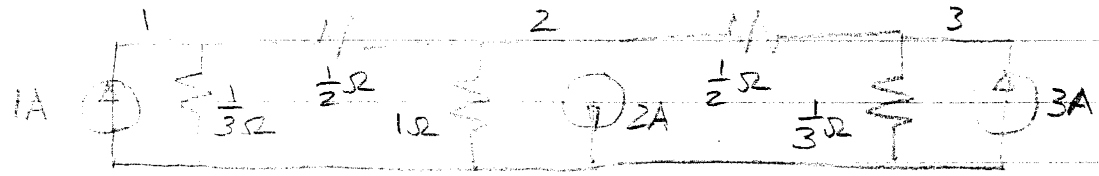
$\frac{11}{2} V_2 = 27.5 \qquad V_2 = 5V$

$V_1 = 2(2V_2 - 9.5) = 1V$

(b)
$$\begin{aligned} 5V_1 - 2V_2 + 0V_3 &= 1 \\ -2V_1 + 5V_2 - 2V_3 &= -2 \\ 0V_1 - 2V_2 + 5V_3 &= 3 \end{aligned}$$

A

$\frac{1}{2}\Omega$ resistors link nodes 1,2 and 2,3. Hence remaining resistors and sources can be determined



8

E

(c) DC analysis is essential to ensure that the bias currents and terminal voltages of the transistors are correct. It can also be used to determine quiescant power supply current and heat dissipation.

4

50

3. a) $A(\omega) = |H(j\omega)|$
$\phi(\omega) = \angle H(j\omega)$

(2) [E]

b) $$H_1(j\omega) = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega CR}$$

[A] $$H_2(j\omega) = \frac{R}{R + 1/j\omega C} = \frac{j\omega CR}{1 + j\omega CR}$$

	$\omega = 0$	$\omega \to \infty$	Type
$H_1(j\omega)$	$A(\omega) \to 1$	$A(\omega) \to 0$	Lowpass
$H_2(j\omega)$	$A(\omega) \to 0$	$A(\omega) \to 1$	Highpass

(10)

(c) $$H(j\omega) = \frac{1}{1 + j\omega CR} - \frac{j\omega CR}{1 + j\omega CR} = \frac{1 - j\omega CR}{1 + j\omega CR}$$

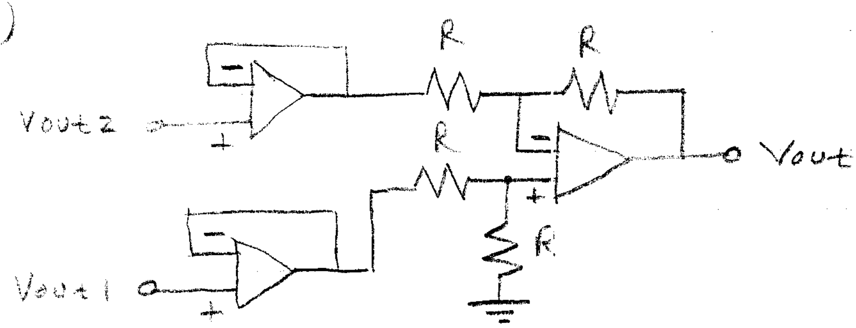
[T]

$$A(\omega) = |H(j\omega)| = \sqrt{\left(\frac{1 + \omega^2 C^2 R^2}{1 + \omega^2 C^2 R^2}\right)} = 1$$

$$\phi(\omega) = \angle H(j\omega) = -2\tan^{-1}(\omega CR)$$

(8) $H(j\omega)$ describes an all-pass filter

(d)



[T]

(6) (Many other equivalent arrangements are possible)

(e) The main filter in a radio is the bandpass filter responsible for channel selectivity.

[E]

(4)

[30]

4 (a) (i) Some elements, such as transistors, have more than 2 terminals, (ii) For a complex circuit like an amplifier, we are often interested in its behaviour between input and output port.

B

(3)

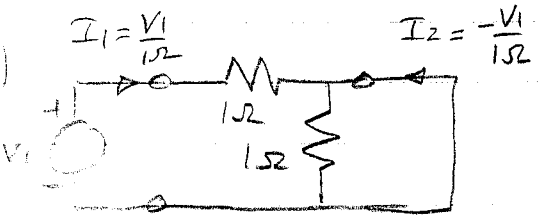
(b) $V_2 = 0$; $y_{11} = \dfrac{I_1}{V_1}$, $y_{21} = \dfrac{I_2}{V_1}$, V_1 at port 1

B

$V_1 = 0$; $y_{12} = \dfrac{I_1}{V_2}$, $y_{22} = \dfrac{I_2}{V_2}$, V_2 at port 2

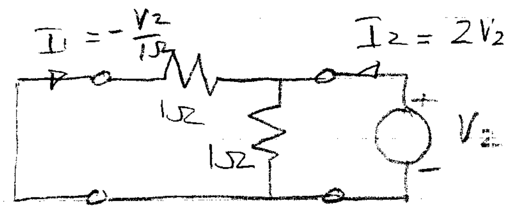
(5)

(c)



$y_{11} = 1S$, $y_{21} = -1S$

A



$y_{12} = -1S$, $y_{22} = 2S$

$$Y = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

(9)

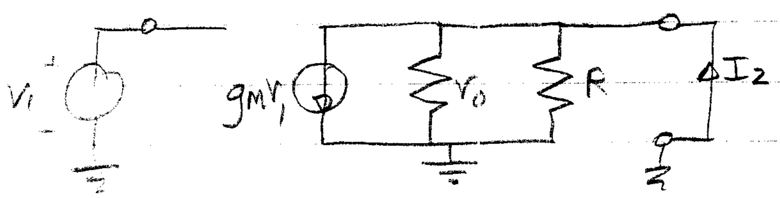
(d) $I_2 = 0 \Rightarrow 0 = y_{21}V_1 + y_{22}V_2$

$$\left.\frac{V_2}{V_1}\right|_{I_2=0} = -\frac{y_{21}}{y_{22}}$$

B

(3)

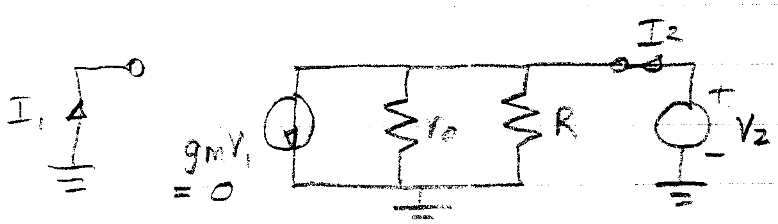
(e)



$I_2 = g_m V_1$

$\therefore\ y_{21} = g_m$

A



$I_2 = (r_o^{-1} + R^{-1})V_2$

$y_{22} = r_o^{-1} + R^{-1}$

(10)

$$\left.\frac{V_2}{V_1}\right|_{I_2=0} = -\frac{g_m}{r_o^{-1} + R^{-1}}$$

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