Geometry & Vectors

Coursework 1

(Hand in the solutions to all questions by Tuesday 26/02/08 16:00)

1) (20 marks)

ABC and DEF are two triangles oriented in space such that the lines $\overleftrightarrow{AD}$, $\overleftrightarrow{BE}$ and $\overleftrightarrow{CF}$ intersect in the same point P. The lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{DE}$ intersect in the point G, the lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{DF}$ intersect in the point H and the lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{EF}$ intersect in the point I.

i) Draw the corresponding figure.

ii) Employ vectors to show that the points G, H and I are collinear.
Hint: Show first by using vectors that the three points A, B and C are collinear, i.e. $C \in \overleftrightarrow{AB}$, if the position vectors $\vec{a} = \overrightarrow{OA}$, $\vec{b} = \overrightarrow{OB}$ and $\vec{c} = \overrightarrow{OC}$ are related as

$$\vec{c} = \lambda\vec{b} + (1-\lambda)\vec{a} \qquad \text{with } \lambda \in \mathbb{R}.$$

2) (15 marks)

The vectors $\vec{\imath}, \vec{\jmath}, \vec{k}$ constitute an orthonormal basis in an Euclidean space. Given are the vectors

$$\vec{u} = -\frac{44}{7}\vec{\imath} + \lambda\vec{\jmath} + \sqrt{11}\vec{k}, \qquad \text{and} \qquad \vec{v} = -2\vec{\imath} + \vec{\jmath} + \sqrt{11}\vec{k} \qquad \text{with } \lambda \in \mathbb{R}.$$

i) Determine the constant λ such that the angle between $\vec{u}$ and $\vec{v}$ becomes $\pi/6$.

ii) Take now $\lambda = 1$ and construct all unit vectors which are perpendicular to both vectors $\vec{u}$ and $\vec{v}$.

iii) Verify the polarization identity for the vectors $\vec{u}$ and $\vec{v}$ with $\lambda = 0$.

iv) Compute $(7/\sqrt{11})\ \vec{u} \times \vec{v}$ for $\lambda = 22/7$.

3) (15 marks)

Consider two lines $\mathcal{L}_1 : \vec{l}_1 = \vec{a} + \lambda\vec{u}$ and $\mathcal{L}_2$: $\vec{l}_2 = \vec{b} + \mu\vec{v}$ with $\lambda, \mu \in \mathbb{R}$. (Do not use components in the task.)

i) Show that if the lines $\mathcal{L}_1$ and $\mathcal{L}_2$ intersect, the relation

$$\vec{v} \cdot (\vec{b} \times \vec{u}) = \vec{v} \cdot (\vec{a} \times \vec{u})$$

holds.

ii) Show that the position vector for the point of intersection is

$$\vec{r} = \vec{a} + \frac{\vec{a} \cdot (\vec{b} \times \vec{v})}{\vec{v} \cdot (\vec{b} \times \vec{u})}\vec{u}. \tag{r}$$

iii) Solve the equation (r) for the vector $\vec{a}$.