UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday 9 June, 2005 9 to 12

PAPER 54

STRING THEORY

*Attempt **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

Minor errors in numerical factors will not be heavily penalized.

The covariant action of the bosonic string in flat space-time is

$$I = \frac{-1}{4\pi\alpha'} \int d\sigma d\tau \sqrt{-\det\gamma}\, \gamma^{\alpha\beta}\, \partial_\alpha X^\mu \partial_\beta X_\mu \, .$$

STATIONERY REQUIREMENTS
Cover sheet
Treasury tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 A closed bosonic string moves in a flat space-time that has one compact circular dimension of radius R. Starting from the fact that, in the conformal gauge, its coordinates satisfy the two-dimensional wave equation,

$$\partial^2 X^\mu = 0$$

obtain the mode expansion of X^μ, paying careful attention to the boundary conditions.

Explain how to obtain the spectrum of physical states using the light-cone gauge.

Find the spectrum of massless states for general values of R. Show that the complete spectrum is invariant under the transformation $R \to \alpha'/R$. Find the extra massless states that arise at the self-dual point, $R = \sqrt{\alpha'}$. Hence, explain how the deformations of R away from the self-dual point may be interpreted in terms of a Higgs model of symmetry breaking.

2 An open bosonic relativistic string has endpoints satisfying Neumann boundary conditions in $p+1$ dimensions and Dirichlet conditions in the remaining directions. Deduce the mode expansions of its coordinates in conformal gauge. Explain the space-time interpretation of these open strings.

Give evidence that, if one ignores a tachyonic instability, the low energy limit of a system of two parallel coincident $D3$-branes describes four-dimensional $U(2)$ Yang–Mills theory. Why does the symmetry break to $U(1) \times U(1)$ when the two $D3$-branes are separated by a transverse distance L?

Calculate the value of L below which an open string joining two D-branes develops a tachyon instability.

3 Show that after choosing the conformal gauge, bosonic string theory reduces to free two-dimensional field theory subject to some constraints. What is the algebra of these constraints? How are these constraints used to define physical states? Determine the physical states in the first three mass levels of the open string. Show that adding world-sheet Majorana fermions with the Dirac action

$$I^{(\psi)} = \frac{i}{4\pi\alpha'} \int d\sigma d\tau\, \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu$$

(where the matrices ρ^α are the two-dimensional Dirac matrices) to the bosonic string action leads to a theory with global two-dimensional supersymmetry. Sketch how this leads to an extension of the bosonic constraint algebra.

4 Answer **one** of the following:

(a) Describe, without derivation, the spectrum of states of the open fermionic string. The tachyon of the fermionic string can be eliminated by a suitable projection. Show that there are equal numbers of bosonic and fermionic states at the first two mass levels *after* this projection has been made.

(b) Obtain the formula for the asymptotic number of states at a large mass level for the *closed* bosonic string, commenting on the thermodynamic consequences of such a spectrum.

[*You may assume that the mass of a closed string state is given by:* $\alpha'(\text{mass})^2 = 2N+2\tilde{N}-4$, *and that states satisfy the level matching condition,* $N - \tilde{N} = 0$, *where* N *and* $\tilde{N}$ *are eigenvalues of the number operators built out of transverse oscillator modes.*]

(c) Give an overview of how Einstein's theory of general relativity emerges from string theory at low energy.

END OF PAPER