Geometry and Vectors

Coursework 1

Hand in the complete solutions to all three questions in the general office (room C123)

DEADLINE: Thursday 05/03/2009 at 16:00

1. The vectors $\vec{\imath}, \vec{\jmath}, \vec{k}$ constitute an orthonormal basis in an Euclidean space. The vectors $\vec{u}$ and $\vec{v}$ are defined by [12 marks]

$$\vec{u} = \lambda\vec{\imath} - 7\vec{\jmath} - \vec{k}, \qquad \text{and} \qquad \vec{v} = 2\vec{\imath} - \vec{\jmath} + 2\vec{k} \qquad \text{with } \lambda \in \mathbb{R}.$$

(i) Determine the constant λ such that the angle between $\vec{u}$ and $\vec{v}$ becomes $\pi/4$.

(ii) Take now $\lambda = -1$ and construct all vectors with length $\sqrt{90}$ which are perpendicular to both vectors $\vec{u}$ and $\vec{v}$.

(iii) Compute the expression $\vec{u} \times \vec{v}$ for $\lambda = 14$.

2. The vectors $\vec{a}, \vec{b}, \vec{c}, \vec{x}$ are arbitrary and $\lambda \in \mathbb{R}$ is a scalar. [12 marks]

(i) For given vectors $\vec{a}, \vec{b}$ and $\vec{c}$ find the general expression for the vector $\vec{x}$, which solves the vector equation

$$\lambda\vec{x} + (\vec{x} \cdot \vec{b})\vec{a} = \vec{c} \qquad \lambda \neq 0.$$

[Hint: Treat the cases $\lambda + \vec{a} \cdot \vec{b} \neq 0$ and $\lambda + \vec{a} \cdot \vec{b} = 0$ separately.]

(ii) Use the result from (i) to solve the vector equation

$$\vec{x} \times \vec{a} = \vec{b}$$

for $\vec{x}$ when $\vec{a}$ and $\vec{b}$ are given.
[Hint: You may use the identity $\vec{u} \times \vec{v} \times \vec{w} = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$.]

3. Given the three position vectors $\vec{a} = \overrightarrow{OA}$, $\vec{b} = \overrightarrow{OB}$, $\vec{c} = \overrightarrow{OC}$ and a point D situated on the line $\overleftrightarrow{BC}$, with $\vec{d}$ denoting the vector $\overrightarrow{AD}$; [26 marks]

(i) draw the corresponding figure;

(*ii*) use vectors to show that the shortest distance from the point A to the point D is given by the expression

$$|\vec{d}| = \frac{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}{|\vec{b} - \vec{c}|};$$

(*iii*) take now the vectors

$$\vec{a} = -\frac{1}{4}\vec{\imath} + \vec{\jmath}, \qquad \vec{b} = \vec{\imath}, \qquad \vec{c} = \frac{5}{4}\vec{\imath} + \frac{3}{2}\vec{\jmath}$$

and compute $|\vec{d}|$;

(*iv*) compute the position vector $\overrightarrow{OD}$.