UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday 7 June, 2004 1.30 to 4.30

PAPER 24

SET THEORY

Attempt **QUESTION 1** *and* **ANY TWO** *other questions.*

There are **eight** *questions in total.*

The questions carry equal weight.

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Prove the independence of each of the following axioms from the remaining axioms of ZF: sumset, power set, replacement, extensionality, foundation, infinity. Prove also the independence of the axiom of choice from a version of ZF with extensionality weakened to allow the existence of many empty sets or *urelemente*.

2 Explain the Erdös-Rado theorem on the existence of infinite monochromatic sets. What is a weakly compact cardinal? A measurable cardinal? A supercompact cardinal?

3 Write an essay on forcing.

4 Prove Kruskal's theorem and Friedman's Finite Form.

5 Let A be an arbitrary set; give it the discrete topology, and A^ω the product topology. Show that games played over A whose payoff set is Borel have winning strategies.

6 Prove the Open Ramsey Theorem; show that the restriction to open partitions cannot be relaxed altogether by exhibiting a partition of ω-sequences from an infinite set with no infinite monochromatic set.

7 What can one do with ultraproducts?

8 What is a BQO?