Geometry & Vectors

Coursework 2
(Hand in the solutions to all questions on Thursday 24/03/05 14:00)

1) (30 marks) Prove that the distance d of a point $P(x_0, y_0)$ from a line $\mathcal{L}$ described by the equation $ax + by + c = 0$ is

$$d = \left| \frac{ax_0 + by_0 + c}{\sqrt{a^2 + b^2}} \right|$$

2) (25 marks) Find the equations (in normal form) of

(i) the ellipse with foci $F(0, \pm 2)$ which passes through the point $P(1,0)$,

(ii) the hyperbola with asymptotes $y = \pm 2x$ and directrices $x = \pm 1$,

(iii) the ellipse consisting of all points P such that $|PA|+|PB|=7$, where A=(0,3) and B=(0,-3).

3) (15 marks) Find the equations of the tangents to the ellipse with equation

$$\frac{x^2}{36} + \frac{y^2}{20} = 1$$

which pass through the point $P(8,0)$.

4) (10 marks) Find the equation of the plane through the points A(3,1,1), B(1,1,0) and C(1,0,3). Determine the point of intersection of this plane with the line

$$\frac{x-1}{2} = \frac{y}{2} = \frac{1-z}{3} \ .$$

5) (10 marks) Determine the equation of the line of intersection of the planes

$$\begin{aligned} \mathcal{P}_1 \ &: \quad 5x + 4y + 7z = 26 \\ \mathcal{P}_2 \ &: \quad 2x + 3y + 2z = 11 \end{aligned}$$

in Cartesian form.

6) (10 marks) Find the equation of the plane which contains the line with equation

$$\frac{x-2}{2} = \frac{y-1}{3} = \frac{z-1}{-2}$$

and which is parallel to the line with equation

$$\frac{x+1}{3} = \frac{y-1}{2} = \frac{z+1}{1} \ .$$