UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday 7 June, 2004 1.30 to 4.30

PAPER 18

SPECTRAL GEOMETRY

Attempt **THREE** *questions.*

There are **four** *questions in total.*

The questions carry equal weight.

You may use without proof results from other areas of Mathematics, such as Analysis or Topology, provided you state them clearly.

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 State and prove what is the spectrum of a flat torus. Prove that two isospectral tori of dimension two are isometric.

2 Define the heat kernel and heat invariants for the Laplacian acting on functions on a Riemannian manifold M. State some geometric invariants that are determined by the heat invariants.

Prove that the heat kernel is defined, and has an appropriate asymptotic expansion, on a neighbourhood of the diagonal in $M \times M$.

3 State Sumada's Theorem concerning certain coverings and resulting isospectral manifolds. Explain, with proof, how to detect whether these manifolds are isometric.

Give an example of a group T with non-isomorphic Gassman subgroups. Deduce the existence, for any $n \in \mathbb{N}$, of n manifolds of dimension 5 all of which have the same spectrum but no two of which are isometric.

4 Identify a pair of Gassman subgroups U_1, U_2 of $PSL(3,2)$. Explain *briefly* why they are Gassman equivalent and not conjugate.

Assuming any result you require about hyperbolic tri-rectangles, construct an infinite family, parameterised by an interval on the real line, of examples of isospectral non-isometric Riemann surfaces of genus 4.

[You may use the fact that $PSL(3,2)$ has generators A and D with commutator $C = [D, A]$ of order 7 such that the permutation actions of A and D on the cosets $U_i C^n$ are given by

Action of	On cosets of U_1	On cosets of U_2
A	(0)(125)(364)	(0)(143)(256)
D	(1)(03)(2645)	(4)(25)(0163)

where, in each case, the coset $U_i C^n$ is denoted by n.]