IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2004

EEE/ISE PART I: MEng, BEng and ACGI

ANALYSIS OF CIRCUITS

Monday, 7 June 10:00 am

Time allowed: 2:00 hours

Corrected Copy

There are FIVE questions on this paper.

Answer THREE questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D. Haigh

Second Marker(s) : G. Weiss

1

a) Give definitions for the voltage between two nodes in a circuit and the current flowing through a branch in a circuit in terms of electrical charge Q, work (or energy) E and time t. [2]

b) State Kirchoff's current law as it applies to a node in a circuit. [1]

c) The following circuit consists of resistors, a DC voltage source and DC current sources:

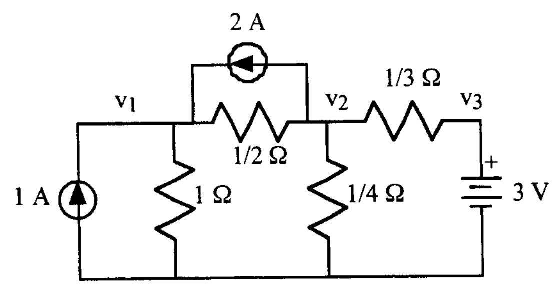


Figure 1.1

It is intended to analyse this circuit using nodal analysis; the nodal voltages v_1, v_2 and v_3 are defined with respect to the ground node.

i) At which nodes should Kirchoff's current law be applied?

ii) Write the Kirchoff's current law equation for each of these nodes.

iii) Solve these equations to determine the unknown nodal voltages.

[10]

d) State the principle of superposition as it applies to the determination of the voltages and currents in a linear circuit containing resistors, voltage sources and current sources. [1]

e) Use the principle of superposition to determine the current i in the following circuit:

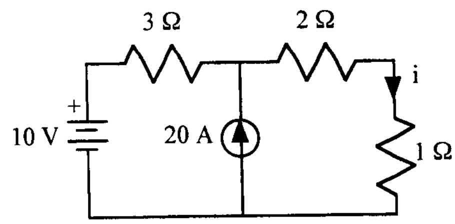


Figure 1.2

[6]

2

a) A 2-terminal element can be defined by a relationship between the voltage *v(t)* between its terminals and the current *i(t)* flowing through the element.

i) Give the relationship between *v(t)* and *i(t)* for the inductor and for the capacitor.
ii) Use these relationships to derive DC equivalent models for the inductor and for the capacitor.
iii) Use the same relationships to make statements about whether the voltage and the current associated with both the inductor and the capacitor can or cannot jump, i.e. change between two different values instantaneously.

[6]

b) In the circuit below, the switch spends a long time in position 1 and then, at time t = 0, it moves to position 2:

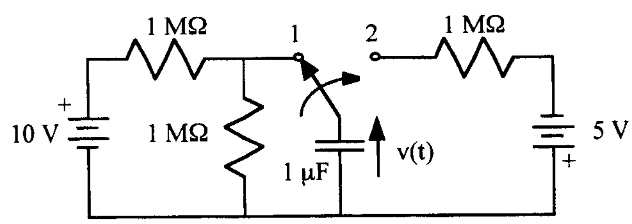


Figure 2.1

For the time interval when $t \geq 0$, determine the following:

i) The time constant of the RC circuit formed.
ii) The value of the capacitor voltage *v(t)* at time $t = 0$ (initial value).
iii) The value of *v(t)* as $t \to \infty$ (final limiting value).
iv) An expression for the capacitor voltage as a function of time *t*.

[8]

c) Consider the circuit status described in part b), where the switch moved from position 1 to position 2 at time t = 0.
Suppose that, at time $t = 2$ s, the switch moves back to position 1.
For the time interval when $t \geq 2$ s, determine the following:

i) The time constant of the RC circuit formed.
ii) The value of capacitor voltage *v(t)* at $t = 2$ s (initial value for this part of the question).
iii) The value of *v(t)* as $t \to \infty$ (final limiting value).
iv) An expression for the capacitor voltage as a function of time *t* for $t \geq 2$ s.

[6]

3

a) Write down the phasors corresponding to the following voltage functions:

i) $v_1(t) = \sqrt{2}\cos(5t - 45^{\circ})$

ii) $v_2(t) = 7\sin(3t + 45^{\circ})$

iii) $v_3(t) = -\cos(6t)$

[3]

b) Give expressions for the impedance of an <u>inductor</u> of inductance value L and of a <u>capacitor</u> of capacitance value C as a function of frequency ω in both <u>rectangular</u> and <u>polar</u> forms.
Give simple equivalent circuit models for both the inductor and for the capacitor for the cases where the frequency $\omega = 0$ and for $\omega \to \infty$.

[4]

c) Consider the following circuit:

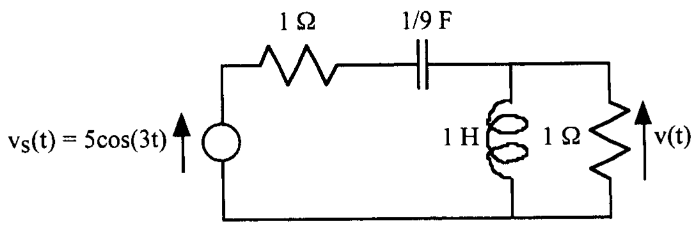


Figure 3.1

i) Draw the phasor equivalent circuit.

ii) Carry out circuit analysis to determine the voltage $v(t)$ in phasor form $\overline{V}$.

iii) Convert the phasor $\overline{V}$ into the corresponding time domain expression $v(t)$.

[8]

d) The following circuit has a periodic voltage source excitation which consists of a fundamental sinusoidal component and a 3rd harmonic component as shown:

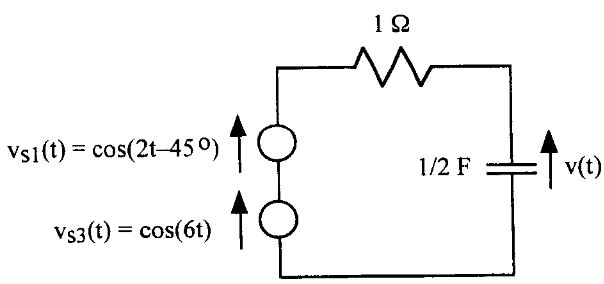


Figure 3.2

Show the two phasor equivalent circuits which can be used to solve for $v(t)$ indicating the impedances of the elements, the voltage source phasor and the working frequency in each case – Do not complete the solution for $v(t)$.

[5]

4

a) The lowpass type of filter response is useful in practice.
State the names of 3 other types of filter response.
Sketch typical amplitude responses $A(\omega)$ of all 4 types against frequency ω.

[4]

b) Consider the lossy parallel tuned circuit shown in Figure 4.1:
Determine the expression for the circuit impedance $Z(j\omega)$.
Convert the impedance into the following form:

$$Z(j\omega) = \frac{R}{1 + jQ_O\left[\dfrac{\omega}{\omega_O} - \dfrac{\omega_O}{\omega}\right]}$$

where the Q-factor Q_o and the resonant frequency ω_o depend on the element values of the circuit.
Express Q_o and ω_o as functions of the element values R, L and C.

[7]

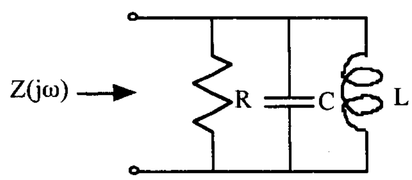


Figure 4.1

c) Determine the transfer function $H(j\omega) = \overline{V_O}/\overline{V_i}$ of the filter circuit in Figure 4.2:

By setting $\omega = 0$ and $\omega \to \infty$, determine the type of response of this filter (in the sense used in part (a) of this question).

[6]

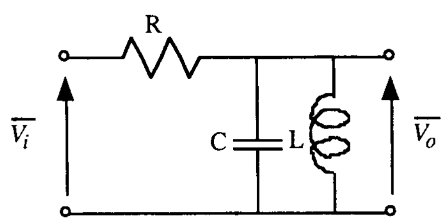


Figure 4.2

d) Define the term 'reactance'.
Sketch a plot of the reactance of the circuit in Figure 4.3 as a function of frequency:

[3]

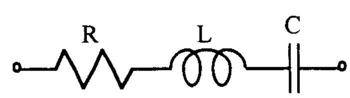


Figure 4.3

5

a) Show circuits that incorporate an operational amplifier and which realise the following functions:

(i) Non-inverting amplifier.

(ii) Inverting summing amplifier with 3 inputs.

(iii) Unity gain buffer amplifier.

In each case, give the expression for the amplifier output voltage in terms of the input voltage, or voltages, and the values of any passive elements, under the assumption that the operational amplifier is ideal.

[6]

b) Determine the transfer function $\overline{V_o}/\overline{V_i}$ of the circuit in Figure 5.1, either by using the gain expression for the non-inverting amplifier configuration or by circuit analysis; the operational amplifier may be assumed to be ideal:

By inspection of the transfer function, determine the gain and phase response of the circuit under the assumptions $R_1 = R_2$ and $C_1 = C_2$ at a frequency ω given by:

$$\omega = \frac{1}{\sqrt{C_1 C_2 R_1 R_2}}.$$

[9]

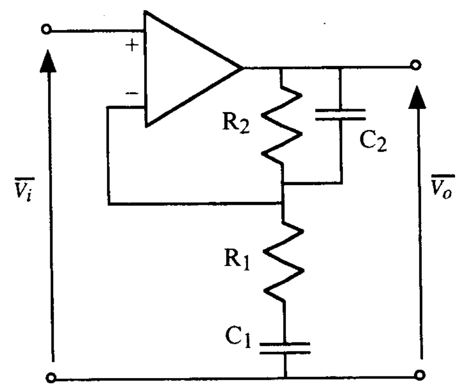


Figure 5.1

c) A circuit has the following transfer function:

$$H(j\omega) = \frac{j\omega}{(j\omega + 0.1)(j\omega + 100)}.$$

Sketch the Bode gain and phase plots for this transfer function.

Determine the approximate gain in dB and the phase shift at a frequency of 3 rad/s.

[5]

Solutions ①
2004
EE1.1

Analysis of Circuits

Q1

[a] a) Voltage between two nodes = E/Q where E is energy needed to move charge Q between nodes.
Current = dQ/dt, ie rate of flow of charge.

[a] b) Net sum of currents at node is zero

[c] c) i) Nodes V_1 and V_2

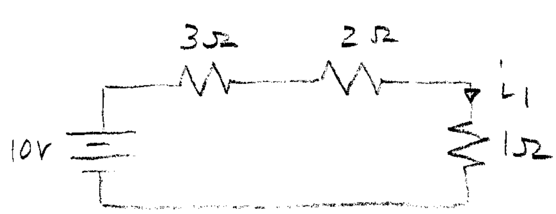
ii) $V_1 + 2(V_1 - V_2) = 3$
$2(V_2 - V_1) + 4V_2 + 3(V_2 - 3) = -2$

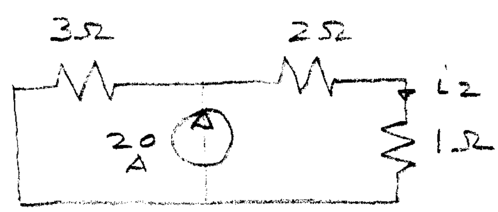
$3V_1 - 2V_2 = 3$ (1)
$-2V_1 + 9V_2 = 7$ (2)

iii) $(2) \times 3/2 \rightarrow -3V_1 + 13\frac{1}{2} V_2 = 10\frac{1}{2}$ (3)
$(3) + (1) \rightarrow 11\frac{1}{2} V_2 = 13\frac{1}{2}$ $V_2 = 27/23$
$V_1 = (3 + 2V_2)/3 = (3 + 54/23)/3$
$= 123/3/23 = 41/23$

[a] d) Determine required voltages and currents with each source in turn active and others deactivated (s/c for voltage source; o/c for current source); add together the results of the analyses.

[c] e)





$i_1 = 10/6 = 5/3$ A

$i_2 = \frac{1}{2} \times 20 = 10$ A

$i_1 + i_2 = 11\frac{2}{3}$ A

[a] Bookwork [c] New application [d] New example

Q2

[a]

a) i) $v = L \frac{di}{dt}$ $\quad$ $i = C \frac{dv}{dt}$

ii) $di/dt = 0; v = 0;$ short-circuit $\quad$ $dv/dt = 0;\ i = 0$ open-circuit

iii) v is finite, so i can not change instantaneously | i is finite, so v can not change instantaneously

[c]

b) i) $\tau = RC = 10^6 \cdot 10^{-6} = 1s$

ii) $V_{01} = 5\ V$

iii) $V_{\infty 1} = -5\ V$

iv) $V_1 = V_\infty - (V_\infty - V_0)\, e^{-t/\tau}$
$= -5 + 10\, e^{-t}$

[c]

c) i) $\tau = R'C = 0.5 \times 10^6 \times 10^{-6} = 0.5\ s$

ii) $V_{02} = -5 + 10 e^{-2} = -3.647\ V$

iii) $V_{\infty 2} = 5\ V$

iv) $V_2 = 5 - (5 - (-3.647))\, e^{-2(t-2)}$
$= 5 - 8.647\, e^{-2t+4}$

Q3

[d] a)

i) $\sqrt{2}\angle -45^\circ$

ii) $7\angle -45^\circ$

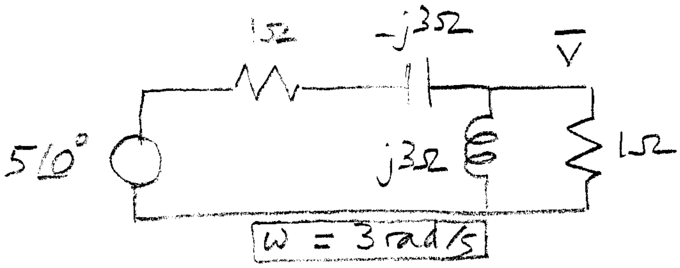
iii) $1\angle 180^\circ$

[a] b)

$$Z_L = j\omega L = \omega L\angle 90^\circ \qquad Z_C = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^\circ$$

	$\omega = 0$	$\omega \to \infty$
Z_L	0 (s/c)	∞ (o/c)
Z_C	∞ (o/c)	0 (s/c)

[c] c)



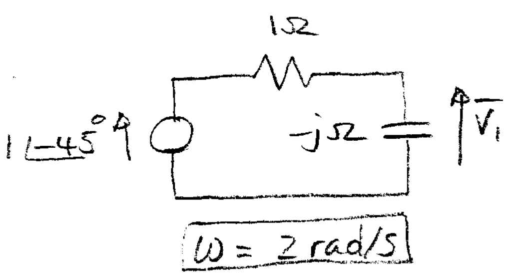
$$\bar{V} = \frac{Z_2}{Z_1+Z_2}\, 5\angle 0^\circ = \frac{1}{1+Z_1Y_2}\times 5 = \frac{5}{1+(1-j3)(1-\frac{j}{3})}$$

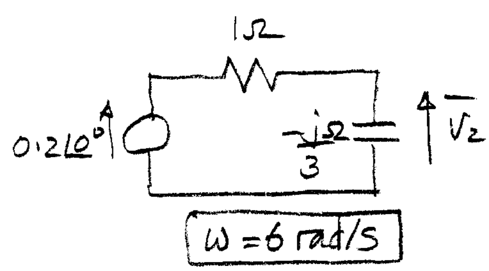
$$= \frac{5}{1+1-1-j3-j/3} = \frac{5}{1-j\frac{10}{3}}$$

$$= \frac{5}{\sqrt{1+\frac{100}{9}}}\ \tan^{-1} 10/3 = \frac{15}{\sqrt{109}}\ \tan^{-1} 10/3 = 1.437\angle 73.30^\circ$$

$$v(t) = 1.437\cos(3t + 73.30^\circ)$$

[c] d)





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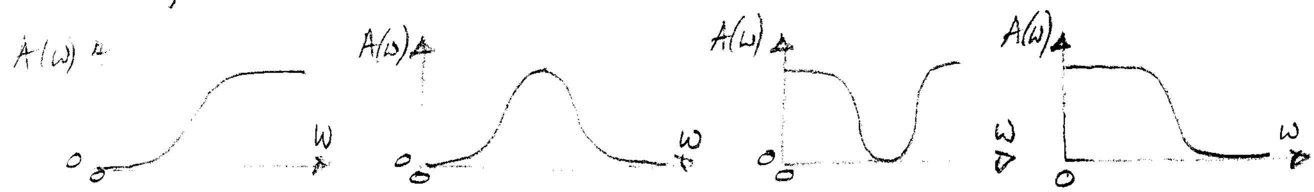
Q 4

a) Highpass Bandpass Bandstop Lowpass

[a]



b)

[c]

$$Z(j\omega) = \frac{1}{\frac{1}{R} + j\omega C + \frac{1}{j\omega L}} = \frac{R}{1 + jR\left(\omega C - \frac{1}{\omega L}\right)} = \frac{R}{1 + jR\sqrt{\frac{C}{L}}\left(\omega\sqrt{LC} - \frac{1}{\omega\sqrt{LC}}\right)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q_0 = R\sqrt{\frac{C}{L}} = \omega_0 CR = \frac{R}{\omega_0 L}$$

[c] c)

$$\frac{\overline{V_o}}{\overline{V_i}} = \frac{Z}{R+Z} = \frac{1}{1+RY} = \frac{1}{1 + R\left(j\omega C + \frac{1}{j\omega L}\right)}$$

$$= \frac{1}{1 + jR\sqrt{\frac{C}{L}}\left(\omega\sqrt{LC} - \frac{1}{\omega\sqrt{LC}}\right)}$$

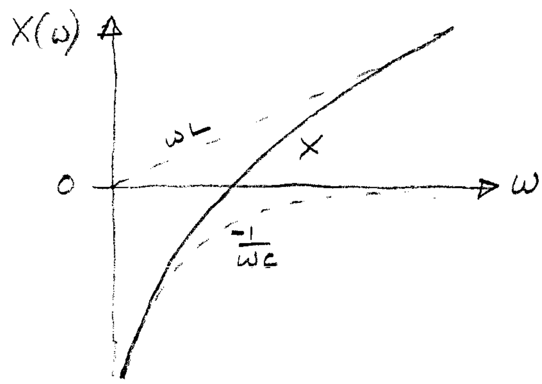
For $\omega = 0$, $A(j\omega) \to 0$
For $\omega \to \infty$, $A(j\omega) \to 0$
∴ Filter type is bandpass

[a] d) Reactance is the imaginary part of impedance Z
where $Z = \overline{V} / \overline{I}$

$Z = R + j(\omega L - 1/\omega C)$

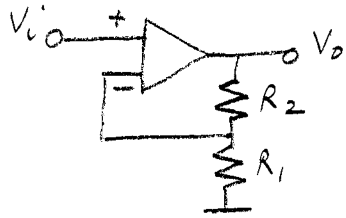
\+ $X = \omega L - 1/\omega C$

[c]

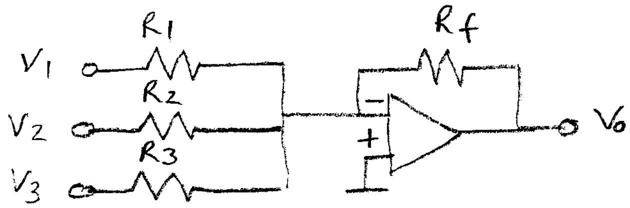


Q5

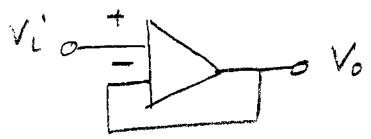
a) [a]



$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_i$$



$$V_o = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$$



$$V_o = V_i$$

b) [d]

$$\frac{V_o}{V_i} = 1 + \frac{Z_2}{Z_1} = 1 + \frac{1}{Z_1 Y_2} = 1 + \frac{1}{\left(R_1 + \frac{1}{j\omega C_1}\right)\left(j\omega C_2 + \frac{1}{R_2}\right)}$$

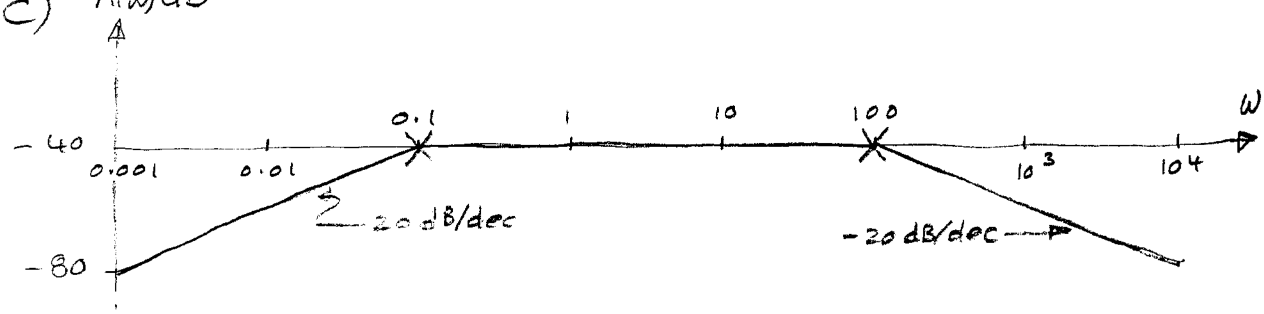
$$= 1 + \frac{j\omega C_1 R_2}{(1 + j\omega C_1 R_1)(1 + j\omega C_2 R_2)}$$

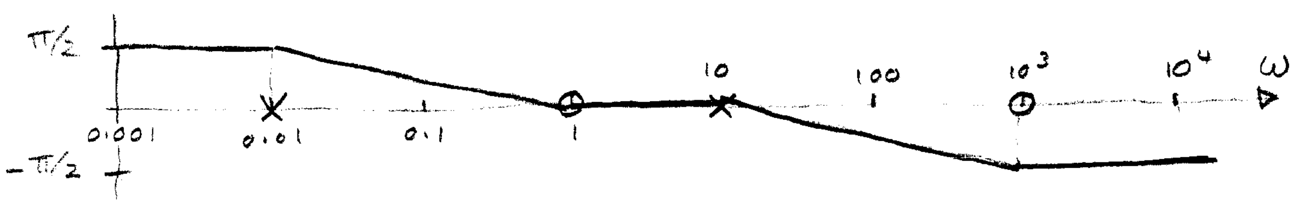
$$= \frac{1 + j\omega(C_1 R_1 + C_2 R_2 + C_1 R_2) - \omega^2 C_1 C_2 R_1 R_2}{1 + j\omega(C_1 R_1 + C_2 R_2) - \omega^2 C_1 C_2 R_1 R_2}$$

At $\omega = \frac{1}{\sqrt{C_1 C_2 R_1 R_2}}$, $\frac{V_o}{V_i} = \frac{C_1 R_1 + C_2 R_2 + C_1 R_2}{C_1 R_1 + C_2 R_2}$

For $R_1 = R_2$ and $C_1 = C_2$, $A(\omega) = 3/2$ $\quad \phi(\omega) = 0°$

c) [d]





At $\omega = 3$ rad/s, $A(\omega) \approx -40$ dB, $\phi(\omega) \approx 0°$

END OF ANSWERS.