

PAPER 53

SUPERSYMMETRY

*Answer **ALL** questions in Section A.*

*Answer any **ONE** of the three questions in Section B.*

Sections A and B carry equal credit.

**You may not start to read the questions
printed on the subsequent pages until
instructed to do so by the Invigilator.**

Section A

1 You are given that

$$\gamma_{pq}\gamma^m\gamma^{pq} = \alpha\gamma^m$$

for Dirac matrices in spacetime dimension D . What is the constant α as a function of D ?

2 You are given that λ is an anticommuting Majorana spinor of $Spin(3,1)$, with charge conjugate $\bar{\lambda}$. Which of the following tensors vanishes identically?

$$(a) \bar{\lambda}\gamma^m\lambda, \quad (b) \bar{\lambda}\lambda, \quad (c) \bar{\lambda}\gamma^{mn}\gamma_5\lambda, \quad (d) \bar{\lambda}\gamma^{mnp}\lambda.$$

3 Given that $\{Q^\alpha, Q^\beta\} = 2(\gamma_m C)^{\alpha\beta} P^m$, for Majorana spinor charge Q and 4-momentum P^m , prove that any state annihilated by one linear combination of the components of Q is annihilated by either two or four linearly independent combinations.

4 Write down the spin content content of the superspin $\frac{1}{2}$ unitary irrep of the super-Poincaré group. How does it decompose into massless superhelicity irreps?

5 You are given that $V(x, \theta)$ is a real general superfield. Why is $\frac{\partial V}{\partial \theta}$ not a superfield? How is this difficulty circumvented?

6 Let Z be a chiral superfield. Write down the most general superspace action for Z for which the component field equations are partial differential equations of order ≤ 2 . State without proof how this action must be restricted in order to ensure renormalizability (in four spacetime dimensions).

7 The differential 1-forms $(dX^m, d\theta^\alpha)$ (for Majorana spinor θ) constitute a basis of 1-forms on superspace. Write down an alternative, supertranslation-invariant, basis. Hence deduce that the 3-form

$$H = (dX^m + i\bar{\theta}\gamma^m d\theta) d\bar{\theta}\gamma_m d\theta$$

is super-Poincaré invariant. Stating without proof any gamma-matrix identity that you may need, show that $dH = 0$.

Section B

8 Write an essay with the title *Spontaneous breaking of supersymmetry*. Your essay should include a discussion of the Witten index and the implications of Goldstone's theorem, and should be illustrated with examples from supersymmetric quantum mechanics and field theories of chiral superfields.

9 Write an essay with the title *Gauge theories in superspace*. Your essay should include a discussion of the superspace analogs of the Maxwell gauge potential, gauge transformations, field strength and Bianchi identity. You should also discuss the minimal coupling to chiral superfields, and include a brief discussion of the non-abelian generalization.

10 Write an essay with the title *Supermembranes*. Your essay should include a discussion of domain walls in the Wess-Zumino model, Dirac's membrane action as an effective worldvolume field theory, and a discussion of its supersymmetric extension, including the need for 'kappa-symmetry'.