IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2003

ANALYSIS OF CIRCUITS

Friday, 30 May 10:00 am

Time allowed: 2:00 hours

There are FIVE questions on this paper.

Answer THREE questions.

Any special instructions for invigilators and information for candidates are on page 1.

Corrected Copy

Examiners responsible First Marker(s) : R. Spence
Second Marker(s) : G. Weiss

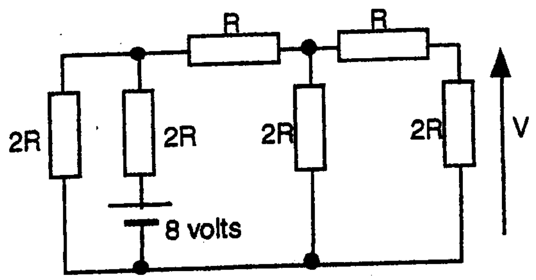
Special information for Invigilators: **none**

Information for candidates:

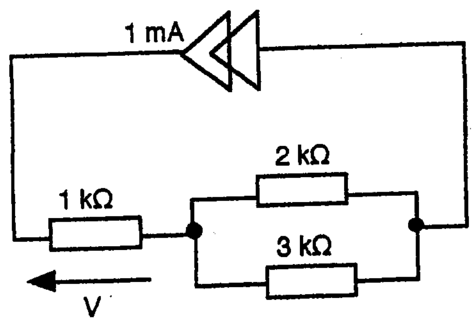
For Question 2, a separate sheet is available on which waveforms can be drawn. If used, this sheet should be tied within the answer book.

The Questions

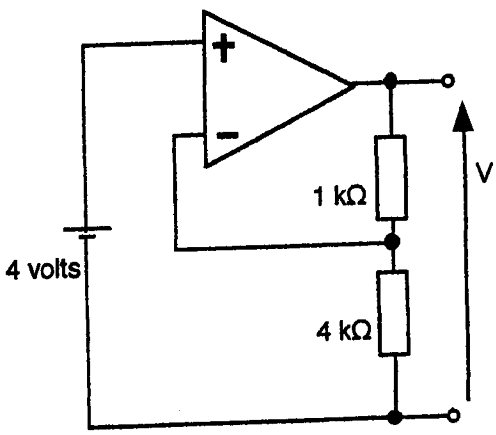
1. Preferably by inspection, but with brief explanation, find the value of the voltage V in each of the five circuits shown below.



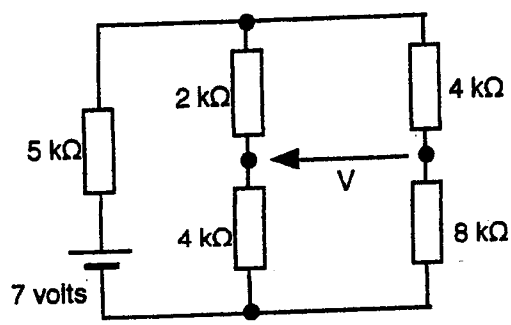
(a) [5]



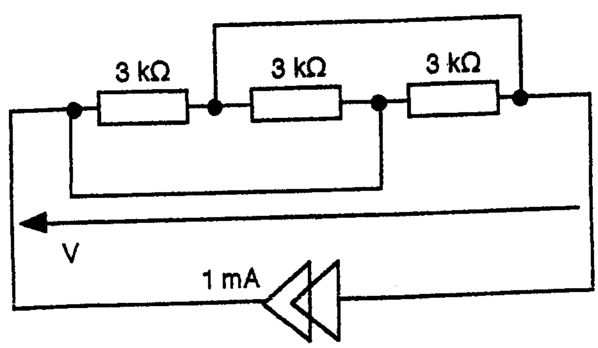
(b) [3]



(c) [4]



(d) [4]



(e) [4]

2. The output voltage of the opamp in the circuits of Figure 2 saturates at +/- 10 V...

(a) The voltage waveform V_i shown below (and repeated on a separ... point X in the circuit of Figure 2a. Under the assumption th... a value of 10 Volts at t=0, derive the waveform of the v... ...e period from t=0 to t=20ms and plot it to the same scale... [10]

(b) Terminal W is now connected to term... 2b, and the voltage waveform V_i is again ap... ...cuit of Figure 2a. On the same plot sketch, with ... the voltage at terminal Z. [10]

Material covered by this question not in syllabus for 2004 exam

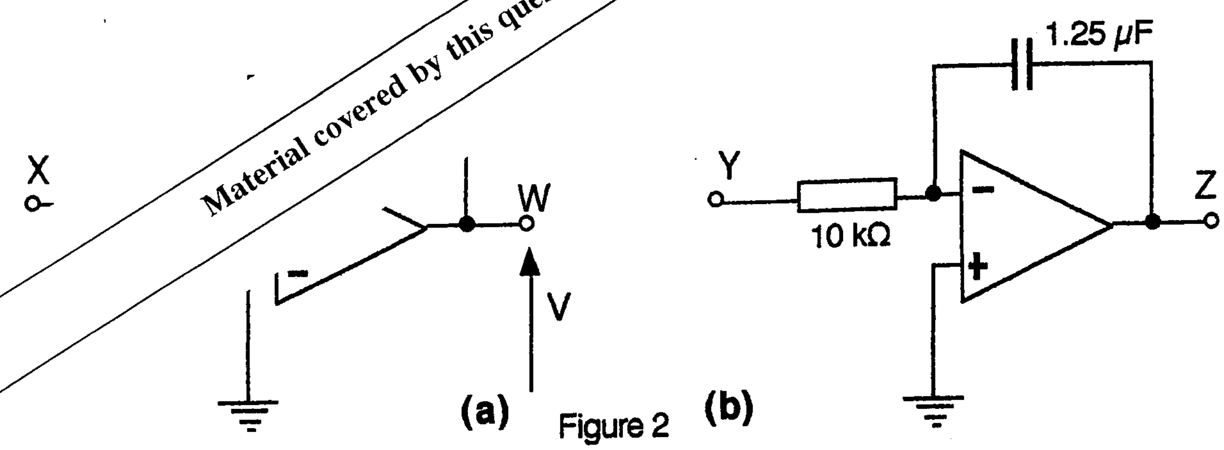
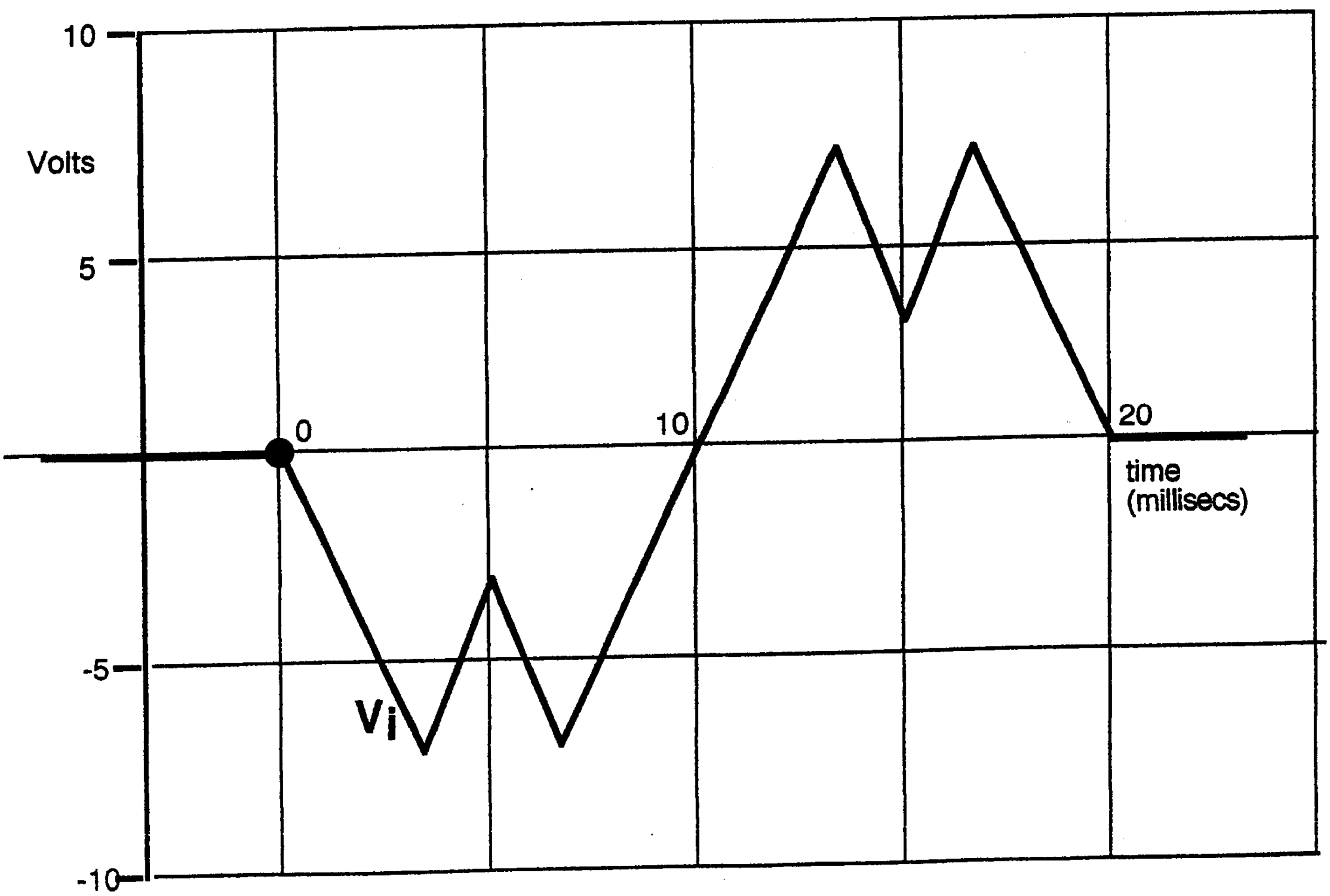


Figure 2



3. (a) Measurements have been made on a circuit comprising the series connection of a resistor, a capacitor and an inductor. The measured magnitude of the impedance of the series connection is plotted against frequency in Figure 3a. Estimate, with explanation, the capacitance of the capacitor, the inductance of the inductor and the combined series resistance of the resistor and inductor. [8]

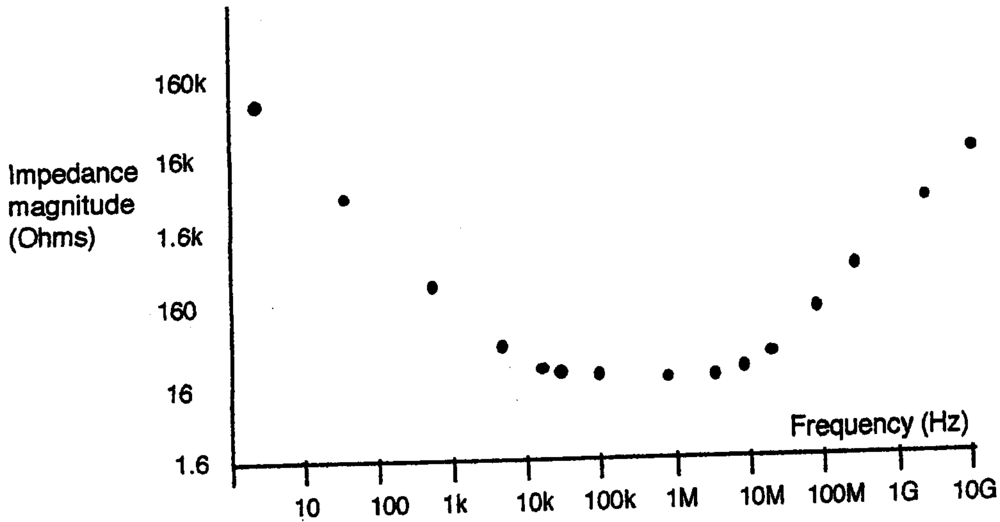


Figure 3a

(b) In the circuit of Figure 3b, V_S is a sinusoidal voltage of radian frequency ω. Derive an expression for the complex voltage V as a function of R, L, C and the radian frequency ω. Hence show that V=0 if $R = (L/C)^{0.5}$. Show that, if this relation between R, L and C holds, the current supplied by the source is in phase with V_S. [12]

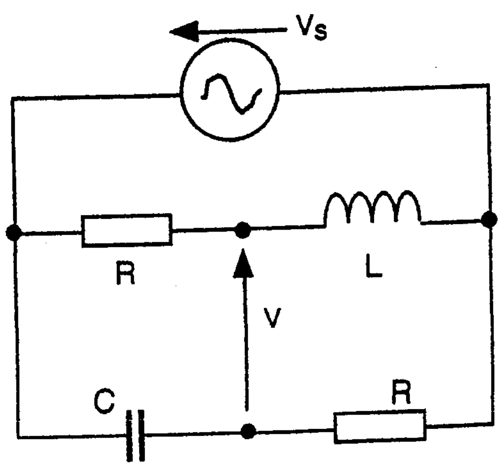


Figure 3b

4. As shown schematically in Figure 4a, the output of a linear amplifier of voltage connected to the input of a feedback circuit of voltage gain B. The voltage are complex and they depend upon the frequency of the signal. The o feedback circuit is connected to the input of the amplifier.

(a) Under the assumption that the gains A and B are n ... tions, derive Barkhausen's criterion for the existence ... oscillation. Express the criterion in terms of A and B ... of the magnitude and phase of A and B. [4]

(b) A circuit designer, in propo ... 4b, has made a mistake in the design of a Wien oscill ... rcuit of Figure 4b will not support sustained sinusoid [8]

(c) Using the ... n in Figure 4b, *and no others*, draw the diagram of a c ... sustained sinusoidal oscillation. [4]

(d ... R and C to ensure oscillation of your new circuit at a ... 0 Hz. [4]

Material covered by this question not in syllabus for 2004 exam

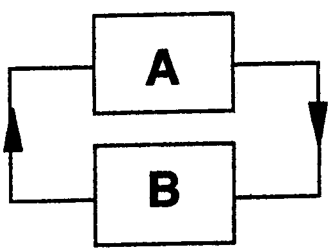


Figure 4a

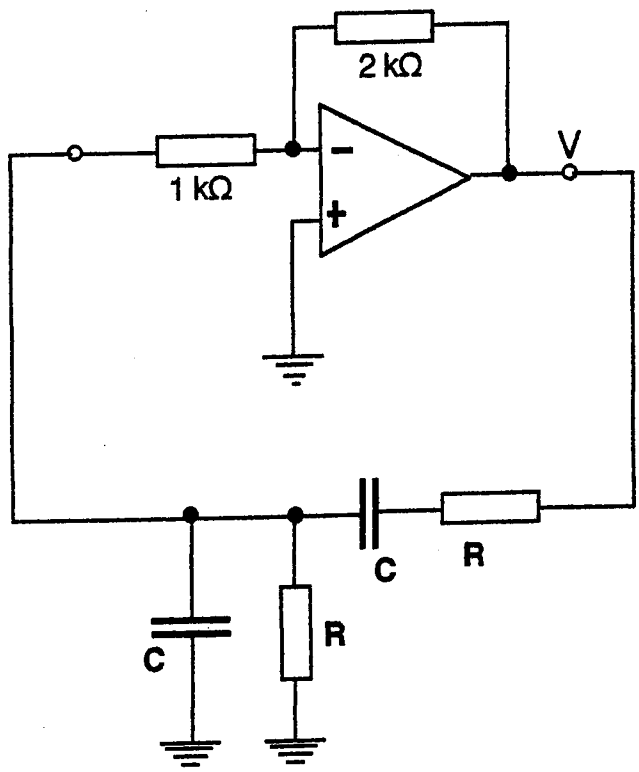


Figure 4b

5. (a) For the circuit of Figure 5, use the Superposition Principle to calculate the voltage V_O. [10]

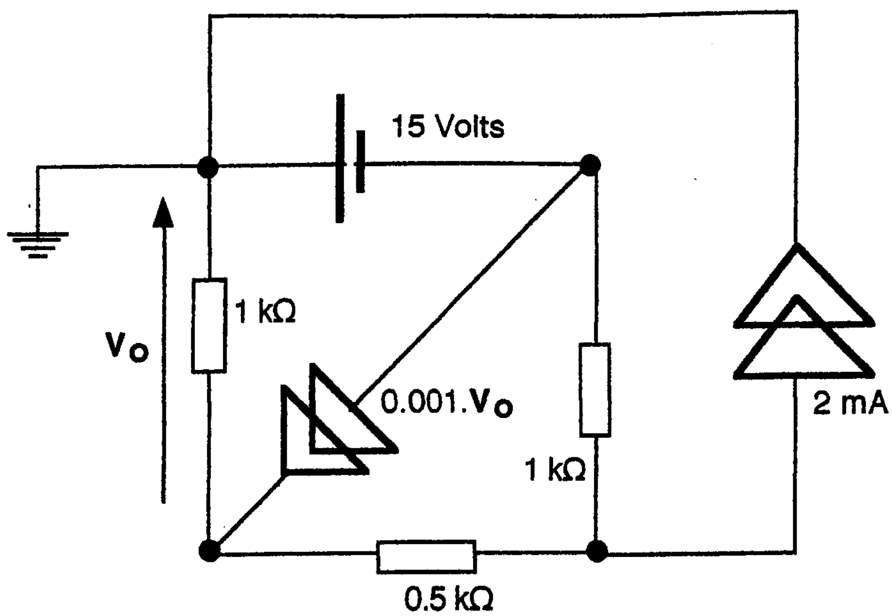
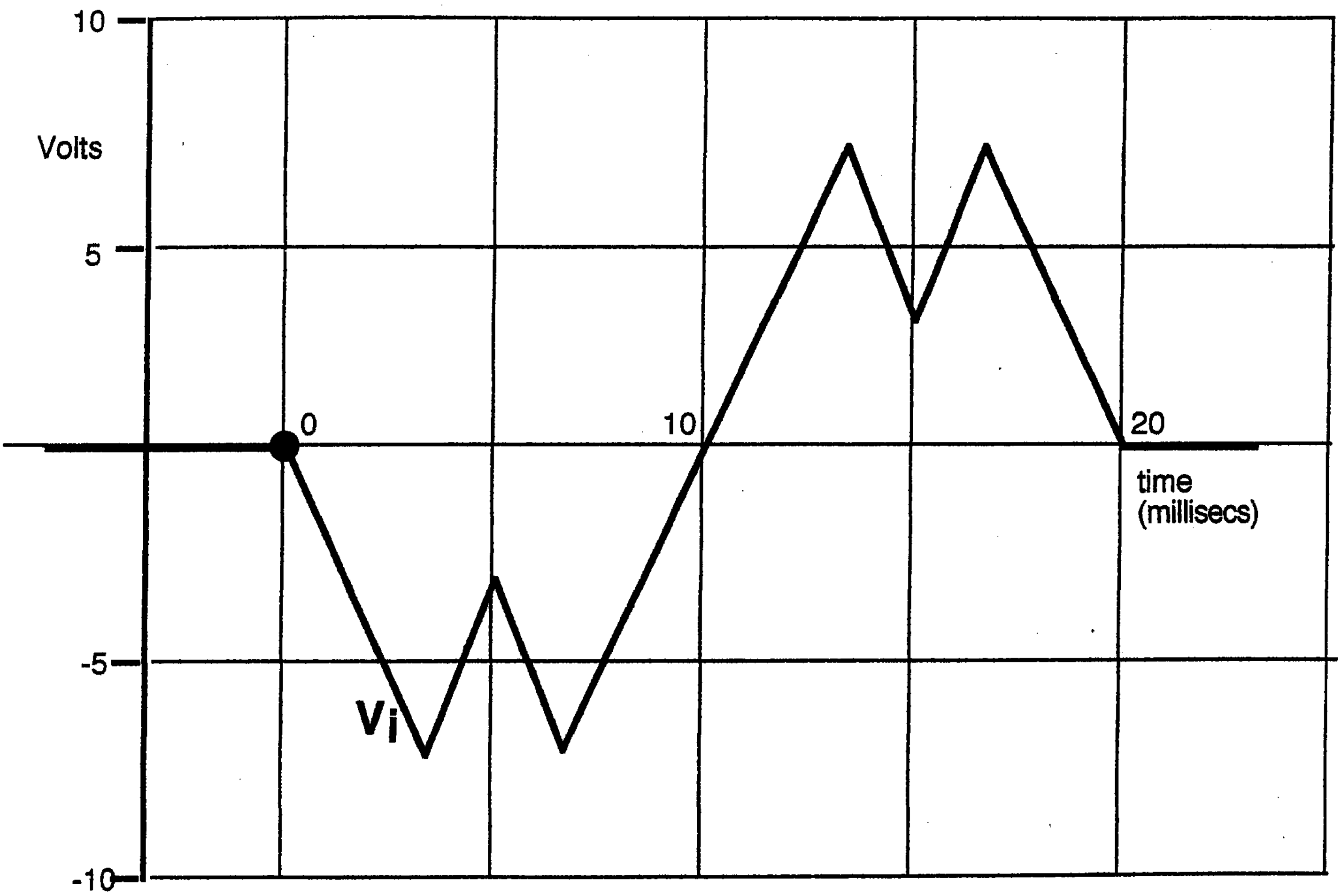


Figure 5

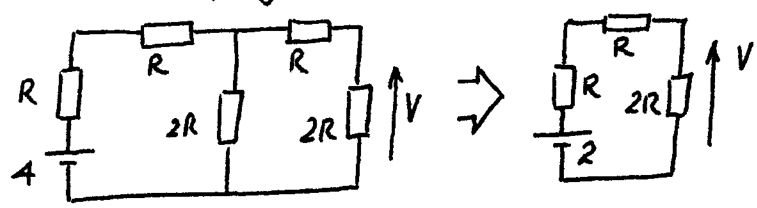
(b) Derive the nodal voltage equations relating the nodal voltages of the circuit of Figure 5 to the independent sources. Solve these equations to find the value of the voltage V_O. [10]

Supplementary sheet for Question 2. To be tied inside the Answer Book



Answer 1

(a) From the left, progressively represent by a Thevenin Equivalent Circuit.



$\therefore V = 1$ volt

(b) Apply Ohms Law to the $1\,k\Omega$ resistor:

$$V = RI = 1k\Omega \times 1\,mA = 1 \text{ Volt}$$

(c) $v_- = 4$ Volts, therefore by voltage divider action $V = 5$ Volts

(d) Because ratio of $2k\Omega$ to $4k\Omega$ is same as $4k\Omega$ to $8k\Omega$, $V = 0$

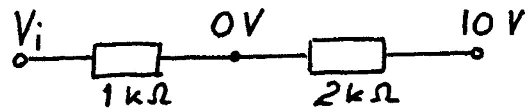
(e) There are three $3k\Omega$ resistors connected in parallel, with a combined resistance of $1\,k\Omega$. Ohms Law gives $V = 1k\Omega \times 1\,mA$
$= 1$ Volt.

Answer 2

(a)

Circuit of Figure 2a is a Schmitt Trigger.

Calculate threshold values of V_i



For zero voltage at +ve input when $V = 10$ (see circuit at right)

$V_i = -5$ Volts. Similarly, when $V = -10$, threshold for V_i is $+5$ Volts.

When V_i first falls below -5 V at $t = 2.5$ ms, V switches from 10 V to -10 V.

Later, when V_i first reaches $+5$ V, V switches back to $+10$ Volts.

(see waveform of V plotted on attached sheet)

(b)

Figure 2b is the circuit of an integrator.

Current into capacitor when $V = 10$ Volts is $10/10\,k\Omega = 1$ mA

Capacitor current $i = -C\dfrac{dV_z}{dt}$ so $10^{-3} = -1.25 . 10^{-6} . \dfrac{dV_z}{dt}$

giving $\dfrac{dV_z}{dt} = -\dfrac{1}{1.25}$ volts per millisecond.

Hence when $t = 2.5$ ms, $V_z = -2$ Volts.

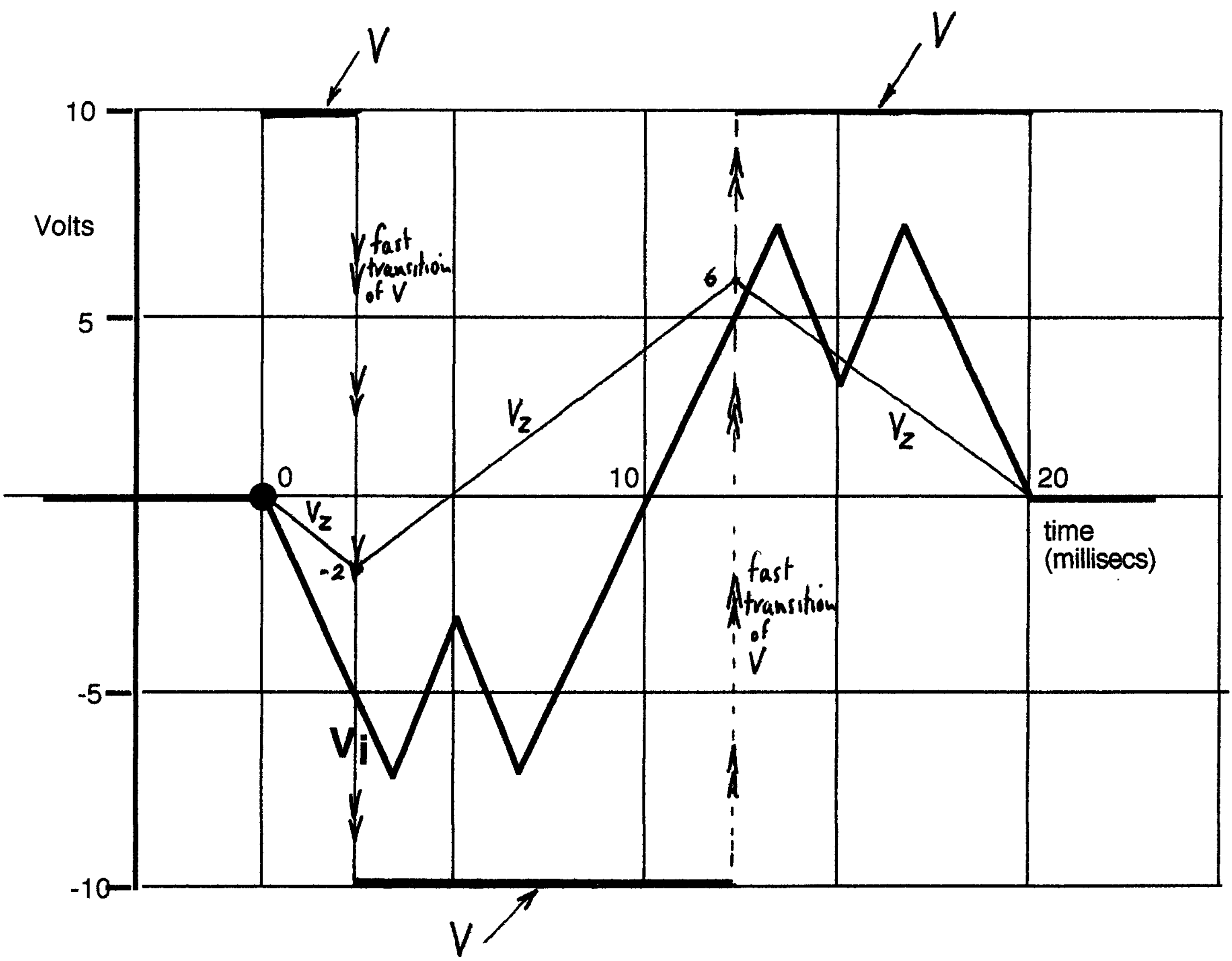
When $V = -10$, $\dfrac{dV_z}{dt} = +\dfrac{1}{1.25}$ volts/millisecond.

Hence between $t = 2.5$ ms and $t = 12.5$ ms, V_z increases linearly by 8 volts to 6 volts.

From $t = 12.5$ ms to $t = 20$ ms, V_z decreases by 6 volts to zero.

(see waveform of V_z plotted on attached sheet)

Supplementary sheet for Question 2. To be tied inside the Answer Book



Answer 3

(a)
Draw the asymptotes as shown on right

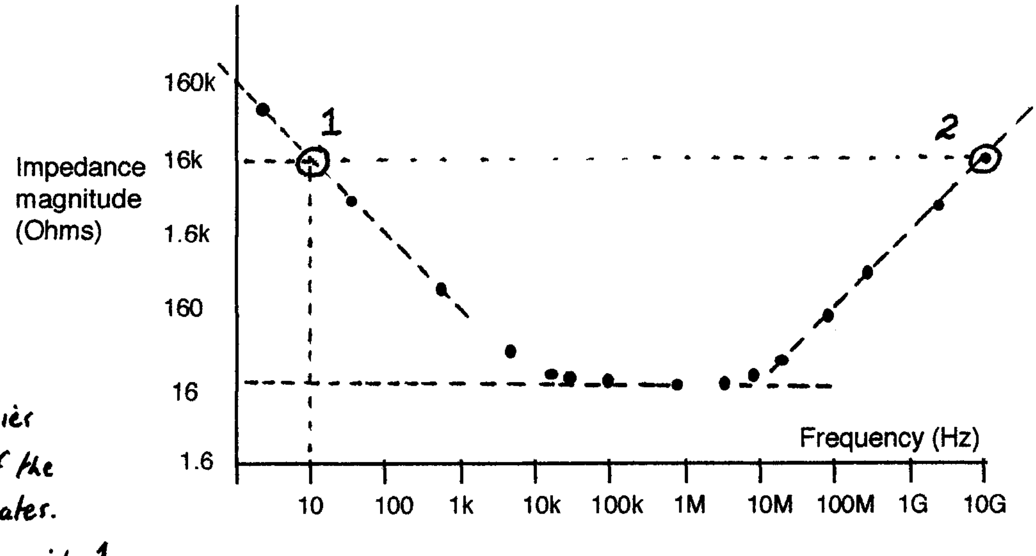


Figure 3a

At low frequencies the impedance of the capacitor dominates.
Take the sample point 1

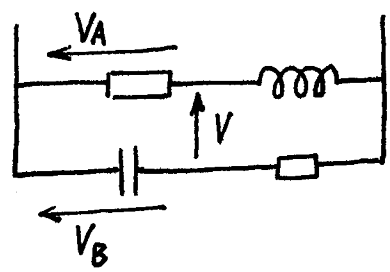
$|Z| = 16\,k\Omega$, $\omega = 2\pi 10$ r/s $\quad \therefore\ C \approx \dfrac{1}{\omega|Z|} = \dfrac{1}{2\pi 10.16.10^3} \approx 1\,\mu F$

At high frequencies the inductor dominates. Take sample point ②

$|Z| = 16\,k\Omega$, $\omega = 2\pi 10^{10}$ r/s $\quad L \approx \dfrac{|Z|}{\omega} = \dfrac{16.10^3}{2\pi 10^{10}} \approx 0{\cdot}255\,\mu H$

At mid frequencies, resistance of resistor dominates From asymptote $R = 16\ \Omega$

(b) By voltage divider principle (see circuit at right)



$V_A = \dfrac{R}{R + j\omega L} V_s \qquad V_B = \dfrac{1/j\omega C}{R + 1/j\omega C} V_s$

So $V = V_B - V_A = V_s \left[\dfrac{1}{1 + j\omega CR} - \dfrac{1}{1 + j\omega L/R} \right]$

Thus, $V = 0$ if $CR = L/R$ i.e., $R = \sqrt{L/C}$

Current in upper branch $= \dfrac{V_s}{R + j\omega L}$ Current in lower branch $= \dfrac{V_s}{R + 1/j\omega C}$

So total current supplied by source $= V_s \left[\dfrac{1}{R + j\omega L} + \dfrac{1}{R + 1/j\omega C} \right]$

$$= V_s \left[\frac{R + 1/j\omega C + R + j\omega L}{R^2 + \frac{L}{C} + j\left[\omega L R - R/\omega C\right]} \right] = \frac{V_s}{R} \left[\frac{2R + j\left(\omega L - 1/\omega C\right)}{R + L/CR + j\left(\omega L - 1/\omega C\right)} \right]$$

Because $\frac{L}{CR} = R$, the ratio of imaginary to real part is the same in both numerator and denominator, the current supplied by source is real: i.e., it is in phase with V_s

Answer 4

(a) Assume a sinusoidal voltage at the input to A, and represented by a complex number V, causing an output voltage AV which is applied to the input of B. If the output of B, equal to ABV, is identical with V, oscillation at the frequency of V will be sustained. Thus, for sustained oscillation,

$$AB = 1 \quad \text{——— (1)}$$

this is the Barkhausen Criterion. Mindful of the fact that both A and B are complex we can write (1) as

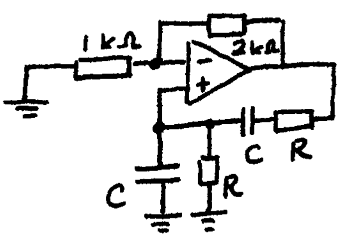
$$(|A|\angle A)(|B|\angle B) = 1\angle 0^\circ$$

giving $|A||B| = 1$ the "magnitude criterion"

and $\angle A + \angle B = 0^\circ$ the "phase criterion"

(b) The circuit of Figure 4b will NOT sustain oscillation because the phase criterion is not satisfied. The amplifier introduces a phase shift of 180° and the feedback circuit can only exhibit a phase shift between −90° and +90°

(c) Using the same components, a Wien oscillator can be realised as shown below



The amplifier has a gain $A = 3 + j0$

The voltage gain of the feedback circuit is

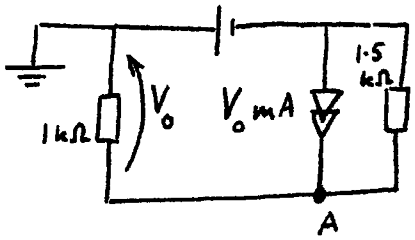
$$B = \frac{\dfrac{1}{1/R + j\omega C}}{\dfrac{1}{1/R + j\omega C} + R + \dfrac{1}{j\omega C}} = \frac{1}{3 + j\left(\omega CR - \frac{1}{\omega CR}\right)}$$

So the feedback circuit has a phase shift of zero at a radian frequency $\omega = 1/CR$ and, at that frequency, a gain of 1/3. Since the amplifier has a gain of 3 and a phase shift of zero, the condition for oscillation is satisfied

(d) If $\omega = 2\pi\, 1590 = 10^4$, $CR = 10^{-4}$. Select $R = 10^4$ ohms so that $C = 10^{-8} = 0.01\,\mu F$

Answer 5

(a) Set the current source to zero



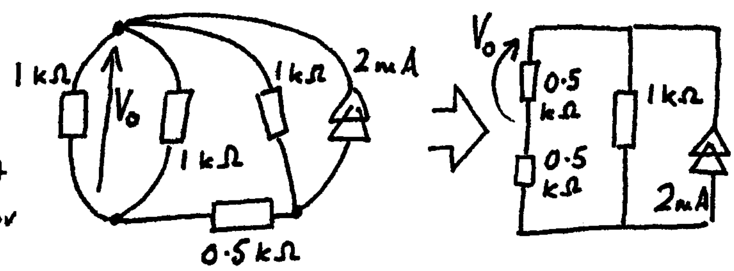
KCL at A:

$$-V_A + \frac{(-15 - V_A)}{1.5} - V_A = 0$$

$$\therefore V_A = -3.75 \text{ Volts}$$

so $V_o = 3.75$ Volts

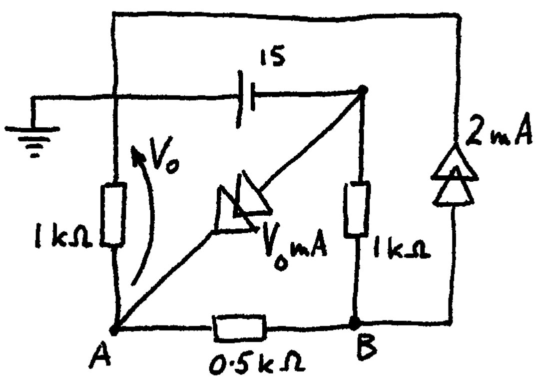
Set the voltage source to zero, whereupon VCCS is equivalent to a 1kΩ resistor



$\therefore V_o = 0.5$ Volt

So, by Superposition, $V_o = 3.75 + 0.5 = 4.25$ Volts

(b)



Note that $V_o = -V_A$

KCL at A (IN) $\quad V_o + \frac{V_B - V_A}{0.5} - \frac{V_A}{1} \Rightarrow -4V_A + 2V_B = 0$ —— ①

KCL at B (IN) $\quad -2 + \frac{(-15 - V_B)}{1} + \frac{(V_A - V_B)}{0.5} \Rightarrow 2V_A - 3V_B = 17$ —— ②

[① × 3] + [② × 2] yields

$$-12V_A + 4V_A = 34 \quad \text{so} \quad V_A = -4.25 \text{ Volts}$$

Therefore $V_o = 4.25$ Volts