UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday 29 May 2003 9 to 12

PAPER 19

SET THEORY

Attempt **FOUR** *questions.*

There are **eight** *questions in total.*

The questions carry equal weight.

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 What is a WQO? State and prove the perfect subsequence lemma. **Either**

(i) Prove Kruskal's theorem and Friedman's Finite Form, **or**

(ii) An **incline** is a structure with two associative and commutative binary operations $+$ and $\cdot$ satisfying

$(\forall xyz)(x \cdot (y + z) = x \cdot y + x \cdot z)$;

$(\forall x)(x + x = x)$;

$(\forall xy)(x + xy = x)$.

We define a relation $\leqslant$ by $x + y \leqslant x$.

Prove that $\leqslant$ is a quasi-order.

Let $(I, +, \cdot)$ be a finitely generated incline. Show that $(I, \leqslant)$ is a WQO.

2 Write an essay on large cardinals.

3 Prove the independence of the axiom of foundation, and extend your technique to prove the independence of the axiom of choice from ZF minus foundation.

4 What are inner models, and what can they be used to prove?

5 Exhibit a recursive partition of $[\mathbb{N}]^n$ with no recursive monochromatic set. Prove the Erdös-Rado theorem on the existence of uncountable monochromatic sets for partitions of n-tuples. What can you say about infinite exponent partition relations?

6 What is a saturated model? Using ultraproducts or otherwise, state and prove a theorem about the existence of saturated models.

By exploiting saturated models, or otherwise, prove the consistency of NFU.

7 (i) Prove the Ehrenfeucht-Mostowski theorem.

(ii) What is AD, the axiom of determinacy? Which games can you prove to be determined? Establish that AD is inconsistent with AC.

8 Prove Fodor's theorem and the Cantor Normal Form theorem.