PAPER 27

SET THEORY AND LOGIC

Attempt no more than **FOUR** *questions.*

There are **NINE** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS *SPECIAL REQUIREMENTS*
Cover sheet   *None*
Treasury tag
Script paper

**You may not start to read the questions
printed on the subsequent pages until
instructed to do so by the Invigilator.**

1   Give two proofs of the Ehrenfeucht-Mostowski theorem, one using Ramsey's theorem and the other using ultraproducts.

2   Prove the independence of the axiom of foundation, and extend your technique to prove the independence of the axiom of choice from ZF minus foundation.

3   Write an essay on the theory of computable functions, setting out all the background necessary to prove Rice's theorem (which you should prove).

4 (a) State and prove the completeness theorem for first order logic.

   (b) State and prove Łoś's theorem. Use it to give an ultraproduct proof that if T is a theory all of whose finite fragments have models then T has a model.

5 (a) State and prove Gödel's theorem on the incompleteness of arithmetic.

   (b) Explain Shepherdson's wall, and prove the consistency of the Axiom of Choice relative to ZF.

6   Prove the consistency of NFU.

7   What is a WQO? A BQO? State and prove Kruskal's theorem on wellquasiorderings of trees. Deduce Friedman's Finite Form from it.

8   Use the method of inner models to show that classical ZF is no stronger than constructive ZF.

9   Prove Frayne's theorem that two first-order structures are elementarily equivalent iff they have isomorphic ultralimits.

END OF PAPER