**MA2600**

# CITY UNIVERSITY
**London**

BSc Honours Degree in Mathematical Science
Mathematical Science with Statistics
Mathematical Science with Computer Science
Mathematical Science with Finance and Economics
Mathematics and Finance

PART 2

## Complex variable

2013

Time allowed: 2 hours

*Full marks may be obtained for correct answers to*
*THREE out of the FOUR questions.*
*All necessary working must be shown.*

**Important note for students regarding past exam papers**
Past exam papers are published for illustrative purposes only. They can be used as a study aid but do not provide a definitive guide to either the format of the next exam, the topics that will be examined or the style of questions that will be set. Students should not expect their own exam to be directly comparable with previous papers. Remember that a degree requires an amount of self-study, reading around topics, and lateral thinking particularly at the higher level modules and for higher marks. Specific guidance for your exam will be given by the lecturer.

1. (a) State Cauchy's integral formula together with the conditions required for it to hold.

[**2**]

(b) Evaluate the following integrals

i.
$$\oint_C \frac{3-z}{1+z^3}\,dz$$
where $C$ is the rectangle with opposite corners at $\pm(2+\frac{i}{4})$. [**6**]

ii.
$$\oint_C \frac{\sin z}{(z-\frac{\pi}{6})^3}dz$$
where $C$ is the unit circle. [**4**]

iii.
$$\oint_C \frac{\sin z}{(z-\frac{\pi}{4})(z-\frac{\pi}{2})^2}dz$$
where $C$ is the unit circle. [**8**]

2. Let $u(x,y) = x^2 - y^2 + e^y \cos x$ for all $x, y \in \mathbb{R}$.

(a) Show that $u_{xx} + u_{yy} = 0$. [**4**]

(b) Define the function $v$ by $v_y = u_x$ and $v_x = -u_y$ for all $x, y \in \mathbb{R}$ and $v(0,0) = 0$. Find $v$. [**10**]

(c) Define the function $f$ of the complex variable $z = x + iy$ whose real part is $u$ and imaginary part is $v$. Is $f$ analytic anywhere? If so give its domain of analyticity. Give $f$ as a function of $z$ only. [**6**]

3. Explaining your method, compute

(a)
$$I = \int_0^{2\pi} \frac{d\theta}{i\cos\theta - i\sin\theta + 1}\,.$$
Simplify your answer until you get a real number. [**10**]

(b)
$$J = \int_{-\infty}^{\infty} \frac{dx}{i + x^3}\,.$$
Simplify your answer until you get a purely imaginary number. [**10**]

4. In this question, we compute

$$F(k) = \int_{-\infty}^{\infty} f(x)\, e^{ikx} \quad , \quad k \in \mathbb{R}\,,$$

where $f(x) = \frac{x}{4+x^2}$.

(a) Describe the contour that should be used to compute this integral (paying attention to the various cases) and state the relevant lemma. Find the poles of $f$ and discuss their contributions in the various cases. [**4**]

(b) Find $F(k)$, $k \in \mathbb{R}$. [**14**]

(c) Deduce the value of

$$\int_{-\infty}^{\infty} f(x)\, \cos(kx)\, dx \quad , \quad k \in \mathbb{R}\,.$$

[**2**]

| | |
|---|---|
| Internal Examiner: | Dr V. Caudrelier |
| External Examiners: | Professor J. Rickard |
| | Professor J.S.W. Lamb |