MA1607

CITY UNIVERSITY

London

BSc Degrees in Mathematical Science
Mathematical Science with Statistics
Mathematical Science with Computer Science
Mathematical Science with Finance and Economics
MMath Degrees in Mathematical Science

PART I EXAMINATION

Geometry and Vectors

May 2006 9:00 am – 11:00 am

Time allowed: 2 hours

Full marks may be obtained for correct answers to FIVE questions of section A and TWO questions of section B.

If more than TWO questions are answered of section B, the best TWO marks will be credited.

Turn over ...

Section **A**

Answer **all six** questions from this section. Each question carries 8 marks.

1. $ABCD$ constitutes a parallelogram. The point E is situated on the line segment $\overline{BD}$, such that $\overline{BE} = \frac{1}{3}\,\overline{BD}$. The point F is the pointin which the diagonal $\overline{BD}$ intersects the line $\overline{AE}$.

Sketch the corresponding figure and show, using vectors, that

$$\overline{AF} = \frac{3}{4}\overline{AE}.$$

2. The vectors $\vec{\imath}, \vec{\jmath}, \vec{k}$ constitute an orthonormal basis. Given are the vectors

$$\vec{u} = 2\vec{\imath} + 2\vec{k} \quad \text{and} \quad \vec{v} = 2\vec{\imath} - \vec{\jmath} + \vec{k}$$

(i) Determine the angle between $\vec{u}$ and $\vec{v}$.

(ii) Construct two vectors of length 3 which are both perpendicular to the plane which contains $\vec{u}$ and $\vec{v}$.

(iii) Verify the triangle identity for the vectors $\vec{u}$ and $\vec{v}$.

3. The vectors $\vec{\imath}, \vec{\jmath}, \vec{k}$ constitute an orthonormal basis. For the four vectors

$$\vec{u} = \vec{\jmath} + 2\vec{k}, \quad \vec{v} = -\vec{\imath} + 3\vec{\jmath}, \quad \vec{w} = \vec{\imath} - \vec{\jmath} \quad \text{and} \quad \vec{x} = \vec{\imath} + \vec{\jmath} - \vec{k}$$

construct a new vector

$$\vec{p} = \lambda(\vec{u} \times \vec{v}) \times (\vec{w} \times \vec{x}).$$

Determine the constant λ such that $\vec{p}$ becomes a unit vector.

4. An ellipse is given in its polar form as

$$r = \frac{k}{1 - e\cos\theta},$$

with eccentricity $e = 4/5$ and $k = 9/5$.

(i) Determine the normal form for this ellipse.

(ii) Find the equations of the tangents to the ellipse which passes through the point $Q(0, 5)$.

5. Given are the two points $A(3, 1, 2)$ and $B(-1, 2, 0)$.

(i) Determine the equation of the line passing through the points A and B. Subsequently find the coordinates of the point in which the line intersects the xy-plane.

(ii) Determine the coordinates of the point in which the line through the points A and B intersects the plane

$$\mathcal{P}: \qquad 2x - 4y - z = 30$$

6. Determine the equation of the line of intersection of the two planes

$$\begin{aligned} \mathcal{P}_1 \ &: \qquad 3x - y + z - 17 = 0 \\ \mathcal{P}_2 \ &: \qquad -x + 2y + 6z + 14 = 0 \end{aligned}$$

in Cartesian form.

Section **B**

Answer **two** questions from this section. Each question carries 26 marks.

7. (*i*) State Euclid's five axioms.

(*ii*) Use the axioms to prove the following:

Given a line $\mathcal{L}$ and a point P which is not on the line, i.e. $P \notin \mathcal{L}$, there is a unique plane $\mathcal{P}$ containing both $\mathcal{L}$ and P.

8. Given are the three points $A(3,-1,0)$, $B(2,1,8)$ and $C(2,3,4)$

(*i*) Determine the equation of the plane $\mathcal{P}$ which contains the three points A, B, C.

(*ii*) Show that the distance of the origin from $\mathcal{P}$ is $34/\sqrt{149}$.

(*iii*) Find the point of intersection between the plane $\mathcal{P}$ and the line

$$\mathcal{L}_1 : \ \frac{x+1}{2} = \frac{y+15}{5} = \frac{z-6}{1}.$$

(*iv*) Given is a second line

$$\mathcal{L}_2 : \ \frac{x-5}{4} = \frac{y+8}{-2} = \frac{z-11}{5}.$$

Do the lines $\mathcal{L}_1$ and $\mathcal{L}_2$ intersect? In case they do, find the point of intersection.

Turn over ...

9. Given are the two spheres

$$\mathcal{S}_1 \;:\; x^2 + y^2 + z^2 = 9$$
$$\mathcal{S}_2 \;:\; x^2 + (y+6)^2 + z^2 = 36$$

and the plane

$$\mathcal{P}: \; \lambda x + \mu y + z = 6.$$

(i) Show that the condition for $\mathcal{P}$ to be the tangent plane to $\mathcal{S}_1$ is

$$\lambda^2 + \mu^2 = 3$$

(ii) Find the condition for $\mathcal{P}$ to be also the tangent plane to $\mathcal{S}_2$.

(iii) Determine the equation of the plane which is the common tangent plane to $\mathcal{S}_1$ and $\mathcal{S}_2$.

(iv) Specify a vector which lies in the plane determined in (iii)

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