City University London

The University for business and the professions

School of Engineering and Mathematical Sciences

Centre for Mathematics

BSc Honours Degrees in Mathematical Science
BSc Honours Degree in Mathematical Science with Finance and Economics

MA 1607 Geometry and Vectors

Part I Examination

Date: 14 May 2004 Time: 1:00 pm – 3:00 pm

Time allowed: 2 hours

Full marks may be obtained for correct answers to ALL of the SIX questions in Section A and TWO of the THREE questions in Section B.

Do not attempt more than TWO questions from Section B.

Number of answer books to be provided: 1 per student

Whether or not calculators are permitted: Yes
Any stats tables etc: No
Whether or not the exam paper can be removed from the exam room: Yes

External Examiner: Professor D J Needham, Professor M E O'Neill
Internal Examiner: Dr O Jones

Section A

Answer **all** questions from this section. Each question carries 8 marks

1. Given the family of lines $x + 4y = k$ find the particular member of the family which is tangent to the parabola $y^2 = 2y - x - 4$.

   Calculate the perpendicular distance of the vertex of the parabola to this tangent.

2. Find the equation of the ellipse with foci at $(-2, 1)$ and $(2, 1)$ and major axis of length 8.

   What happens to the ellipse if its eccentricity tends to zero?

3. A line is drawn and passes through the points $(2, 1, 3)$ and $(4, 1, 2)$. Find the coordinates of the point where the line meets the plane $z = 0$.

   What are the coordinates of the point where the line meets the plane $x + 2y - 3z = 6$?

4. Find the perpendicular distance of the point $(2, 3, -1)$ from the line with equation

$$\frac{x-13}{10} = \frac{y-8}{3} = \frac{z+7}{-8}.$$

Turn over ...

5. Show that the lines

$$L_1: \quad \frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{3} \qquad \text{and} \qquad L_2: \quad \frac{x-2}{1} = \frac{y-5}{-1} = \frac{z}{1}$$

intersect and find the coordinates of their common point and the equation of their common plane.

6. If

$$\underline{a} = 2\underline{i} + 3\underline{j} + \underline{k}, \quad \underline{b} = \underline{i} - \underline{j} + 4\underline{k}, \quad \underline{c} = 3\underline{i} - \underline{j} - 2\underline{k}, \quad \underline{d} = \underline{i} + 3\underline{j} - \underline{k}$$

find

$$(\underline{a} \cdot \underline{b})\underline{c}, \quad \underline{a} \cdot \underline{b} \times \underline{c} \quad \text{and} \quad (\underline{a} \times \underline{b}) \times \underline{c}.$$

Verify that

$$(\underline{a} \times \underline{b}) \cdot (\underline{c} \times \underline{d}) = (\underline{a} \cdot \underline{c})(\underline{b} \cdot \underline{d}) - (\underline{a} \cdot \underline{d})(\underline{b} \cdot \underline{c}).$$

Turn over ...

Section B

Answer **two** questions from this section. Each question carries 26 marks.

7. (a) Given

$$\underline{u} = 2\underline{i} + 3\underline{j} + \underline{k}, \quad \underline{v} = \underline{i} - \underline{j} + 2\underline{k}, \quad \underline{w} = \underline{i} + 3\underline{j} - \underline{k}, \quad \underline{r} = a\underline{u} + b\underline{v},$$

where a and b are scalar constants, find

(i) the angle between $\underline{u}$ and $\underline{v}$,

(ii) the relation between a and b if $\underline{r}$ is orthogonal to $\underline{w}$,

(iii) the values of a and b if $\underline{r}$ is a unit vector and (ii) holds.

(b) The position vector $\underline{r}$ of a particle at time t is given by

$$\underline{r} = a\cos\omega t\ \underline{i} + a\sin\omega t\ \underline{j} + b\cos\mu t\ \underline{k}$$

where a, b, ω and μ are constants.

Find the acceleration vector at time t and deduce that this acceleration vector is never normal to the position vector.

(c) A triangle PQR has vertices at points $P(1,0,2)$, $Q(3,4,2)$ and $R(4,1,1)$. Find the perpendicular distance of the point $(2,1,6)$ from the plane of the triangle.

8. Find the equation of the sphere, centre $A(5,-10,5)$ which touches the plane $9x + 12y + 20z = 0$.

A sphere is drawn, centre the origin and passing through A. Show that the circle in which it cuts the first sphere lies in the plane $10x-20y+10z = 299$.

Hence find the centre and radius of the circle.

Turn over ...

9. (a) $ABCD$ is a parallelogram. E is a point on diagonal BD such that $BE = \frac{1}{3}BD$ and F is the midpoint of side AD. If $\underline{AB} = \underset{\sim}{b}$ and $\underline{AC} = \underset{\sim}{c}$ show that $\underline{EF} = \frac{1}{6}(\underset{\sim}{c} - 5\underset{\sim}{b})$.

(b) Find the magnitude of the shortest distance between the two lines

$$\frac{x-1}{4} = \frac{y-1}{3} = \frac{z-2}{-2} \quad \text{and} \quad \frac{x}{4} = \frac{y-5}{0} = \frac{z-15}{-1}.$$

Internal Examiner:	Dr O.K. Jones
External Examiners:	Professor D.J. Needham
	Professor M.E. O'Neill