Geometry & Vectors

Coursework 2
(Hand in the solutions to all questions by Tuesday 27/03/07 16:00)

1) (20 marks)

i) For given vectors $\vec{a}, \vec{b}$ and $\vec{c}$ and scalar $p \in \mathbb{R}$, find the general expression for $\vec{x}$, which solves the vector equation

$$p\vec{x} + (\vec{x} \cdot \vec{b})\vec{a} = \vec{c} \qquad p \neq 0.$$

ii) For given vectors $\vec{a}$ and $\vec{b}$ solve the vector equation

$$\vec{x} \times \vec{a} = \vec{b}$$

for $\vec{x}$. Hint: Use the result from i).

iii) For given vectors $\vec{a}$ and $\vec{b}$ and scalars $\lambda, \mu \in \mathbb{R}$, solve the simultaneous vector equations

$$\lambda\vec{x} + \mu\vec{y} = \vec{a} \qquad \text{and} \qquad \vec{x} \times \vec{y} = \vec{b}$$

for $\vec{x}$ and $\vec{y}$. Hint: Use the result from ii).

2) (10 marks)

Prove that the tangent on a parabola $y^2 = 4ax$ at the point $P(x_0, y_0)$ is given by the equation

$$y = \frac{y_0}{2x_0}x + 2\frac{x_0}{y_0}a.$$

Do not differentiate, but rather use the fact that the tangent is defined as the line which intersects the parabola in precisely one point.

3) (10 marks)

A circle with radius 1 and with center locatd on the y-axis is inscribed in the parabola $y = 2x^2$. This means the circle and the parabola have the same tangent lines at the point of intersection. Determine the point of intersection.

4) (10 marks)

Given are the four points $A(3, 2, 1)$, $B(4, 5, 5)$, $C(4, 2, -2)$ and $D(6, 5, -1)$. Find the point of intersection of the line $\overleftrightarrow{AB}$ with $\overleftrightarrow{CD}$ and also the point of intersection between the lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$.