

MATHEMATICAL TRIPOS Part III

Monday 4 June 2001 9 to 12

PAPER 64

STRING THEORY

Attempt FOUR questions. The questions are of equal weight.

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator. 2

1 Derive the classical equations of motion for a relativistic string from the Polyakov action, commenting on the invariances of the action and their significance.

2 In the covariant quantization of the Nambu-Goto action, show that there is a physical state of the form $(L_{-2} + \mu L_{-1}^2)|k\rangle$ if the dimension of space-time, d = 26. What is its norm? Give an outline account of how the No-Ghost Theorem leads to the elimination of negative norm squared states for $d \leq 26$.

3 Give a sketch of the path-integral approach to the quantization of the Polyakov action for the bosonic string.

4 Discuss quantization of a closed string when a number of the dimensions of space are compactified to form circles of radius R_j , $1 \leq j \leq M$. Show that, if any of the radii R_j takes a certain value, the theory acquires additional massless vector particles.

5 Explain how the incorporation of anticommuting fields into string theory leads to the presence in the theory of space-time fermions.