City University
London

MA2600: Complex analysis set task

1. Using Cauchy's integral formula, evaluate

$$J = \oint_C \frac{\cos 2z}{z^2(z^2 - z + 1)}\, dz$$

where C is the circle of radius 2 centered at the origin. (Please present your answer in the form $J = A\cos(2z_+) + B\cos(2z_-) + C$ where A, B, C, $z_\pm$ should be found.)

2. Let f be defined by

$$f(z) = \frac{2}{z^2 - (4+i)z + 4i}\,.$$

   (a) Find the poles z_1 and z_2 of f such that $|z_1| < |z_2|$.

   (b) Give the series expansion of f in the following three regions, stating in each case if it is a Taylor or Laurent series.

      i. $|z| < |z_1|$.
      ii. $|z_1| < |z| < |z_2|$.
      iii. $|z - z_1| < |z_2 - z_1|$.

3. Explaining your method, compute

   (a)

$$I = \int_0^{2\pi} \frac{d\theta}{2i\cos\theta + 2i\sin\theta + 1}\,.$$

   (b)

$$J = \int_{-\infty}^{\infty} \frac{x^2 dx}{1 + x^6}\,.$$

   For the integral J, simplify your answer until you get an expression involving real numbers only.