Geometry & Vectors

Coursework 2

(Hand in the solutions to all questions by Friday 24/03/06 15:00)

(each question carries 10 marks)

1) Prove that the tangent on the parabola $y^2 = 4ax$, with $a \in \mathbb{R}$, at the point $P(x_0, y_0)$ is given by the equation

$$y = \frac{y_0}{2x_0}x + \frac{2x_0}{y_0}a.$$

2) An ellipse is parameterized by the equation

$$2x^2 + 3y^2 - 4x + 5y + 4 = 0 \ .$$

In the x, y-plane find the centre of the ellipse, the length of the major and minor axis, the location of the foci and vertices, its eccentricity and the equation of the directrix.

3) Given are the two points $A(3, 1, 2)$ and $B(-1, 2, 0)$.

i) Determine the equation of the line passing through the points A and B. Subsequently find the coordinates of the point in which the line intersects the xz-plane.

ii) Determine the coordinates of the point in which the line through the points A and B intersects the plane

$$\mathcal{P}: \qquad 2x - 4y - z = 5.$$

4) Given are the two lines

$$\mathcal{L}_1 \ : \qquad \frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{3}$$

$$\mathcal{L}_2 \ : \qquad x - 2 = 5 - y = z.$$

i) Do the two lines intersect? In case they do, find the coordinates of the point $P = \mathcal{L}_1 \cap \mathcal{L}_2$.

ii) Determine equation of their common plane $\mathcal{P}$, i.e. $\mathcal{L}_1 \in \mathcal{P}$, $\mathcal{L}_2 \in \mathcal{P}$.

5) Determine the equation of the line $\mathcal{L}$ of intersection of the two planes

$$\mathcal{P}_1 \ : \qquad 3x - y + z - 11 = 0$$

$$\mathcal{P}_2 \ : \qquad -x + 2y + 6z + 7 = 0$$

in Cartesian form, i.e. $\mathcal{L} = \mathcal{P}_1 \cap \mathcal{P}_2$.